The fundamental objects that we deal with in calculus are functions. This chapter prepares the way for calculus by discussing the basic ideas concerning functions, their graphs, and ways of transforming and combining them. We stress that a function can be represented in different ways: by an equation, in a table, by a graph, or in words. We look at the main types of functions that occur in calculus and describe the process of using these functions as mathematical models of real-world phenomena. We also discuss the use of graphing calculators and graphing software for computers.

1.1 Four Ways to Represent a Function

Functions arise whenever one quantity depends on another. Consider the following four situations.

A. The area A of a circle depends on the radius r of the circle. The rule that connects r and A is given by the equation $A = \pi r^2$. With each positive number r there is associated one value of A, and we say that A is a *function* of r.

B. The human population of the world P depends on the time t. The table gives estimates of the world population $P(t)$ at time t, for certain years. For instance,

$$P(1950) \approx 2{,}560{,}000{,}000$$

But for each value of the time t there is a corresponding value of P, and we say that P is a function of t.

C. The cost C of mailing a first-class letter depends on the weight w of the letter. Although there is no simple formula that connects w and C, the post office has a rule for determining C when w is known.

D. The vertical acceleration a of the ground as measured by a seismograph during an earthquake is a function of the elapsed time t. Figure 1 shows a graph generated by seismic activity during the Northridge earthquake that shook Los Angeles in 1994. For a given value of t, the graph provides a corresponding value of a.

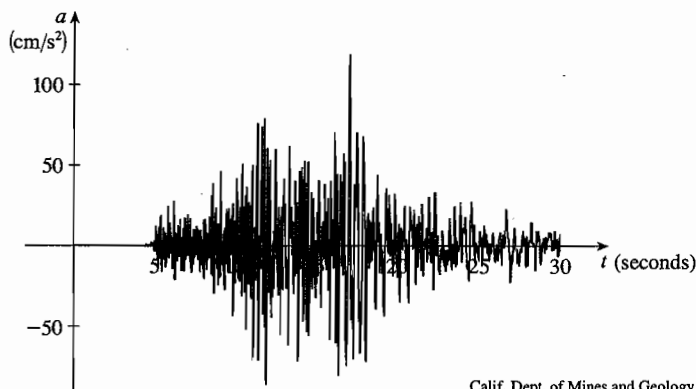| Year	Population (millions)
1900	1650
1910	1750
1920	1860
1930	2070
1940	2300
1950	2560
1960	3040
1970	3710
1980	4450
1990	5280
2000	6080

FIGURE 1
Vertical ground acceleration during the Northridge earthquake

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Each of these examples describes a rule whereby, given a number $(r, t, w, $ or $t)$, another number $(A, P, C, $ or $a)$ is assigned. In each case we say that the second number is a function of the first number.

A **function** f is a rule that assigns to each element x in a set A exactly one element, called $f(x)$, in a set B.

We usually consider functions for which the sets A and B are sets of real numbers. The set A is called the **domain** of the function. The number $f(x)$ is the **value of f at x** and is read "f of x." The **range** of f is the set of all possible values of $f(x)$ as x varies throughout the domain. A symbol that represents an arbitrary number in the *domain* of a function f is called an **independent variable**. A symbol that represents a number in the *range* of f is called a **dependent variable**. In Example A, for instance, r is the independent variable and A is the dependent variable.

It's helpful to think of a function as a **machine** (see Figure 2). If x is in the domain of the function f, then when x enters the machine, it's accepted as an input and the machine produces an output $f(x)$ according to the rule of the function. Thus, we can think of the domain as the set of all possible inputs and the range as the set of all possible outputs.

The preprogrammed functions in a calculator are good examples of a function as a machine. For example, the square root key on your calculator computes such a function. You press the key labeled $\sqrt{}$ (or $\sqrt{x}$) and enter the input x. If $x < 0$, then x is not in the domain of this function; that is, x is not an acceptable input, and the calculator will indicate an error. If $x \geq 0$, then an *approximation* to $\sqrt{x}$ will appear in the display. Thus, the $\sqrt{x}$ key on your calculator is not quite the same as the exact mathematical function f defined by $f(x) = \sqrt{x}$.
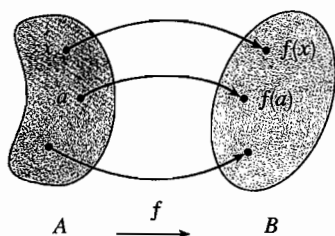
Another way to picture a function is by an **arrow diagram** as in Figure 3. Each arrow connects an element of A to an element of B. The arrow indicates that $f(x)$ is associated with x, $f(a)$ is associated with a, and so on.

The most common method for visualizing a function is its graph. If f is a function with domain A, then its **graph** is the set of ordered pairs

$$\{(x, f(x)) \mid x \in A\}$$

(Notice that these are input-output pairs.) In other words, the graph of f consists of all points (x, y) in the coordinate plane such that $y = f(x)$ and x is in the domain of f.

The graph of a function f gives us a useful picture of the behavior or "life history" of a function. Since the y-coordinate of any point (x, y) on the graph is $y = f(x)$, we can read the value of $f(x)$ from the graph as being the height of the graph above the point x (see Figure 4). The graph of f also allows us to picture the domain of f on the x-axis and its range on the y-axis as in Figure 5.

FIGURE 2
Machine diagram for a function f
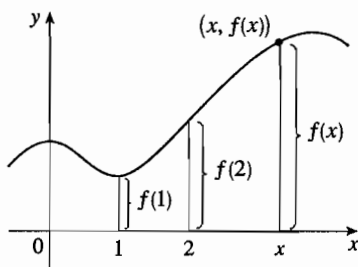

FIGURE 3
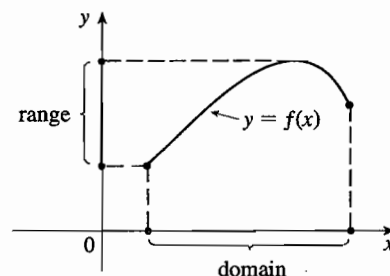Arrow diagram for f

FIGURE 4

FIGURE 5

EXAMPLE 1 The graph of a function f is shown in Figure 6.
(a) Find the values of $f(1)$ and $f(5)$.
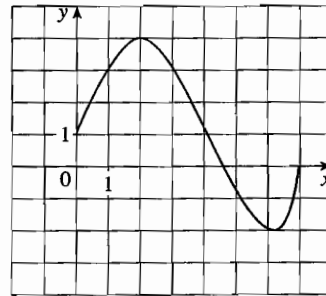(b) What are the domain and range of f?

