GUIDELINES FOR WRITTEN HOMEWORK

1. THE PURPOSE OF WRITTEN HOMEWORK

The purpose of written homework in this course is to develop your skills in understanding and communicating mathematics. It is not to give you busy work or drill. Don't think of your homework paper as a certificate proving that you have done the assignment. Think of it as an exercise in learning, and reporting what you have learned. There is a lot of truth in the saying, "If you can't explain it, you don't understand it".

2. YOUR RESPONSIBILITY

Communicate with the reader. Do not write to the instructor (who already knows how to do the problem), but explain your solution to someone who needs help, perhaps a classmate who has been sick. Start at the beginning and be clear, logical and complete.

The ultimate test of what you write is this: *can someone learn from your paper? easily?* Remember, the reader will see only what you wrote, not what you meant to say. So it must all be there, and accurate.

Make your paper "reader-friendly".

3. COMMON ERRORS ON HOMEWORK

Here are some common errors students make on homework:

incorrect math	undefined terms
bad English	same symbol used for different quantities
too many words and too little math	omitting reasons
abuse of the equal sign	not answering the question
	bad format

To help you avoid these errors, let's discuss each one and give some examples.

(A) *Incorrect mathematics*

Mathematics is the most precise of all subjects. *Every statement you make must be correct.* A solution will be ruined by a single false step such as

$$\frac{a+3}{b+3} = \frac{a}{b} + 1 \qquad (x+y)^2 = x^2 + y^2 \qquad (x^5)^2 = x^{25}$$

If you are not sure about a statement, check it for some special cases. For example, if you suspect that $n^2 > 2^n$, try a few values of n. You will soon see that $5^2 < 2^5$, so the statement is false.

<p style="text-align:center">Be very careful not to make false assertions.</p>

(B) *English*

Good communication requires good English. A correct solution garbled by bad English may be worthless to the reader. The rules of grammar, spelling, and punctuation apply to mathematics as well as to all writing.

For clear communication, present one idea at a time. Since an idea is expressed by a complete sentence,

<p style="text-align:center">Write in complete sentences.</p>

Examples

Non-sentence	*Sentence*
dog	The dog crossed the street.
$a + b$	$a + b = 15.$
since x is positive	Since x is positive, $x > -2$.
Take a number m. Where m is odd.	Take a number m, where m is odd.
$(x+y)^2 x^2 + 2y + y^2$	$(x+y)^2 = x^2 + 2xy + y^2$

Keep your sentences short. Long sentences are hard to follow and tend to become garbled. Break up long sentences into several short ones. End each sentence with a period.

To avoid pronouns, *give names to quantities and use those names.* Don't maximize "it", maximize $f(x)$.

Do not use arrows. They are a lazy person's way of avoiding writing well, and they confuse the reader.
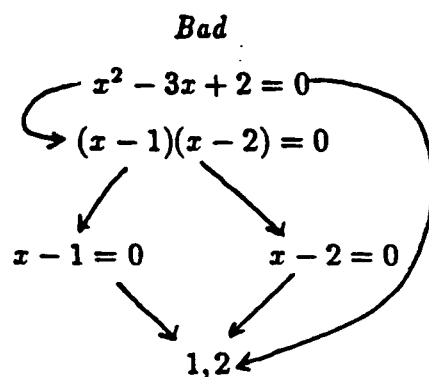
Bad	Good
	Let us solve $x^2 - 3x + 2 = 0$.
	Factor:
	$$(x-1)(x-2) = 0.$$
	Hence, either
	$$x - 1 = 0, \quad \text{so } x = 1$$
	or
	$$x - 2 = 0, \quad \text{so } x = 2.$$

The "Bad" column shows the following equations connected by arrows:
$$x^2 - 3x + 2 = 0$$
$$(x-1)(x-2) = 0$$
$$x - 1 = 0 \qquad x - 2 = 0$$
$$1, 2$$

(C) *Write math:* Strike a balance between words and symbols.

The notation of mathematics is clean and precise. Try to use mathematical symbols instead of words.

