1. The graph of $f(x) = \cos x$ is shifted to the right by 4 units, then stretched horizontally by a factor of 2, and finally shifted up by 2 units to obtain the graph of $h(x)$. Then, $h(x) =$

$\cos x \Rightarrow \cos(x-4) \rightarrow \cos\left(\frac{1}{2}x-4\right)$

$\rightarrow \cos\left(\frac{1}{2}x-4\right)+2$

A. $4+\cos\left(\frac{x}{2}-4\right)$

B $2+\cos\left(\frac{x}{2}-2\right)$

C. $4+\cos(x-4)$

D. $2-\cos\left(\frac{x}{2}-2\right)$

E. None of the above

2. The domain of the function $f(x) = \dfrac{1}{\sqrt{1-|4-2x|}}$ is

$|4-2x| < 1$

$-1 < 4-2x < 1$

$-5 < -2x < -3$

$5/2 > x > 3/2$

A. $-\frac{1}{2} < x < \frac{1}{2}$

B. $\frac{3}{2} < x < \frac{5}{2}$

C. $-\frac{1}{2} \le x \le \frac{1}{2}$

D. $\frac{3}{2} \le x < \frac{5}{2}$

E. $x > \frac{3}{2}$

3. Given that $\cos\theta = -\frac{4}{9}$ and $\pi < \theta < \frac{3\pi}{2}$, it follows that $\sin\theta =$

$\sin\theta = \pm\sqrt{1-\cos^2\theta} = \pm\sqrt{1-\frac{16}{81}}$

$= \pm\sqrt{\frac{65}{81}} = \pm\frac{\sqrt{65}}{9}$

θ in 3rd quadrant $\Rightarrow \sin\theta < 0$

A. $-\frac{\sqrt{65}}{9}$

B. $\frac{\sqrt{65}}{9}$

C. $\frac{9}{\sqrt{65}}$

D. $-\frac{9}{\sqrt{65}}$

E. $\frac{4}{\sqrt{65}}$

4. The center C and radius r of the circle given by $6x^2 + 6y^2 - 24x + 36y = 48$ are

A. $C = (1, 3),\ r = \sqrt{48}$

B. $C = (2, -3),\ r = \sqrt{21}$

C. $C = (-2, -3),\ r = \sqrt{48}$

D. $C = (2, 3),\ r = \sqrt{21}$

E. $C = (3, 2),\ r = \sqrt{21}$

$x^2 + y^2 - 4x + 6y = 8$

$(x-2)^2 + (y+3)^2 = 8 + 13 = 21$

center : $(2, -3)$

radius $= \sqrt{21}$

5. An equation of the line through $(3, 1)$ and perpendicular to $3x + 4y = 6$ is

A. $y - \frac{4}{3}x - 3 = 0$

B. $y - \frac{4}{3}x + 3 = 0$

C. $y - 3x + 8 = 0$

D. $y + \frac{3}{4}x - 3 = 0$

E. $y + \frac{3}{4}x + 3 = 0$

$3x + 4y = 6$ has slope $-\frac{3}{4}$

$\perp$ has slope $m = \frac{4}{3}$

$y = \frac{4}{3}(x-3) + 1 = \frac{4}{3}x - 3$

i.e. $y - \frac{4}{3}x + 3 = 0$

6. Given that $f(x) = \sqrt{4 - x^2}$ and $g(x) = \sqrt[4]{x^2 - 9}$, the domain of $f \circ g$ is

A. $[-5, -3] \cup [3, 5]$

B. $[-3, 3]$

C. $(-3, 3) \cup (5, \infty)$

D. $(-5, 3)$

E. $[-5, 5]$

domain$(g) = \{x : x^2 \geq 9\} = \{x : |x| \geq 3\}$

$f \circ g(x) = \sqrt{4 - (\sqrt[4]{x^2 - 9})^2}$

$= \sqrt{4 - \sqrt{x^2 - 9}}$; we need $\sqrt{x^2-9} \leq 4$, i.e. $x^2 - 9 \leq 16$; i.e. $x^2 \leq 25$

However, we also need $|x| \geq 3$ and thus domain$(f \circ g) = \{x : |x| \geq 3\} \cap \{x : |x| \leq 5\} = [-5, -3] \cup [3, 5]$

7. Which of the following statements are true?

F I. $4^x \cdot 4^y = 16^{x+y}$

T II. $(5 \cdot 8)^x = 5^x \cdot 8^x$

T III. $\left(\frac{10}{17}\right)^x = \frac{10^x}{17^x}$

A. Only I

B. Only III

C. Only I and III

D. Only II and III

E. They are all false

8. The inverse of the function $f(x) = 2 - e^{-x^3}$ is $f^{-1}(x) =$

$$x = 2 - e^{-[f^{-1}(x)]^3}$$

Hence, $e^{-[f^{-1}(x)]^3} = 2 - x$, i.e.