FIGURE 6

SOLUTION
(a) We see from Figure 6 that the point $(1, 3)$ lies on the graph of f, so the value of f at 1 is $f(1) = 3$. (In other words, the point on the graph that lies above $x = 1$ is 3 units above the x-axis.)

When $x = 5$, the graph lies about 0.7 unit below the x-axis, so we estimate that $f(5) \approx -0.7$.

(b) We see that $f(x)$ is defined when $0 \leqslant x \leqslant 7$, so the domain of f is the closed interval $[0, 7]$. Notice that f takes on all values from -2 to 4, so the range of f is

$$\{y \mid -2 \leqslant y \leqslant 4\} = [-2, 4]$$

EXAMPLE 2 Sketch the graph and find the domain and range of each function.
(a) $f(x) = 2x - 1$ (b) $g(x) = x^2$

SOLUTION
(a) The equation of the graph is $y = 2x - 1$, and we recognize this as being the equation of a line with slope 2 and y-intercept -1. (Recall the slope-intercept form of the equation of a line: $y = mx + b$. See Appendix B.) This enables us to sketch the graph of f in Figure 7. The expression $2x - 1$ is defined for all real numbers, so the domain of f is the set of all real numbers, which we denote by $\mathbb{R}$. The graph shows that the range is also $\mathbb{R}$.

$y = 2x - 1$

FIGURE 7

(b) Since $g(2) = 2^2 = 4$ and $g(-1) = (-1)^2 = 1$, we could plot the points $(2, 4)$ and $(-1, 1)$, together with a few other points on the graph, and join them to produce the graph (Figure 8). The equation of the graph is $y = x^2$, which represents a parabola (see Appendix C). The domain of g is $\mathbb{R}$. The range of g consists of all values of $g(x)$, that is, all numbers of the form x^2. But $x^2 \geqslant 0$ for all numbers x and any positive number y is a square. So the range of g is $\{y \mid y \geqslant 0\} = [0, \infty)$. This can also be seen from Figure 8.
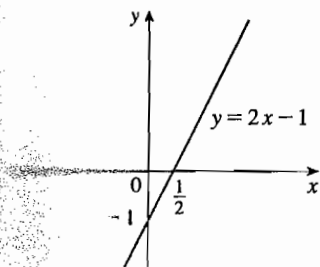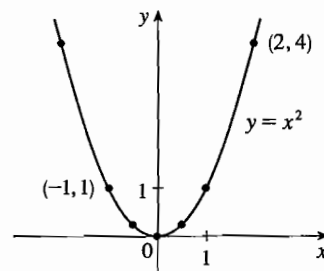
$(2, 4)$

$y = x^2$

$(-1, 1)$

FIGURE 8

▐▐ Representations of Functions

There are four possible ways to represent a function:

- verbally   (by a description in words)
- numerically   (by a table of values)
- visually   (by a graph)
- algebraically   (by an explicit formula)

If a single function can be represented in all four ways, it is often useful to go from one representation to another to gain additional insight into the function. (In Example 2, for instance, we started with algebraic formulas and then obtained the graphs.) But certain functions are described more naturally by one method than by another. With this in mind, let's reexamine the four situations that we considered at the beginning of this section.

A. The most useful representation of the area of a circle as a function of its radius is probably the algebraic formula $A(r) = \pi r^2$, though it is possible to compile a table of values or to sketch a graph (half a parabola). Because a circle has to have a positive radius, the domain is $\{r \mid r > 0\} = (0, \infty)$, and the range is also $(0, \infty)$.

B. We are given a description of the function in words: $P(t)$ is the human population of the world at time t. The table of values of world population on page 11 provides a convenient representation of this function. If we plot these values, we get the graph (called a *scatter plot*) in Figure 9. It too is a useful representation; the graph allows us to absorb all the data at once. What about a formula? Of course, it's impossible to devise an explicit formula that gives the exact human population $P(t)$ at any time t. But it is possible to find an expression for a function that *approximates* $P(t)$. In fact, using methods explained in Section 1.5, we obtain the approximation

$$P(t) \approx f(t) = (0.008079266) \cdot (1.013731)^t$$

and Figure 10 shows that it is a reasonably good "fit." The function f is called a *mathematical model* for population growth. In other words, it is a function with an explicit formula that approximates the behavior of our given function. We will see, however, that the ideas of calculus can be applied to a table of values; an explicit formula is not necessary.
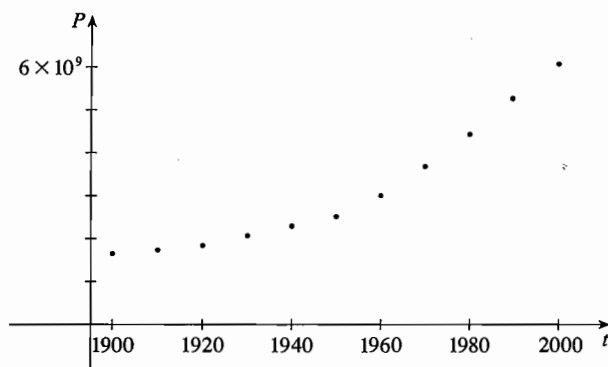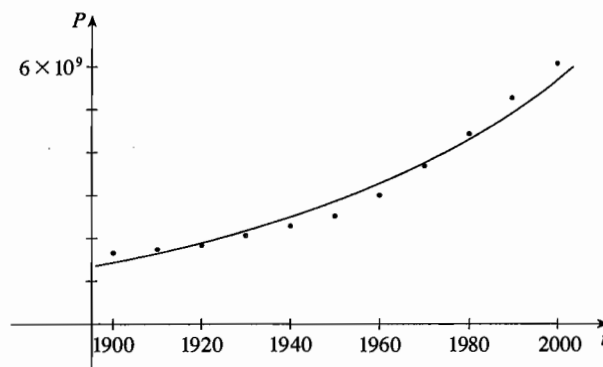