Words	Math
If you start with any number and subtract one from the square of the number, then the answer you will get is the same as the answer you will get if you multiply one more than the number by one less than it.	Let x be any number. Then $$x^2 - 1 = (x+1)(x-1).$$
[Ugh! 43 words]	

$x = 2^n + 2^n$ which is a power of 2 plus itself, doubling it, or raising 2 to one higher power.	$x = 2^n + 2^n = 2 \cdot 2^n = 2^{n+1}$.
If you multiply an odd number by 2, it becomes even. (A number cannot "become" another number.)	Twice an odd number is even.
the answer that you get when you add a and b together	$a + b$

Note: In grade school "$3 + 5$" is an arithmetic problem. In higher mathematics "$3 + 5$" is another name for "8". It is not a problem so there is no "answer".

Guideline: If your homework paper looks like an English composition, it is probably unsatisfactory. Try to translate most of the words into mathematical notation. Usually, your paper should contain about half math, half words.

Don't go overboard and cut out all words. Then, even though your math may be correct, it will be brutal to read. Always include some words to guide the reader through your work, for example:

First, we compute the area.

There are two cases, n even and n odd. If n is even,...

Move all terms to the left side and factor.

In short, *tell the reader what you are doing.*

(D) *Honor the equal sign.*

Quantities on either side of an equal sign must be equal. Oranges do not equal apples, numbers do not equal sets, etc. The equal sign has a precise meaning; it is not a miscellaneous punctuation mark such as a dash —

Bad	Good
$n = \text{even} = 2n$	If n is even, then $n = 2k$
(Note: n does not equal $2n$.)	for some k.
$n^2 = 16 = n = \pm 4$	$n^2 = 16$, hence $n = \pm 4$.
(16 does not equal ± 4.)	
An $n - gon = (n-2)180°$.	The sum of the interior angle
(A polygon does not equal	of an $n - gon$ is $(n-2)180°$.
equal a sum of degrees.)	
Case 1 = odd	Case 1: n odd
$\cos A = 1/2 = 60°$	$\cos A = 1/2$, so $A = 60°$.

(E) *Use different letters for different things.*

For example, write an equation showing that n is a power of 2.

Wrong	Right
$n = 2^n$	$n = 2^k$ for some k

In fact, there is no whole number n for which $n = 2^n$, e.g.

$$1 \neq 2^1 \qquad 2 \neq 2^2 \qquad 3 \neq 2^3 \qquad 4 \neq 2^4 \qquad \text{etc.}$$

An odd number can be written as $2k + 1$. How do you write two odd numbers n_1 and n_2?

Wrong	Right
$n_1 = 2k + 1, \quad n_2 = 2k + 1$	$n_1 = 2k_1 + 1, \quad n_2 = 2k_2 + 1$

The k's will be different unless $n_1 = n_2$.

(F) *Define your terms.*

State at the beginning what your symbols mean. The reader is just a reader, not a mind reader.

Problem 1. State the Pythagorean Theorem.

Answer $a^2 + b^2 = c^2$.

Whoa! What are a, b, and c? The reader cannot tell what this statement means, not even that it concerns right triangles.

(G) *Give reasons.*

Problem 2. Prove the Pythagorean Theorem.

Proof Let a right triangle have legs of length a and b, and hypotenuse of length c. Then $a^2 + b^2 = c^2$. (End of "proof".)

Comment. The writer gives no reasons for the conclusion. She just states the theorem without proving it.

Problem 3. Solve the equation $x^3 - 15x^2 - 28x - x = 85$.

Solution First combine the two terms in x:

$$x^3 - 15x^2 - 29x = 85$$

Now subtract 85 from both sides.

$$x^3 - 15x^2 - 29x - 85 = 0$$

Hence $x = 17$.

Comment. Wow! Where did $x = 17$ come from? Out of a hat? Is the writer a genius?

This student explains two very easy steps that hardly need explanation. But when he comes to the heart of the problem, he pole vaults over it.

Use common sense. Indicate the main points in a problem. Help the reader through the hard parts.

(H) *Be explicit.*

Problem 4. Can an even number be divisible by an odd number?