$-[f^{-1}(x)]^3 = \ln(2-x)$ and $[f^{-1}(x)]^3 = -\ln(2-x)$

Finally,

$$f^{-1}(x) = \sqrt[3]{-\ln(2-x)} = -\sqrt[3]{\ln(2-x)}$$

A. $2 - \frac{1}{3}\ln x$

B. $\frac{1}{3}\ln x - 2$

C. $-\sqrt[3]{\ln(2-x)}$

D. $-\frac{1}{3}\ln 2 - x$

E. $\sqrt[3]{\ln(x-2)}$

9. $\displaystyle\lim_{x\to 2} \frac{\frac{1}{x+1} - \frac{1}{3}}{x-2} = \lim_{x \to 2} \frac{\frac{3-(x+1)}{3(x+1)}}{x-2}$

$$= \lim_{x\to 2} \frac{2-x}{3(x+1)(x-2)} = \lim_{x\to 2} \frac{-\cancel{(x-2)}}{3(x+1)\cancel{(x-2)}}$$

$$= \lim_{x\to 2} \frac{-1}{3(x+1)} = -\frac{1}{9}$$

A. 3

B. $-\frac{1}{9}$

C. $\frac{2}{3}$

D. $\frac{3}{2}$

E. $\frac{1}{9}$

10. If f and g are continuous at $x = 2$ with $g(2) = 3$ and $\lim_{x \to 2} \frac{2f(x) - g(x)}{2g(x) - f(x)} = -1$, then f(2) is

$\frac{2f(2) - g(2)}{2g(2) - f(2)} = -1$ by continuity, IF $f(2) \neq 2g(2)$

Then, $2f(2) - 3 = -[2 \times 3 - f(2)]$ and

$f(2) = 3 - 6 = -3 \quad (\neq 2 \times 3)$

A. $= -6$

B. $= -3$

C. $= 73$

D. $= 1$

E. impossible to determine

11. $\lim_{x \to -2^+} e^{\frac{x^2+2x}{x^2-2x}} = e^{\lim_{x \to -2^+} \frac{x^2+2x}{x^2-2x}} = e^{\frac{0}{8}} = 1$

A. ∞

B. e

C. 0

D. 1

E. $-\infty$

12. Given the functions f and g defined by the table below,

x	-1	0	1	2
$f(x)$	0	3	1	-1
$g(x)$	1	-1	2	-3

the value of $f \circ g^{-1}$ at $x = -1$ is

$f \circ g^{-1}(-1) = f(g^{-1}(-1)) = f(0) = 3$

A. -1

B. 0

C. 1

D. 2

E. 3

13. If $7 - 2r \le G(r) \le (r+2)^2 + 12$ for $r \in [-\pi, \pi)$, then $\lim\limits_{r\to -3} G(r)$

A. cannot be determined

B. $= 12$

C. $= 0$

D. $= 13$

E. $= 7$

$\lim\limits_{r \to -3} (7 - 2r) = 7 + 6 = 13$

$\lim\limits_{r \to -3} \left[(r+2)^2 + 12\right] = 1 + 12 = 13$

Squeeze Thm. $\Rightarrow \lim\limits_{r \to -3} G(r) = 13$

14. The only possible real value for $\lim\limits_{x\to -2} \dfrac{5x^2 + ax + 3(a-1)}{x^2 + x - 2}$ is

A. 0

B. $\dfrac{23}{5}$

C. $\dfrac{13}{5}$

D. 1

E. $\dfrac{37}{3}$

$\lim\limits_{x \to -2} \dfrac{5x^2 + ax + 3(a-1)}{x^2 + x - 2}$ is real only if

$\lim\limits_{x \to -2} (5x^2 + ax + 3(a-1)) = 0$, since $\lim\limits_{x \to -2} (x^2 + x - 2) = 0$.

Then, $5(-2)^2 - 2a + 3a - 3 = 0$ gives $a = 3 - 20 = -17$

and $\lim\limits_{x \to -2} \dfrac{5x^2 - 17x - 54}{x^2 + x - 2} = \lim\limits_{x \to -2} \dfrac{(5x-27)\cancel{(x+2)}}{\cancel{(x+2)}(x-1)}$

$= \lim\limits_{x \to -2} \dfrac{5x - 27}{x - 1} = \dfrac{-37}{-3} = \dfrac{37}{3}$

15. The function $J(s) = \dfrac{1}{1 - e^{1/s}}$ is discontinuous at

A. $s = -1$ only

B. $s = 0$ only

C. $s = -1$ and $s = 0$ only

D. $s = -1$, $s = 0$, and $s = 1$ only

E. no real number s

$s = 0$ only, since $e^{1/s} = 1$ requires $\frac{1}{s} = 0$, which is impossible.