FIGURE 9

FIGURE 10

The function P is typical of the functions that arise whenever we attempt to apply calculus to the real world. We start with a verbal description of a function. Then we may be able to construct a table of values of the function, perhaps from instrument readings in a scientific experiment. Even though we don't have complete knowledge of the values of the function, we will see throughout the book that it is still possible to perform the operations of calculus on such a function.

|||| A function defined by a table of values is called a *tabular* function.

w (ounces)	$C(w)$ (dollars)
$0 < w \le 1$	0.37
$1 < w \le 2$	0.60
$2 < w \le 3$	0.83
$3 < w \le 4$	1.06
$4 < w \le 5$	1.29
$\vdots$	$\vdots$

C. Again the function is described in words: $C(w)$ is the cost of mailing a first-class letter with weight w. The rule that the U.S. Postal Service used as of 2002 is as follows: The cost is 37 cents for up to one ounce, plus 23 cents for each successive ounce up to 11 ounces. The table of values shown in the margin is the most convenient representation for this function, though it is possible to sketch a graph (see Example 10).

D. The graph shown in Figure 1 is the most natural representation of the vertical acceleration function $a(t)$. It's true that a table of values could be compiled, and it is even possible to devise an approximate formula. But everything a geologist needs to know—amplitudes and patterns—can be seen easily from the graph. (The same is true for the patterns seen in electrocardiograms of heart patients and polygraphs for lie-detection.) Figures 11 and 12 show the graphs of the north-south and east-west accelerations for the Northridge earthquake; when used in conjunction with Figure 1, they provide a great deal of information about the earthquake.
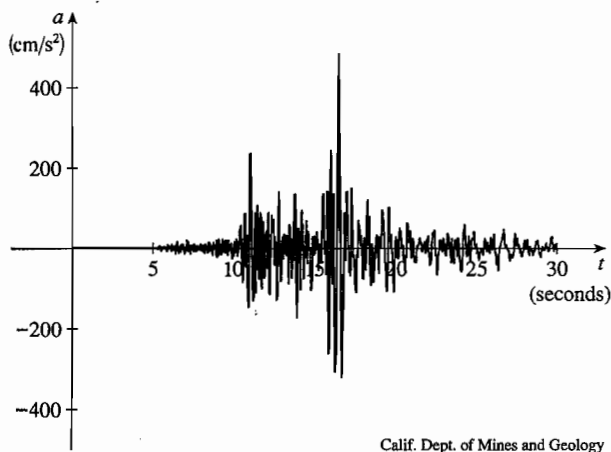

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FIGURE 11 North-south acceleration for the Northridge earthquake

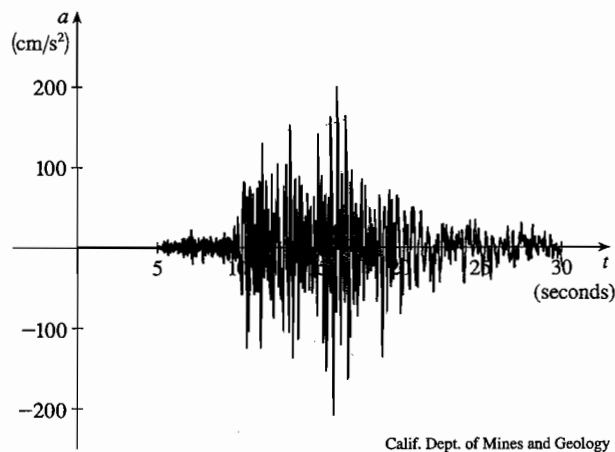Calif. Dept. of Mines and Geology

FIGURE 12 East-west acceleration for the Northridge earthquake

In the next example we sketch the graph of a function that is defined verbally.

EXAMPLE 3 When you turn on a hot-water faucet, the temperature T of the water depends on how long the water has been running. Draw a rough graph of T as a function of the time t that has elapsed since the faucet was turned on.

SOLUTION The initial temperature of the running water is close to room temperature because of the water that has been sitting in the pipes. When the water from the hot-water tank starts coming out, T increases quickly. In the next phase, T is constant at the temperature of the heated water in the tank. When the tank is drained, T decreases to the temperature of the water supply. This enables us to make the rough sketch of T as a function of t in Figure 13.
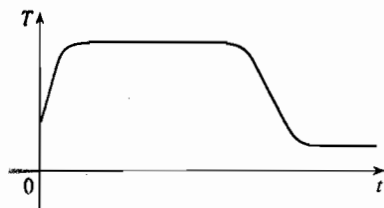

FIGURE 13

A more accurate graph of the function in Example 3 could be obtained by using a thermometer to measure the temperature of the water at 10-second intervals. In general, scientists collect experimental data and use them to sketch the graphs of functions, as the next example illustrates.

EXAMPLE 4 The data shown in the margin come from an experiment on the lactonization of hydroxyvaleric acid at 25°C. They give the concentration $C(t)$ of this acid (in moles per liter) after t minutes. Use these data to draw an approximation to the graph of the concentration function. Then use this graph to estimate the concentration after 5 minutes.

t	$C(t)$
0	0.0800
2	0.0570
4	0.0408
6	0.0295
8	0.0210

SOLUTION We plot the five points corresponding to the data from the table in Figure 14. The curve-fitting methods of Section 1.2 could be used to choose a model and graph it. But the data points in Figure 14 look quite well behaved, so we simply draw a smooth curve through them by hand as in Figure 15.
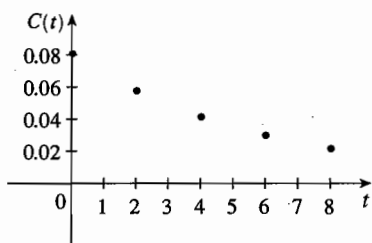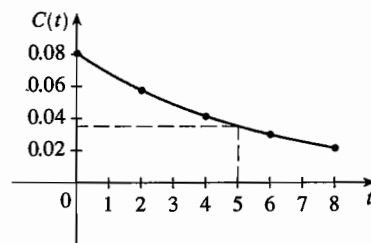

FIGURE 14

FIGURE 15

Then we use the graph to estimate that the concentration after 5 minutes is

$$C(5) \approx 0.035 \text{ mole/liter}$$

In the following example we start with a verbal description of a function in a physical situation and obtain an explicit algebraic formula. The ability to do this is a useful skill in solving calculus problems that ask for the maximum or minimum values of quantities.

