Math 460: Homework # 6. Due October 11

Note: Problems 1, 2 and 4 all use Geometer's Sketchpad.

1. (Use Geometer's Sketchpad.) Let ABC be a triangle. In class I showed you how to construct a triangle with three given sides, using only the "Circle by center and radius" and "Segment" commands. Use this method to construct a second triangle DEF whose sides are equal to the medians of ABC. (Notice that when you change the shape of ABC, the shape of DEF changes along with it.) Do not hide the objects used in your construction. Find an equation relating the areas of ABC and DEF. Print out a picture. Then change the shape of ABC and print a second picture showing that the equation still holds.

2. (Use Geometer's Sketchpad.) Construct a triangle and its three medians. The medians divide the triangle into 6 smaller triangles—use a custom tool to construct their centroids (hide the lines you use to do this). Now connect these 6 centroids to form a hexagon. Find the ratio of the area of this hexagon and the area of the original triangle. (Sketchpad will give you a decimal approximation to this ratio: try to find the exact ratio as a fraction. It is **not** 9/25.) Print out a picture.

3. (10 points) In this problem we prove Theorem 24. Let $\triangle ABC$ be a triangle, and draw the line ℓ through A parallel to BC, the line m through B parallel to AC, and the line n through C parallel to AB. Let D be the intersection of ℓ and m, E the intersection of ℓ and n, and F the intersection of m and n.

   (i) Prove that the altitudes of $\triangle ABC$ are the same lines as the perpendicular bisectors of $\triangle DEF$.

   (ii) Use (i) to prove that the altitudes of $\triangle ABC$ are concurrent.

4. (See Figure 1.) Prove that the bisectors of angles 1, 2 and 3 are concurrent. (Hint: use a strategy similar to the proof of Theorem 23.) Make a picture to illustrate your proof with Geometer's Sketchpad.

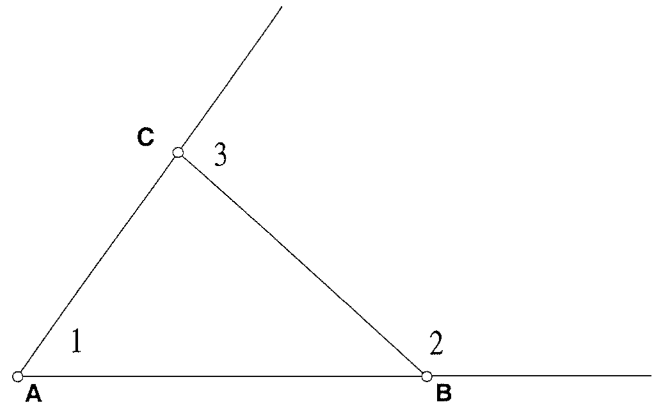

Figure 1

5. The medians of a triangle divide it into 6 small triangles. Prove that they all have the same area.

6. Let ABC be a triangle with centroid G. Let l be the line through G parallel to AB, and let D and E be the points where l intersects AC and BC respectively. Prove that the area of CDE is $4/9$ of the area of ABC.

7. (See Figure 2.) Given: M, N and P are the midpoints of AB, AC and BC respectively, MD is parallel to AP, and $MD = AP$. To prove: $CD = NB$. (Hint: there are three parallelograms in this picture. Do *not* draw in any extra lines.)

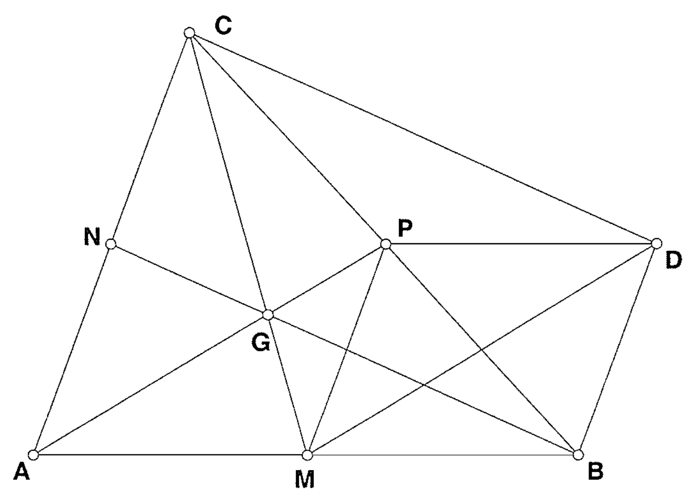

Figure 2

8. (See Figure 3.) Given: $\angle C = \angle D$ and $\triangle APR \cong \triangle BQT$. To prove $\triangle ADF \cong \triangle BCE$

