Activity Set 10.2 AREAS ON GEOBOARDS

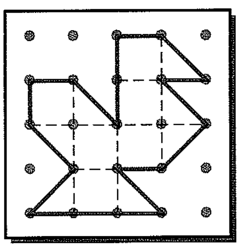
PURPOSE

To use rectangular geoboards to develop area concepts.

MATERIALS

A geoboard and rubber bands (optional). There is a template for making a rectangular geoboard on Material Card 15 or you may wish to use the geoboard in the Virtual Manipulatives.

INTRODUCTION



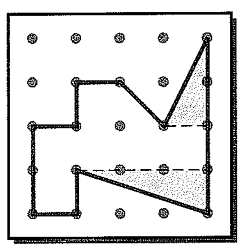
The rows and columns of nails on a rectangular geoboard, like the one shown at the left, outline 16 small squares. In this activity set, these small squares will serve as the unit squares for the areas of the geoboard figures. Finding the area of a geoboard polygon means determining the number of unit squares and partial squares needed to cover the polygon.

It takes 6 unit squares and 7 halves of unit squares to cover the figure on the geoboard, to the left so its area is $9\frac{1}{2}$ unit squares or $9\frac{1}{2}$ square units.

In the following activities, you will examine different methods for determining areas of polygons and discover how the formulas for the areas of rectangles, parallelograms, and triangles are related.

1. In the figure for part a, the upper shaded triangle has an area of 1 square unit because it is half of a 2 by 1 rectangle. Similarly, the lower shaded triangle has an area of $1\frac{1}{2}$ square units because it is half of a 3 by 1 rectangle. Determine the areas of these figures by subdividing them into squares, halves of squares, and triangles that are halves of rectangles.

*a.



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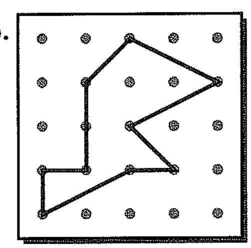
*b.



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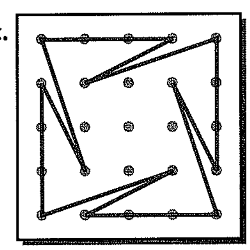
c.



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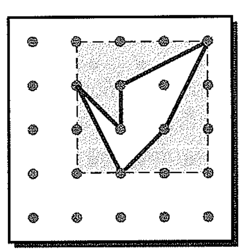
2. Sometimes it is easier to find the area outside a figure than the area inside. The hexagon in part a has been enclosed inside a square. What is the area of the shaded region? Subtract this area from the area of the 3 by 3 square to find the area of the hexagon. Use this technique to find the areas of the other figures.

*a.



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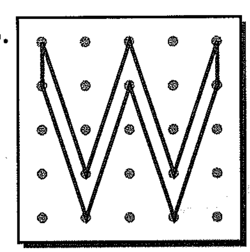
*b.



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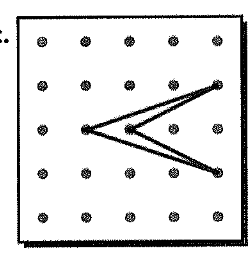
c.



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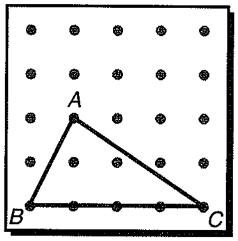
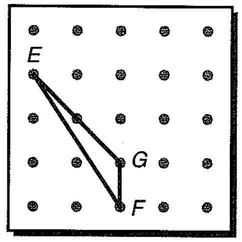
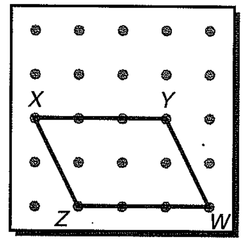
3. The *height, or altitude, of a triangle* is the perpendicular distance from a vertex to the line containing the opposite side. The *height of a parallelogram* is the perpendicular distance from a line containing one side to the line containing the opposite side. Using the length between two consecutive vertical or horizontal pins as the unit length, determine the heights, base length, and area of each triangle and each parallelogram.