EXAMPLE 5 A rectangular storage container with an open top has a volume of 10 m³. The length of its base is twice its width. Material for the base costs $10 per square meter; material for the sides costs $6 per square meter. Express the cost of materials as a function of the width of the base.
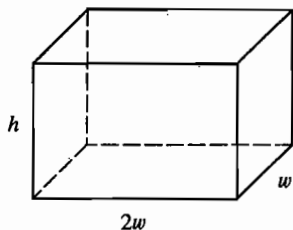
SOLUTION We draw a diagram as in Figure 16 and introduce notation by letting w and $2w$ be the width and length of the base, respectively, and h be the height.

The area of the base is $(2w)w = 2w^2$, so the cost, in dollars, of the material for the base is $10(2w^2)$. Two of the sides have area wh and the other two have area $2wh$, so the cost of the material for the sides is $6[2(wh) + 2(2wh)]$. The total cost is therefore

$$C = 10(2w^2) + 6[2(wh) + 2(2wh)] = 20w^2 + 36wh$$

FIGURE 16

To express C as a function of w alone, we need to eliminate h and we do so by using the fact that the volume is 10 m³. Thus

$$w(2w)h = 10$$

which gives

$$h = \frac{10}{2w^2} = \frac{5}{w^2}$$

Substituting this into the expression for C, we have

$$C = 20w^2 + 36w\left(\frac{5}{w^2}\right) = 20w^2 + \frac{180}{w}$$

Therefore, the equation

$$C(w) = 20w^2 + \frac{180}{w} \qquad w > 0$$

expresses C as a function of w.

EXAMPLE 6 Find the domain of each function.

(a) $f(x) = \sqrt{x + 2}$

(b) $g(x) = \dfrac{1}{x^2 - x}$

SOLUTION

If a function is given by a formula and the domain is not stated explicitly, the convention is that the domain is the set of all numbers for which the formula makes sense and defines a real number.

(a) Because the square root of a negative number is not defined (as a real number), the domain of f consists of all values of x such that $x + 2 \geq 0$. This is equivalent to $x \geq -2$, so the domain is the interval $[-2, \infty)$.

(b) Since

$$g(x) = \frac{1}{x^2 - x} = \frac{1}{x(x - 1)}$$

and division by 0 is not allowed, we see that $g(x)$ is not defined when $x = 0$ or $x = 1$. Thus, the domain of g is

$$\{x \mid x \neq 0, x \neq 1\}$$

which could also be written in interval notation as

$$(-\infty, 0) \cup (0, 1) \cup (1, \infty)$$

The graph of a function is a curve in the xy-plane. But the question arises: Which curves in the xy-plane are graphs of functions? This is answered by the following test.

The Vertical Line Test A curve in the xy-plane is the graph of a function of x if and only if no vertical line intersects the curve more than once.

The reason for the truth of the Vertical Line Test can be seen in Figure 17. If each vertical line $x = a$ intersects a curve only once, at (a, b), then exactly one functional value is defined by $f(a) = b$. But if a line $x = a$ intersects the curve twice, at (a, b) and (a, c), then the curve can't represent a function because a function can't assign two different values to a.
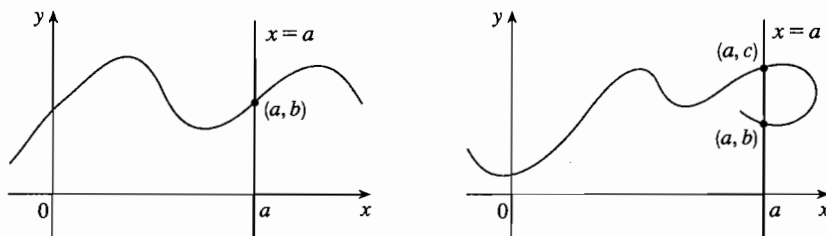

FIGURE 17

For example, the parabola $x = y^2 - 2$ shown in Figure 18(a) is not the graph of a function of x because, as you can see, there are vertical lines that intersect the parabola twice. The parabola, however, does contain the graphs of *two* functions of x. Notice that the equation $x = y^2 - 2$ implies $y^2 = x + 2$, so $y = \pm\sqrt{x + 2}$. Thus, the upper and lower halves of the parabola are the graphs of the functions $f(x) = \sqrt{x + 2}$ [from Example 6(a)] and $g(x) = -\sqrt{x + 2}$. [See Figures 18(b) and (c).] We observe that if we reverse the roles of x and y, then the equation $x = h(y) = y^2 - 2$ does define x as a function of y (with y as the independent variable and x as the dependent variable) and the parabola now appears as the graph of the function h.
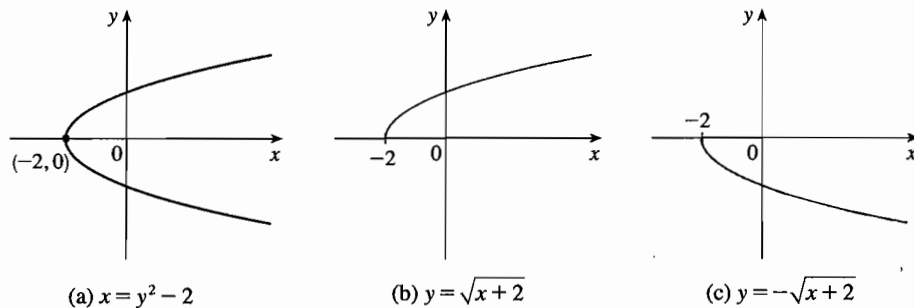
FIGURE 18

(a) $x = y^2 - 2$ (b) $y = \sqrt{x + 2}$ (c) $y = -\sqrt{x + 2}$

||| **Piecewise Defined Functions**

The functions in the following four examples are defined by different formulas in different parts of their domains.

