MA 161 **Exam 2** **Fall 2007**

Name____

10-digit PUID____

RECITATION Division and Section Numbers________________________

Recitation Instructor____________________________________

Instructions:

1. Fill in all the information requested above and on the scantron sheet.
2. This booklet contains 12 problems, each worth $8\frac{1}{3}$ points. The maximum score is 100 points.
3. For each problem mark your answer on the scantron sheet and also circle it in this booklet.
4. Work only on the pages of this booklet.
5. Books, notes, calculators or any electronic devices are not to be used on this test.
6. At the end turn in your exam and scantron sheet to your recitation instructor.

$x < 0 \Rightarrow \frac{1}{x} = \frac{-1}{\sqrt{x^2}}$

1. $\lim_{x \to -\infty} \sqrt{4x^2 + 2x} + 2x = \lim_{x \to -\infty} \frac{(\sqrt{4x^2+2x}+2x)(\sqrt{4x^2+2x}-2x)}{\sqrt{4x^2+2x}-2x}$

$= \lim_{x \to -\infty} \frac{\cancel{4x^2}+2x-\cancel{(2x)^2}}{\sqrt{4x^2+2x}-2x} = \lim_{x \to -\infty} \frac{(2x)\left(\frac{1}{x}\right)}{(\sqrt{4x^2+2x}-2x)\left(\frac{-1}{\sqrt{x^2}}\right)}$

$= \lim_{x \to -\infty} \frac{2}{-\sqrt{\frac{4x^2+2x}{x^2}}-2} = \lim_{x \to -\infty} \frac{2}{-\sqrt{4+\frac{2}{x}}-2}$

$= \frac{2}{-\sqrt{4+0}-2} = \frac{2}{-2-2} = -\frac{1}{2}$

A. 0
B. ∞
C. $-\infty$
D. $\frac{1}{2}$
E. $-\frac{1}{2}$

2. Let $f(x) = \frac{1}{\sqrt{x}}$. Which of the following equals $f'(4)$?

T I. $\lim_{h \to 0} \frac{\frac{1}{\sqrt{4+h}} - \frac{1}{\sqrt{4}}}{h} = f'(4)$

F II. $\lim_{x \to 4} \frac{\frac{1}{\sqrt{4}} - \frac{1}{\sqrt{x}}}{x-4} = -f'(4)$

T III. $\frac{-1}{16} = f'(4)$

$f'(x) = -\frac{1}{2}x^{-3/2}$

$\Rightarrow f'(4) = -\frac{1}{2}(4)^{-3/2}$

$= \left(-\frac{1}{2}\right)\left(\frac{1}{2^3}\right) = \frac{-1}{16}$

$f'(4) = \lim_{x \to 4} \frac{f(x)-f(4)}{x-4} = \lim_{x \to 4} \frac{\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{4}}}{x-4}$

$f'(4) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{\sqrt{4+h}} - \frac{1}{\sqrt{4}}}{h}$

A. I. only
B. II. only
C. III. only
D. I. and III. only
E. I. and II. and III.

3. For what values of x does the graph of

$$y = 2x^3 + 3x^2 - 36x + \ln(2)$$

have horizontal tangents?

$$\frac{dy}{dx} = 6x^2 + 6x - 36 = 6(x+3)(x-2) = 0$$

$$\iff x = -3 \text{ or } x = 2$$

A. $x = -2, \ -3$

B. $x = 2, \ -3$

C. $x = -2, \ 3$

D. $x = 2, \ 3$

E. None of the above.

4. $\frac{d}{dx}\left(\frac{e^x}{1+x}\right) = \frac{(e^x)'(x+1) - (e^x)(1+x)'}{(1+x)^2}$

$$= \frac{e^x(x+1) - e^x}{(1+x)^2}$$

$$= \frac{xe^x}{(1+x)^2}$$

A. $\frac{e^x - 1}{(1+x)^2}$

B. $\frac{xe^x}{(1+x)^2}$

C. e^x

D. $\frac{e^x + 1 + x}{(1+x)^2}$

E. $\frac{e^x - 1 - x}{(1+x)^2}$.

5. If $f(x) = \sqrt{x}g(x)$, $g(9) = 12$ and $g'(9) = 2$, then $f'(9) =$

$$f'(x) = (\sqrt{x})' g(x) + (\sqrt{x})\, g'(x) = \frac{g(x)}{2\sqrt{x}} + (\sqrt{x})g'(x)$$