Vague	*Explicit*
Yes, there is nothing to prevent it.	Yes; for example, 10 is divisible by 5.

Problem 5. Are there any fractions of the form $\dfrac{2}{n}$ that are less than 0.01?

Vague	*Explicit*
Yes. You can show that if n is large enough, then $\dfrac{2}{n} < 0.01$.	Yes. Take $n = 2000$. Then $\dfrac{2}{n} = \dfrac{2}{2000} = \dfrac{1}{1000} < 0.01$.

Comment. Don't say something that "can be done". Do it!

(I) *Answer the question.*

When you finish a problem, go back and read it again. Be sure you have given a clear answer to the question asked.

Problem 6. I drive from here to Chicago, 125 miles at 50 mph, then from Chicago to Minneapolis, 550 miles, at 55 mph. How long does it take me to drive from here to Minneapolis?

Non-answer	*Answer*
$125 + 550 = 675$	The time to travel from here to Chicago is 125 miles/50 mph = 2.5 hours. The time from Chicago to Minneapolis is 550 miles/55 mph = 10 hours. Thus, the total time is 12.5 hours.

Problem 7. Explain why the sum of any two consecutive integers is not divisible by 4.

Non-answer	Answer
$1 + 2 = 3$, not divisible by 4	Let n and $n + 1$ be two con-
$2 + 3 = 5$, not divisible by 4	secutive integers. Their sum is
	$n + (n + 1) = 2n + 1$
(Two special cases do not	which is odd, hence not
prove a general statement.)	divisible by 4.

(J) *Be aware of format.*

Be considerate to the reader. Make your paper pleasant to read.

For example, don't cram. It's silly to squeeze your whole assignment into a couple of inches at the top of the paper, and then leave the rest of the page blank. Look for excuses to leave some space. Break up long paragraphs. Write equations on separate lines. All of this will pay off by improving your chalkboard skills as well.

Problem 6. Find the maximum area of a rectangle whose perimeter is 100.

Solution 1

$$A = x(50 - x) = 50x - x^2 \frac{dA}{dx} = 50 - 2x, 50 - 2x = 0$$
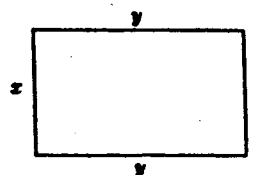$$x = 25. \text{ Hence } A = 25(50 - 25) = 625$$

Comment. The mathematics is correct, but the presentation is terrible. The solution is so crammed it's hard to read. Terms are undefined an no explanations are given. Here is a better solution.

Solution 2 The area of the rectangle is $A = xy$, where x is the length and y is the width. Since the perimeter is 100 ft,

$$2x + 2y = 100.$$

$$x + y = 50.$$

$$y = 50 - x$$

Hence

$$A = x(50 - x) = 50x - x^2$$

This function is differentiable for all x, so it can have a maximum only where its derivative is zero. Set $\dfrac{dA}{dx} = 0$:

$$50 - 2x = 0$$

$$x = 25$$

Because $\dfrac{d^2A}{dx^2} = -2$ at $x = 25$, this value of x yields a maximum for A. The maximum area is

$$A = 25(50 - 25) = 625 \ \text{ft}^2$$

4. Good writing

Students often say, "I can understand the math, but I can't write it". Like most people, they don't realize that good writing is not easy and requires practice.

Keep this in mind: almost nobody can write anything well the first time. Even professional writers constantly revise and re-write. Therefore

Do not submit your first draft.

First write out the assignment, and make sure that the math is correct. Then re-write using these guidelines, e.g. everything in sentences, good notation, good format, clear and unambiguous, etc.

Even after a second draft, your paper may not be acceptable. Here are three ways to check whether it is:

(1) Put your paper away for a few hours or until the next day. Then read it pretending you've never seen it before. Does it make sense? Can you follow it easily?

(2) Read your paper *out loud*. This is a great way to catch errors and garbled writing.

(3) Have another student read your paper. In short,

Read over what you write

The principles of written homework expressed in these Guidelines will probably require more thought and care than you have put into your assignments in previous mathematics courses. But they will make you a clearer thinker and a better student.