EXAMPLE 7 A function f is defined by

$$f(x) = \begin{cases} 1 - x & \text{if } x \leq 1 \\ x^2 & \text{if } x > 1 \end{cases}$$

Evaluate $f(0)$, $f(1)$, and $f(2)$ and sketch the graph.

SOLUTION Remember that a function is a rule. For this particular function the rule is the following: First look at the value of the input x. If it happens that $x \leq 1$, then the value of $f(x)$ is $1 - x$. On the other hand, if $x > 1$, then the value of $f(x)$ is x^2.

Since $0 \leq 1$, we have $f(0) = 1 - 0 = 1$.

Since $1 \leq 1$, we have $f(1) = 1 - 1 = 0$.
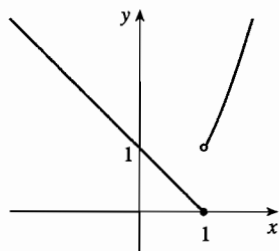
Since $2 > 1$, we have $f(2) = 2^2 = 4$.

How do we draw the graph of f? We observe that if $x \leq 1$, then $f(x) = 1 - x$, so the part of the graph of f that lies to the left of the vertical line $x = 1$ must coincide with the line $y = 1 - x$, which has slope -1 and y-intercept 1. If $x > 1$, then $f(x) = x^2$, so the part of the graph of f that lies to the right of the line $x = 1$ must coincide with the graph of $y = x^2$, which is a parabola. This enables us to sketch the graph in Figure 19. The solid dot indicates that the point $(1, 0)$ is included on the graph; the open dot indicates that the point $(1, 1)$ is excluded from the graph.

FIGURE 19

The next example of a piecewise defined function is the absolute value function. Recall that the **absolute value** of a number a, denoted by $|a|$, is the distance from a to 0 on the real number line. Distances are always positive or 0, so we have

$$|a| \geq 0 \qquad \text{for every number } a$$

For example,

$$|3| = 3 \qquad |-3| = 3 \qquad |0| = 0 \qquad |\sqrt{2} - 1| = \sqrt{2} - 1 \qquad |3 - \pi| = \pi - 3$$

In general, we have

$$|a| = a \qquad \text{if } a \geq 0$$
$$|a| = -a \quad \text{if } a < 0$$

(Remember that if a is negative, then $-a$ is positive.)

EXAMPLE 8 Sketch the graph of the absolute value function $f(x) = |x|$.

SOLUTION From the preceding discussion we know that

FIGURE 20

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Using the same method as in Example 7, we see that the graph of f coincides with the line $y = x$ to the right of the y-axis and coincides with the line $y = -x$ to the left of the y-axis (see Figure 20).
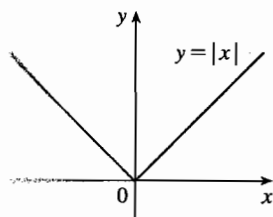
EXAMPLE 9 Find a formula for the function f graphed in Figure 21.
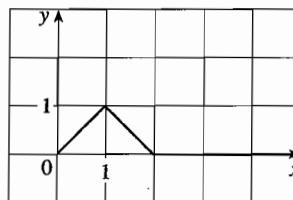

FIGURE 21

SOLUTION The line through $(0, 0)$ and $(1, 1)$ has slope $m = 1$ and y-intercept $b = 0$, so its equation is $y = x$. Thus, for the part of the graph of f that joins $(0, 0)$ to $(1, 1)$, we have

$$f(x) = x \qquad \text{if } 0 \leq x \leq 1$$

The line through $(1, 1)$ and $(2, 0)$ has slope $m = -1$, so its point-slope form is

$$y - 0 = (-1)(x - 2) \qquad \text{or} \qquad y = 2 - x$$

So we have

$$f(x) = 2 - x \qquad \text{if } 1 < x \leq 2$$

We also see that the graph of f coincides with the x-axis for $x > 2$. Putting this information together, we have the following three-piece formula for f:

$$f(x) = \begin{cases} x & \text{if } 0 \leqslant x \leqslant 1 \\ 2 - x & \text{if } 1 < x \leqslant 2 \\ 0 & \text{if } x > 2 \end{cases}$$

EXAMPLE 10 In Example C at the beginning of this section we considered the cost $C(w)$ of mailing a first-class letter with weight w. In effect, this is a piecewise defined function because, from the table of values, we have

$$C(w) = \begin{cases} 0.37 & \text{if } 0 < w \leqslant 1 \\ 0.60 & \text{if } 1 < w \leqslant 2 \\ 0.83 & \text{if } 2 < w \leqslant 3 \\ 1.06 & \text{if } 3 < w \leqslant 4 \end{cases}$$
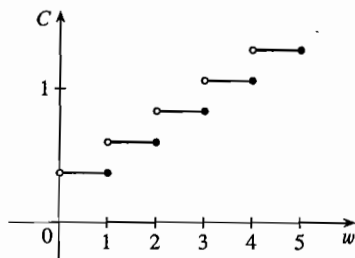

FIGURE 22

The graph is shown in Figure 22. You can see why functions similar to this one are called **step functions**—they jump from one value to the next. Such functions will be studied in Chapter 2.

▐▌▌ Symmetry

If a function f satisfies $f(-x) = f(x)$ for every number x in its domain, then f is called an **even function**. For instance, the function $f(x) = x^2$ is even because

$$f(-x) = (-x)^2 = x^2 = f(x)$$

FIGURE 23
An even function

The geometric significance of an even function is that its graph is symmetric with respect to the y-axis (see Figure 23). This means that if we have plotted the graph of f for $x \geqslant 0$, we obtain the entire graph simply by reflecting about the y-axis.