So, $$f'(9) = \frac{g(9)}{2\sqrt{9}} + (\sqrt{9})\, g'(9) = \frac{12}{6} + 3 \times 2 = 8$$

A. $\frac{1}{3}$

B. $\frac{13}{6}$

C. 12

D. 8

E. 6

6. A table of values for f, g, f' and g' is given.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	1	2	7	6
2	4	3	2	5
3	5	4	4	2

If $h(x) = f(g(x))$, then $h'(2) = f'(g(2)) \cdot g'(2)$

$= f'(3) \cdot 5$

$= 4 \cdot 5 = 20$

A. 6

B. 10

C. 20

D. 21

E. 25

7. If $g(x) = \log_3(x^4)$, then $g'(x) =$

Let $y = \log_3 x^4$. Then, $3^y = x^4$ and

$(3^y)(\ln 3)\, y' = 4x^3$. Finally,

$$y' = g'(x) = \frac{4x^3}{(3^y)(\ln 3)} = \frac{4\cancel{x^3}}{(x^{\cancel{4}})(\ln 3)}$$

$$= \frac{4}{(\ln 3)x}$$

A. $\dfrac{4}{x \ln 3}$.

B. $\dfrac{4}{x}$.

C. $\dfrac{4 \ln 3}{x}$.

D. $\dfrac{1}{x^4 \ln 3}$.

E. $\dfrac{1}{4x^3 \ln 3}$

8. Assume y is a differentiable function of x. If $\sqrt{xy} = x^2y - 6$, then the slope of the tangent line at the point $(1, 9)$ is

The slope is $\left.\frac{dy}{dx}\right|_{\substack{x=1\\y=9}} = y'(1)$.

$$\frac{d}{dx}\sqrt{xy} = \frac{d}{dx}(x^2y - 6)$$

$$\Longrightarrow \frac{y + xy'}{2\sqrt{xy}} = 2xy + x^2y'.$$ For $x = 1$ & $y = 9$,

$$\frac{9 + y'}{2\sqrt{9}} = 2 \times 9 + y',$$ i.e. $9 + y' = 6(18 + y')$

Hence, $9 - 6 \times 18 = 6y' - y'$, and $5y' = -99$

Thus, $y' = \dfrac{-99}{5}$

A. $\dfrac{-99}{5}$

B. 40

C. -45

D. $\dfrac{-99}{2}$

E. $\dfrac{81}{5}$

9. What is the derivative of $x^{\cos(x)}$ at $x = \pi/2$?

$$x^{\cos x} = e^{\ln x^{\cos x}} = e^{(\cos x)(\ln x)}$$

Thus, $\frac{d}{dx} x^{\cos x} = e^{(\cos x)(\ln x)} \frac{d}{dx}\left[(\cos x)(\ln x)\right]$

$$= x^{\cos x}\left[-(\sin x)(\ln x) + \frac{\cos x}{x}\right]$$

and $\left(\frac{d}{dx} x^{\cos x}\right)\Big|_{x=\frac{\pi}{2}} = x^{\cos\frac{\pi}{2}}\left[-(\sin\tfrac{\pi}{2})(\ln\tfrac{\pi}{2}) + \frac{\cos\frac{\pi}{2}}{\pi/2}\right]$

$$= x^0\left[-(1)\ln\frac{\pi}{2} + \frac{0}{\pi/2}\right] = -\ln\frac{\pi}{2} = \ln\frac{2}{\pi}$$

A. Undefined.

B. $\frac{\pi}{2}$.

C. $\frac{\pi}{2} + 1$.

D. $\ln\left(\frac{2}{\pi}\right)$.

E. $\ln\left(\frac{\pi}{2}\right)$.

10. The half-life of a certain element is 20 years. Suppose we have a 50-mg sample. After how long will only 2 mg remain?

$$A = A_0 e^{-kt} = 50 e^{-\frac{\ln 2}{\tau}t}$$

$$2 = 50 e^{-\frac{\ln 2}{20}t} \iff \ln\left(\frac{1}{25}\right) = -\frac{t\ln 2}{20}$$

$$\iff t = 20\frac{\ln(1/25)}{-\ln 2} = 20\frac{\ln(1/25)}{\ln(1/2)}$$

A. $20\frac{\ln(1/2)}{\ln(1/50)}$ years.

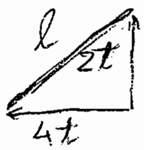
B. $20\frac{\ln(25)}{\ln(1/2)}$ years.