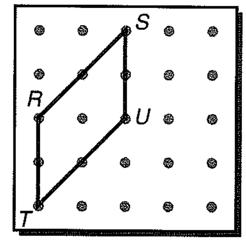
Height from vertex *A*: ______
Base $\overline{BC}$ length: ______
Area: ______



Height from vertex *E*: ______
Base $\overline{FG}$ length: ______
Area: ______



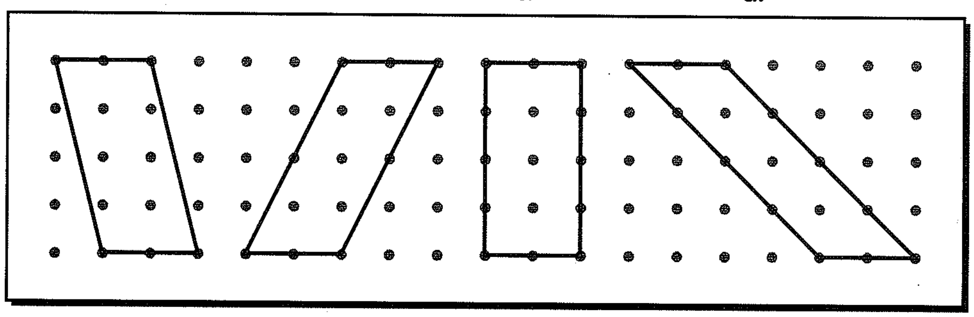
Height from side $\overline{ZW}$: ______
Base $\overline{ZW}$ length: ______
Area: ______



Height from side $\overline{RS}$: ______
Base $\overline{RS}$ length: ______
Area: ______

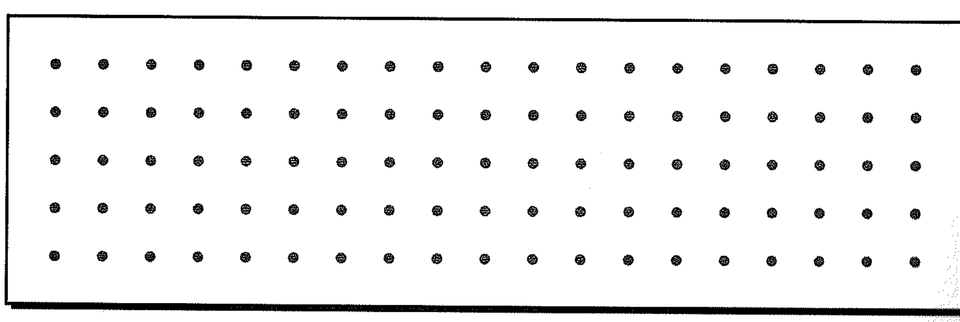
4. Use your geoboard techniques to determine the area and the perimeter of each of these parallelograms.

a. **b.** **c.** **d.**



	a.	b.	c.	d.
Area:	______	______	______	______
Perimeter:	______	______	______	______

e. Each of the preceding parallelograms has one pair of sides of length 2 and the same height. Make a conjecture about how to determine the area of a parallelogram if you know the length of a side and the height from that side. Sketch three additional parallelograms here with different areas to test your conjecture. If your conjecture appears to be valid, record it in the provided space at the top of the next page.

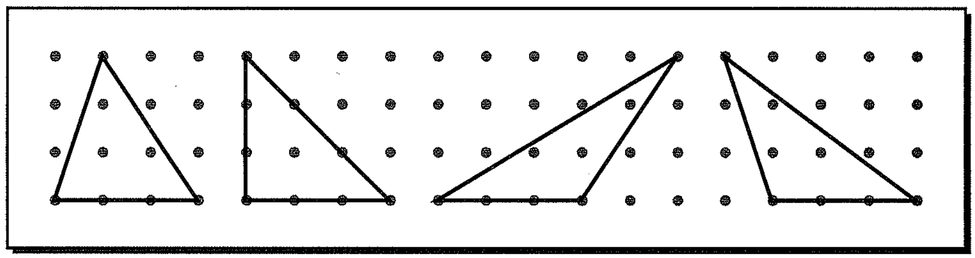


Conjecture:

f. It is a common thought that the area of some polygons is directly connected to the perimeter of the polygon. That is, when the perimeter of a polygon increases or decreases, so does the area. Is this true for parallelograms? Explain your thinking.

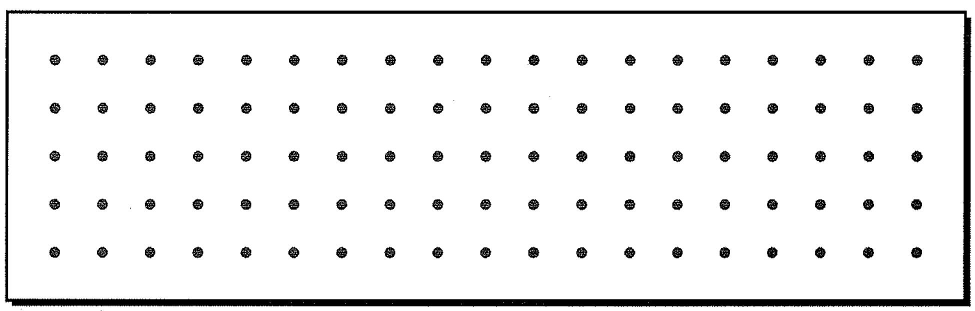
5. Use geoboard techniques to determine the areas of these triangles.