If f satisfies $f(-x) = -f(x)$ for every number x in its domain, then f is called an **odd function**. For example, the function $f(x) = x^3$ is odd because

$$f(-x) = (-x)^3 = -x^3 = -f(x)$$

The graph of an odd function is symmetric about the origin (see Figure 24). If we already have the graph of f for $x \geqslant 0$, we can obtain the entire graph by rotating through $180°$ about the origin.

FIGURE 24
An odd function

EXAMPLE 11 Determine whether each of the following functions is even, odd, or neither even nor odd.

(a) $f(x) = x^5 + x$ (b) $g(x) = 1 - x^4$ (c) $h(x) = 2x - x^2$

SOLUTION

(a)
$$f(-x) = (-x)^5 + (-x) = (-1)^5 x^5 + (-x)$$
$$= -x^5 - x = -(x^5 + x)$$
$$= -f(x)$$

Therefore, f is an odd function.

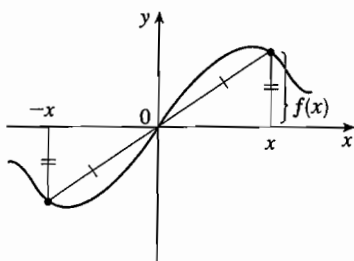(b)
$$g(-x) = 1 - (-x)^4 = 1 - x^4 = g(x)$$

So g is even.

(c) $$h(-x) = 2(-x) - (-x)^2 = -2x - x^2$$

Since $h(-x) \neq h(x)$ and $h(-x) \neq -h(x)$, we conclude that h is neither even nor odd.

The graphs of the functions in Example 11 are shown in Figure 25. Notice that the graph of h is symmetric neither about the y-axis nor about the origin.
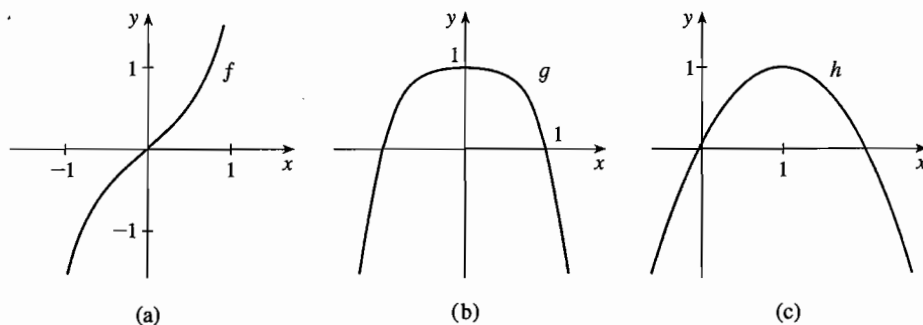
FIGURE 25

(a) (b) (c)

▓▓ Increasing and Decreasing Functions

The graph shown in Figure 26 rises from A to B, falls from B to C, and rises again from C to D. The function f is said to be increasing on the interval $[a, b]$, decreasing on $[b, c]$, and increasing again on $[c, d]$. Notice that if x_1 and x_2 are any two numbers between a and b with $x_1 < x_2$, then $f(x_1) < f(x_2)$. We use this as the defining property of an increasing function.

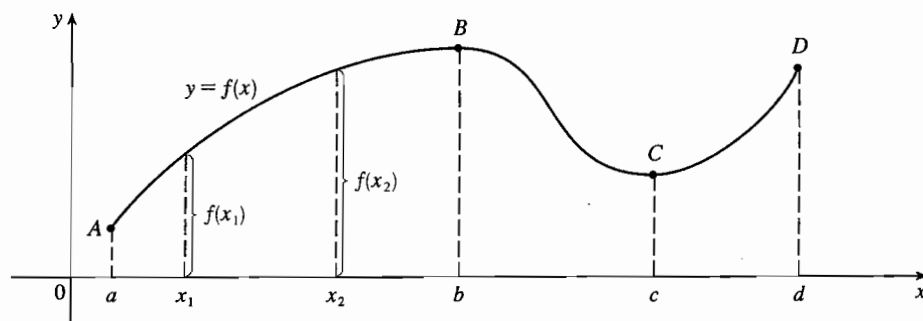**FIGURE 26**

A function f is called **increasing** on an interval I if

$$f(x_1) < f(x_2) \qquad \text{whenever } x_1 < x_2 \text{ in } I$$

It is called **decreasing** on I if

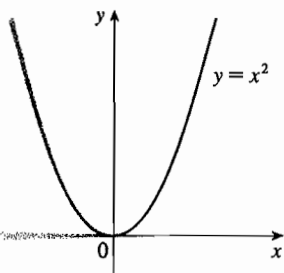$$f(x_1) > f(x_2) \qquad \text{whenever } x_1 < x_2 \text{ in } I$$

In the definition of an increasing function it is important to realize that the inequality $f(x_1) < f(x_2)$ must be satisfied for *every* pair of numbers x_1 and x_2 in I with $x_1 < x_2$.

You can see from Figure 27 that the function $f(x) = x^2$ is decreasing on the interval $(-\infty, 0]$ and increasing on the interval $[0, \infty)$.

FIGURE 27

▐▌▌ 1.1 Exercises

1. The graph of a function f is given.
  (a) State the value of $f(-1)$.
  (b) Estimate the value of $f(2)$.
  (c) For what values of x is $f(x) = 2$?
  (d) Estimate the values of x such that $f(x) = 0$.
  (e) State the domain and range of f.
 (f) On what interval is f increasing?
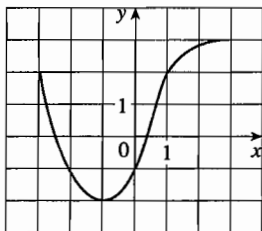
2. The graphs of f and g are given.
  (a) State the values of $f(-4)$ and $g(3)$.
  (b) For what values of x is $f(x) = g(x)$?
  (c) Estimate the solution of the equation $f(x) = -1$.
  (d) On what interval is f decreasing?
  (e) State the domain and range of f.
  (f) State the domain and range of g.
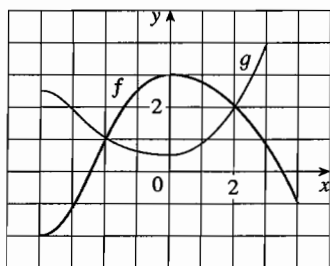
**3.** Figures 1, 11, and 12 were recorded by an instrument operated by the California Department of Mines and Geology at the University Hospital of the University of Southern California in Los Angeles. Use them to estimate the ranges of the vertical, north-south, and east-west ground acceleration functions at USC during the Northridge earthquake.