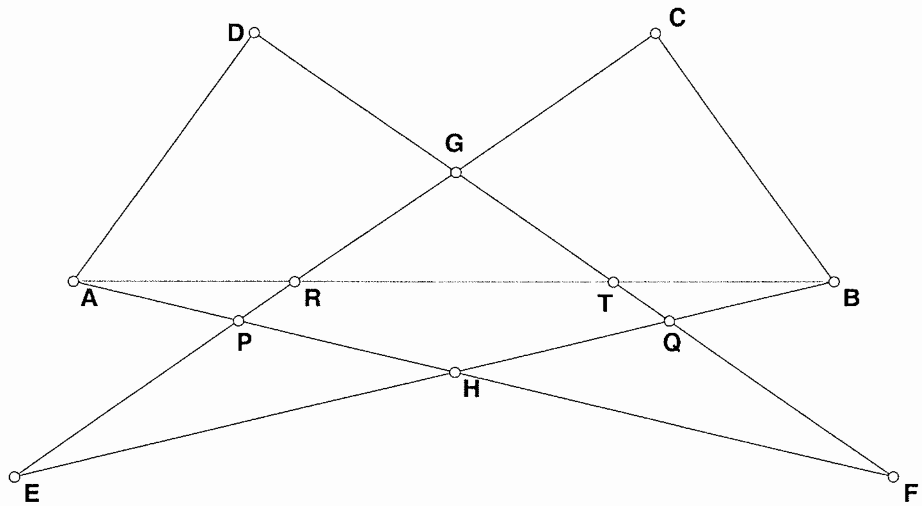

Figure 3

9. (See figure 4.) Prove that

$$\frac{A'B}{A'C}\frac{B'C}{B'A}\frac{C'A}{C'B} = 1.$$

(Hint: Use a strategy similar to Problem 4 from Assignment 5.)

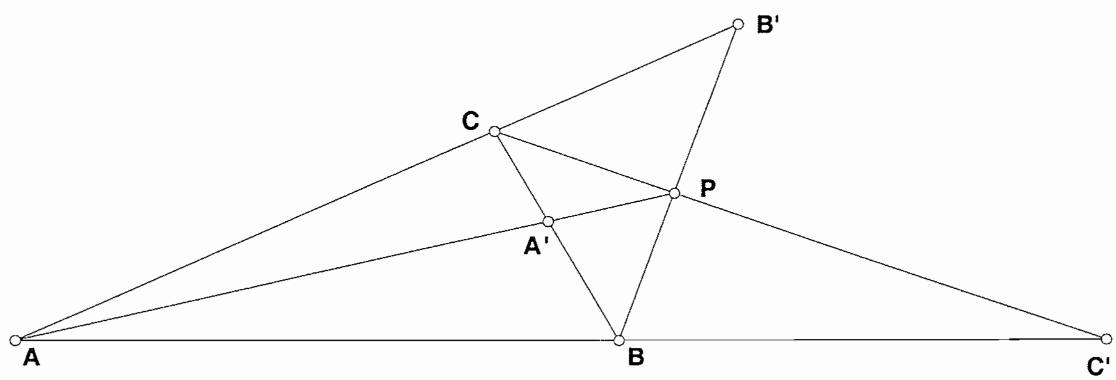

Figure 4

10. We discuss a paradox which Prof. Lipman introduced some weeks ago: a prrof that all triangles are isosceles. Here is a sketch of the proof. Let ABC be any triangle, and D the midpoint of BC. Let the perpendicular to BC at D meet the angle bisector at A at the point E. We then draw perpendiculars from E to the sides AB and AC, defining in this way points F and G. Then draw BE and C.

    Consider triangles AEF and AEG, one can see that they are congruent and we deduce that $AF = AG$ and $EF = EG$. Also triangles BDE and CDE are congruent, and so $BE = CE$, and since BEF and CEG are congruent [they are right triangles], we have that $BF = CG$. We combine these steps and see that $AB = AF + FB$ is equal to $AC = AG + GC$: the triangle is isosceles!

Assignment. This has two parts. First, write a "two column' proof of this based on the outline (the figure comes from a book in our library).

The second part is more interesting. In this course we have introduced a tool which you should recall which would not only show you why the proof is wrong, but also make a conjecture about the full truth.