a. **b.** **c.** **d.**



Area: ______ Area: ______ Area: ______ Area: ______

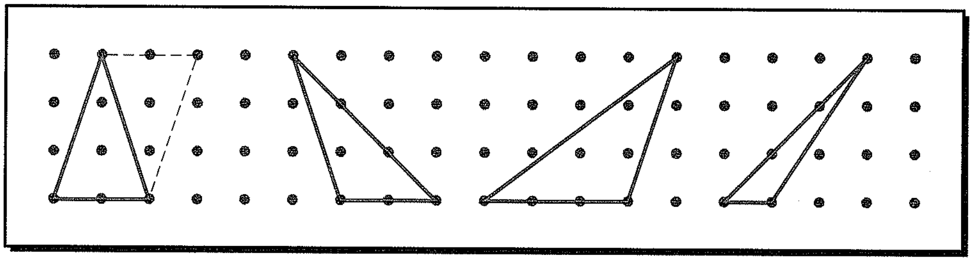
Make a conjecture about the area of triangles with bases of equal length and common heights. Sketch three additional noncongruent triangles with bases of length 5 and altitudes of length 4 to test your conjecture. If your conjecture appears to be valid, record it in the provided space below.



Conjecture:

6. One approach to discovering the formula for the area of a triangle is to use the formula for the area of a parallelogram. The triangle in part a at the top of the next page is enclosed in a parallelogram that shares two sides with the triangle. Enclose each triangle (parts b, c and d) in a parallelogram that shares two sides with it. Then record the areas of the triangle and the area of the parallelogram.

*a. b. *c. d.



Parallelogram area: ______ Triangle area: ______

Parallelogram area: ______ Triangle area: ______

Parallelogram area: ______ Triangle area: ______

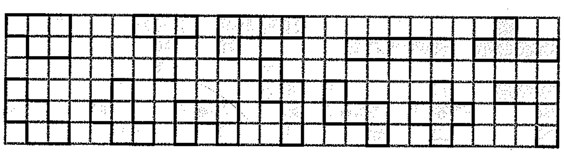
Parallelogram area: ______ Triangle area: ______

***7.** The formula for the area of a triangle is one-half of the base length times the height to that base $[A = (\frac{1}{2})bh]$. Based on the observations you have made in activities 3, 4, 5 and 6, write a statement that explains why this formula works.

JUST FOR FUN

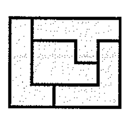
PENTOMINOES

Pentominoes are polygons that can be formed by joining five squares along their edges. Surprisingly, there are 12 such polygons. Pentominoes are used in puzzles, games, tessellations, and other problem-solving activities.

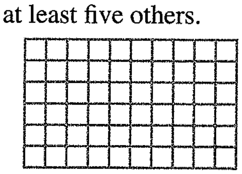


A complete set of pentomino pieces is on Material Card 34. Material Card 35 has a grid that can be used for questions 2, 3, and 4. (*Note:* Pieces can be turned over for the activities.)

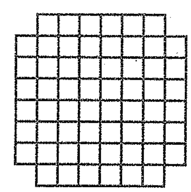
1. Selecting the appropriate pieces from your pentomino set, form the 4 by 5 rectangular puzzle shown here. Outline this rectangle with pencil, and see how many other combinations of pieces you can find that will exactly fill the outline. There are at least five others.



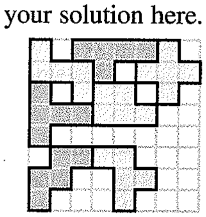
2. The best-known challenge is to use all 12 pieces to form a 6 by 10 rectangle. Supposedly there are over 2000 different solutions. Find one solution and record it on this grid.



3. It is possible to form several puzzles by constructing an 8 by 8 grid of squares and then removing 4 unit squares. In one of these, the unit squares are missing from the corners, as shown here. Use all 12 pieces to cover a grid with this shape. Record your solution here.



***4. Pentomino Game (2 players):** This game is played on an 8 by 8 grid. The players take turns placing a pentomino on uncovered squares of the grid. Play continues until someone is unable to play or all the pieces have been used. The player who makes the last move wins the game. The first seven plays of a game are shown on this board. Player A has made four moves, and player B has made three moves. Find a way player B can make the eighth move (with one of the remaining pentominoes) so that no more moves can be made. Play a few games and look for strategies.



***5.** It is possible to place 5 different pentomino pieces on an 8 by 8 grid so that no other piece from the same set can be placed on the grid. Find 5 such pieces.