4. In this section we discussed examples of ordinary, everyday functions: Population is a function of time, postage cost is a function of weight, water temperature is a function of time. Give three other examples of functions from everyday life that are described verbally. What can you say about the domain and range of each of your functions? If possible, sketch a rough graph of each function.

5–8 ▐▌▌ Determine whether the curve is the graph of a function of x. If it is, state the domain and range of the function.

5.

6.

7.

8.
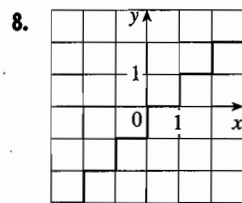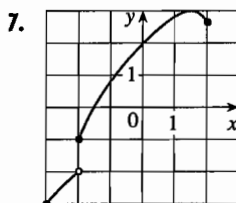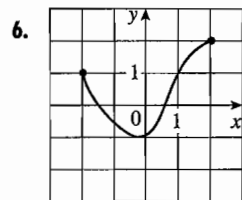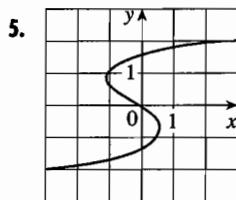
9. The graph shown gives the weight of a certain person as a function of age. Describe in words how this person's weight varies over time. What do you think happened when this person was 30 years old?
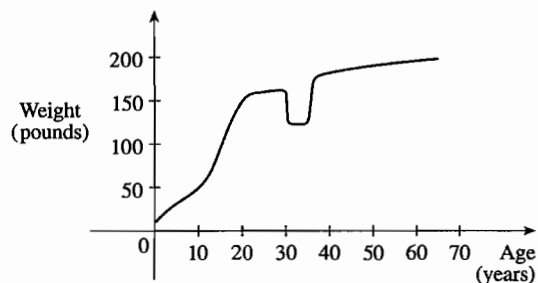
10. The graph shown gives a salesman's distance from his home as a function of time on a certain day. Describe in words what the graph indicates about his travels on this day.
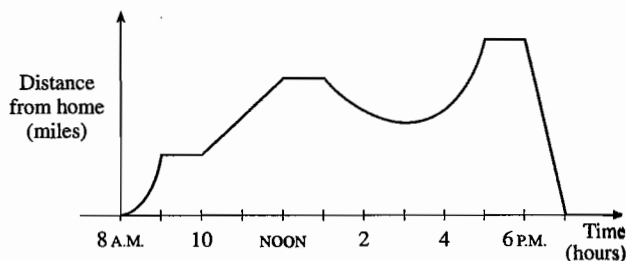
11. You put some ice cubes in a glass, fill the glass with cold water, and then let the glass sit on a table. Describe how the temperature of the water changes as time passes. Then sketch a rough graph of the temperature of the water as a function of the elapsed time.

12. Sketch a rough graph of the number of hours of daylight as a function of the time of year.

13. Sketch a rough graph of the outdoor temperature as a function of time during a typical spring day.

14. You place a frozen pie in an oven and bake it for an hour. Then you take it out and let it cool before eating it. Describe how the temperature of the pie changes as time passes. Then sketch a rough graph of the temperature of the pie as a function of time.

15. A homeowner mows the lawn every Wednesday afternoon. Sketch a rough graph of the height of the grass as a function of time over the course of a four-week period.

16. An airplane flies from an airport and lands an hour later at another airport, 400 miles away. If t represents the time in minutes since the plane has left the terminal building, let $x(t)$ be the horizontal distance traveled and $y(t)$ be the altitude of the plane.
(a) Sketch a possible graph of $x(t)$.
(b) Sketch a possible graph of $y(t)$.
(c) Sketch a possible graph of the ground speed.
(d) Sketch a possible graph of the vertical velocity.

17. The number N (in thousands) of cellular phone subscribers in Malaysia is shown in the table. (Midyear estimates are given.)

t	1991	1993	1995	1997
N	132	304	873	2461

(a) Use the data to sketch a rough graph of N as a function of t.
(b) Use your graph to estimate the number of cell-phone subscribers in Malaysia at midyear in 1994 and 1996.

18. Temperature readings T (in °F) were recorded every two hours from midnight to 2:00 P.M. in Dallas on June 2, 2001. The time t was measured in hours from midnight.

t	0	2	4	6	8	10	12	14
T	73	73	70	69	72	81	88	91

(a) Use the readings to sketch a rough graph of T as a function of t.
(b) Use your graph to estimate the temperature at 11:00 A.M.

19. If $f(x) = 3x^2 - x + 2$, find $f(2)$, $f(-2)$, $f(a)$, $f(-a)$, $f(a + 1)$, $2f(a)$, $f(2a)$, $f(a^2)$, $[f(a)]^2$, and $f(a + h)$.

20. A spherical balloon with radius r inches has volume $V(r) = \frac{4}{3}\pi r^3$. Find a function that represents the amount of air required to inflate the balloon from a radius of r inches to a radius of $r + 1$ inches.

21–22 IIII Find $f(2 + h)$, $f(x + h)$, and $\dfrac{f(x + h) - f(x)}{h}$, where $h \neq 0$.

21. $f(x) = x - x^2$

22. $f(x) = \dfrac{x}{x + 1}$

23–27 IIII Find the domain of the function.