C. $20\frac{\ln(1/25)}{\ln(1/2)}$ years.

D. $20\frac{\ln(1/2)}{\ln(1/25)}$ years.

E. $20\frac{\ln(1/2)}{\ln(25)}$ years.

11. Two people start from the same point at the same time. One walks north at 2 mi/h and the other walks west at 4 mi/h. How fast is the distance between them changing after 30 minutes?



$$(2^2+4^2)t^2=\ell^2 \iff \ell = 2\sqrt{5}\,t$$

$$\frac{d\ell}{dt}=2\sqrt{5}=\frac{10}{\sqrt{5}}$$

A. $\dfrac{20}{\sqrt{5}}$ mph.

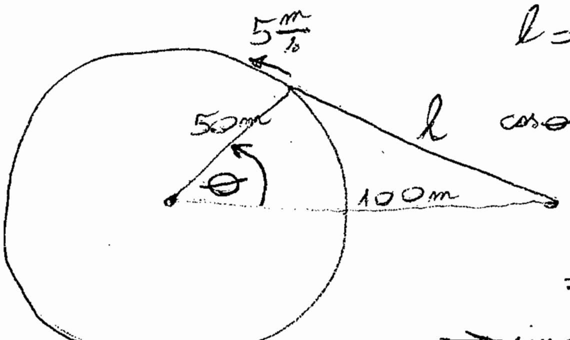
B. $\dfrac{10}{\sqrt{5}}$ mph.

C. $\dfrac{6}{\sqrt{5}}$ mph.

D. $\dfrac{5}{\sqrt{5}}$ mph.

E. $\dfrac{2}{\sqrt{5}}$ mph.

12. A runner sprints around a circular track of radius 50 m at a constant speed of 5 m/s. The runner's friend is standing at a distance 100 m from the center of the track. How fast is the distance between the friends changing when the distance between them is 120 m and the runner is running away from the friend?



$$\ell = 120$$
$$\Downarrow$$
$$\cos\theta = \frac{120^2-50^2-100^2}{-10,000} = \frac{144-25-100}{-100} = -\frac{19}{100}$$
$$\Rightarrow \sin\theta = \sqrt{1-\frac{361}{10,000}}$$

$$C = 2\pi r = 100\pi \text{ m}$$

$$T = \frac{100\pi}{5} = 20\pi \text{ (sec)}$$

$$\frac{d\theta}{dt} = \frac{2\pi}{20\pi} = \frac{1}{10}$$ and Law of Cosines:

$$\ell^2 = 50^2+100^2-2(50)(100)\cos\theta$$

$$\Rightarrow 2\ell\frac{d\ell}{dt} = 10,000\,(\sin\theta)\frac{d\theta}{dt}$$

$$\Rightarrow \frac{d\ell}{dt} = \frac{(10,000)\left(\frac{\sqrt{9639}}{100}\right)\left(\frac{1}{10}\right)}{240}\left(\frac{m}{s}\right)$$

A. $\dfrac{(5,000)\left(\frac{\sqrt{9639}}{100}\right)\left(\frac{1}{10}\right)}{240}$ m/s.

B. $\dfrac{(5,000)\left(\frac{\sqrt{9425}}{100}\right)\left(\frac{1}{10}\right)}{240}$ m/s.

C. $\dfrac{(10,000)\left(\frac{\sqrt{9639}}{100}\right)\left(\frac{1}{5}\right)}{240}$ m/s.

D. $\dfrac{(10,000)\left(\frac{\sqrt{9425}}{100}\right)\left(\frac{1}{10}\right)}{240}$ m/s.

E. $\dfrac{(10,000)\left(\frac{\sqrt{9639}}{100}\right)\left(\frac{1}{10}\right)}{240}$ m/s.