23. $f(x) = \dfrac{x}{3x - 1}$

24. $f(x) = \dfrac{5x + 4}{x^2 + 3x + 2}$

25. $f(t) = \sqrt{t} + \sqrt[3]{t}$

26. $g(u) = \sqrt{u} + \sqrt{4 - u}$

27. $h(x) = \dfrac{1}{\sqrt[4]{x^2 - 5x}}$

28. Find the domain and range and sketch the graph of the function $h(x) = \sqrt{4 - x^2}$.

29–40 IIII Find the domain and sketch the graph of the function.

29. $f(x) = 5$

30. $F(x) = \frac{1}{2}(x + 3)$

31. $f(t) = t^2 - 6t$

32. $H(t) = \dfrac{4 - t^2}{2 - t}$

33. $g(x) = \sqrt{x - 5}$

34. $F(x) = |2x + 1|$

35. $G(x) = \dfrac{3x + |x|}{x}$

36. $g(x) = \dfrac{|x|}{x^2}$

37. $f(x) = \begin{cases} x & \text{if } x \leq 0 \\ x + 1 & \text{if } x > 0 \end{cases}$

38. $f(x) = \begin{cases} 2x + 3 & \text{if } x < -1 \\ 3 - x & \text{if } x \geq -1 \end{cases}$

39. $f(x) = \begin{cases} x + 2 & \text{if } x \leq -1 \\ x^2 & \text{if } x > -1 \end{cases}$

40. $f(x) = \begin{cases} -1 & \text{if } x \leq -1 \\ 3x + 2 & \text{if } |x| < 1 \\ 7 - 2x & \text{if } x \geq 1 \end{cases}$
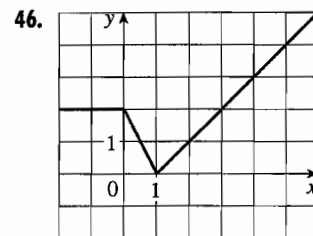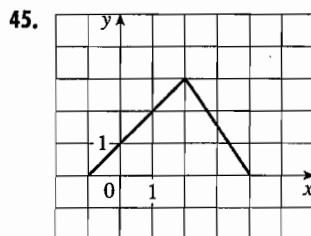
41–46 IIII Find an expression for the function whose graph is the given curve.

41. The line segment joining the points $(-2, 1)$ and $(4, -6)$

42. The line segment joining the points $(-3, -2)$ and $(6, 3)$

43. The bottom half of the parabola $x + (y - 1)^2 = 0$

44. The top half of the circle $(x - 1)^2 + y^2 = 1$

45.

46.

**47–51** ▐▐▐ Find a formula for the described function and state its domain.

47. A rectangle has perimeter 20 m. Express the area of the rectangle as a function of the length of one of its sides.

48. A rectangle has area 16 m². Express the perimeter of the rectangle as a function of the length of one of its sides.

49. Express the area of an equilateral triangle as a function of the length of a side.

50. Express the surface area of a cube as a function of its volume.

51. An open rectangular box with volume 2 m³ has a square base. Express the surface area of the box as a function of the length of a side of the base.

52. A Norman window has the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 30 ft, express the area A of the window as a function of the width x of the window.

53. A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 12 in. by 20 in. by cutting out equal squares of side x at each corner and then folding up the sides as in the figure. Express the volume V of the box as a function of x.
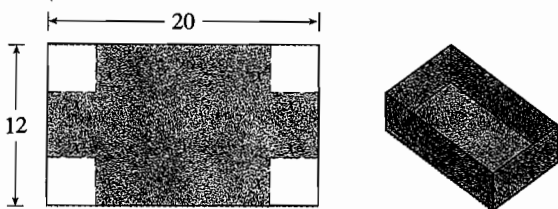

54. A taxi company charges two dollars for the first mile (or part of a mile) and 20 cents for each succeeding tenth of a mile (or part). Express the cost C (in dollars) of a ride as a function of the distance x traveled (in miles) for $0 < x < 2$, and sketch the graph of this function.

55. In a certain country, income tax is assessed as follows. There is no tax on income up to $10,000. Any income over $10,000 is taxed at a rate of 10%, up to an income of $20,000. Any income over $20,000 is taxed at 15%.
(a) Sketch the graph of the tax rate R as a function of the income I.
(b) How much tax is assessed on an income of $14,000? On $26,000?
(c) Sketch the graph of the total assessed tax T as a function of the income I.

56. The functions in Example 10 and Exercises 54 and 55(a) are called *step functions* because their graphs look like stairs. Give two other examples of step functions that arise in everyday life.

57–58 ▐▐▐ Graphs of f and g are shown. Decide whether each function is even, odd, or neither. Explain your reasoning.

57.

58.
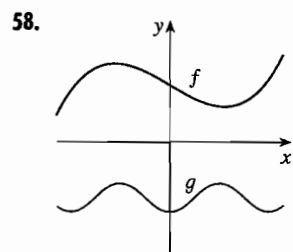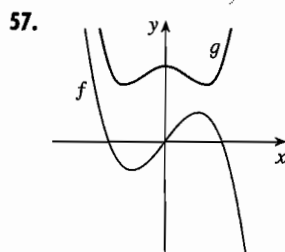
59. (a) If the point $(5, 3)$ is on the graph of an even function, what other point must also be on the graph?
(b) If the point $(5, 3)$ is on the graph of an odd function, what other point must also be on the graph?

60. A function f has domain $[-5, 5]$ and a portion of its graph is shown.
(a) Complete the graph of f if it is known that f is even.
(b) Complete the graph of f if it is known that f is odd.
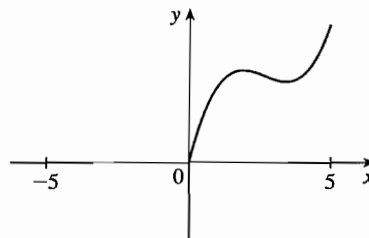
61–66 ▐▐▐ Determine whether f is even, odd, or neither. If f is even or odd, use symmetry to sketch its graph.

61. $f(x) = x^{-2}$

62. $f(x) = x^{-3}$

63. $f(x) = x^2 + x$

64. $f(x) = x^4 - 4x^2$

65. $f(x) = x^3 - x$

66. $f(x) = 3x^3 + 2x^2 + 1$