Euclid's *Elements*

Book I

DEFINITIONS

1. A **point** is that which has no part.
2. A **line** is breadthless length.
3. The extremities of a line are points.
4. A **straight line** is a line which lies evenly with the points on itself.
5. A **surface** is that which has length and breadth only.
6. The extremities of a surface are lines.
7. A **plane surface** is a surface which lies evenly with the straight lines on itself.
8. A **plane angle** is the inclination to one another of two lines in a plane which meet one another and do not lie in a straight line.
9. And when the lines containing the angle are straight, the angle is called **rectilineal.**
10. When a straight line set up on a straight line makes the adjacent angles equal to one another, each of the equal angles is **right**, and the straight line standing on the other is called a **perpendicular** to that on which it stands.
11. An **obtuse angle** is an angle greater than a right angle.
12. An **acute angle** is an angle less than a right angle.
13. A **boundary** is that which is an extremity of anything.
14. A **figure** is that which is contained by any boundary of boundaries.
15. A **circle** is a plane figure contained by one line such that all the straight lines falling from one point among those lying within the figure are equal to one another;
16. And the point is called the **center** of the circle.
17. A **diameter** of the circle is any straight line drown through the center and terminated in both directions by the circumference of the circle, and such a straight line also bisects the circle.
18. A **semicircle** is the figure contained by the diameter and the circumference cut off by it. And the center of the semicircle is the same as that of the

circle.

19. **Rectilineal figures** are those which are contained by straight lines: **trilateral** figures (**triangles**) being those contained by three; **quadrilateral**, those contained by four; and **multilateral**, those contained by more than four straight lines.
20. Of trilateral figures, an **equilateral triangle** is that which has its three sides equal, an **isosceles triangle**, that which has two of its sides equal, and a **scalene triangle**, that which has its three sides unequal.
21. Further, of trilateral figures, a **right-angled triangle** is that which has a right angle, and **obtuse-angled triangle** is that which has an obtuse angle, and an **acute-angled triangle** is that which has its three angles acute.
22. Of quadrilateral figures, a **square** is that which is both equilateral and right-angled; an **oblong** is that which is right-angled but not equilateral; a **rhombus** is that which is equilateral but not right-angled; and a **rhomboid** is that which has its opposite sides and angles equal to one another but is neither equilateral nor right-angled. And let quadrilateral other than these be called **trapezia**.
23. **Parallel** straight lines are straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction.

POSTULATES

1. (It is possible) to draw a straight line from any point to any point.
2. (It is possible) to produce (continue) a finite straight line continuously in a straight line.
3. (It is possible) to describe a circle with any center and distance (radius).
4. All right angles are equal to one another.
5. If a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

COMMON NOTIONS

1. Things which are equal to the same thing are also equal to one another.
2. If equals be added to equals, the wholes are equal.
3. If equals be subtracted from equals, the remainders are equal.
4. Things which coincide with one another are equal to one another.
5. The whole is greater than the part.

Book I Propositions

Proposition 1.

On a given finite straight line to construct an equilateral triangle.

Proposition 2.

To place at a given point (as an extremity) a straight line equal to a given straight line.

Proposition 3.

Given two unequal straight lines, to cut off from the greater a straight line equal to the less.

Proposition 4.

If two triangles have the two sides equal to two sides respectively, and have angles contained by the equal straight lines equal, they will also have the base equal to the base, the triangle will be equal to the triangle, and the remaining angles will be equal to the remaining angles respectively, namely those which the equal sides subtend. (SAS)

Proposition 5.

In isosceles triangles the angles at the base are equal to one another, and, if the equal straight lines be produced further, the angles under the base will be equal to one another.

Proposition 6.

If in a triangle two angles be equal to one another, the sides which subtend the equal angles will also be equal to one another.

Proposition 7.

Given two straight lines constructed on a straight line (from its extremities) and meeting in a point, there cannot be constructed on the same straight line (from its extremities), and on the same side of it, two other straight lines meeting in another point and equal to the former two respectively, namely each to that which has the same extremity with it.

Proposition 8.

If two triangles have the two sides equal to two sides respectively, and have also the base equal to the base, they will also have the angles equal which are contained by the equal straight lines. (SSS)

Proposition 9.

To bisect a given rectilineal angle.

Proposition 10.

To bisect a given finite straight line.

Proposition 11.

To draw a straight line at right angles to a given straight line from a given point on it.

Proposition 12.

To a given infinite straight line, from a given point which is not on it, to draw a perpendicular straight line.

Proposition 13.

If a straight line set up on a straight line make angles, it will make either two right angles or angles equal to two right angles.

Proposition 14.

If with any straight line, and at a point on it, two straight lines not lying on the same side make the adjacent angles equal to two right angles, the two straight lines will be in a straight line with one another.

Proposition 15.

If two straight lines cut one another, they make the vertical angles equal to one another.

Proposition 16.

In any triangle, if one of the sides be produced, the exterior angle is greater than either of the interior and opposite angles.

Proposition 17.

In a triangle two angles taken together in any manner are less than two right angles.

Proposition 18.

In any triangle the greater side subtends the greater angle.

Proposition 19.

In any triangle the greater angle is subtended by the greater side.

Proposition 20.

In any triangle two sides taken together in any manner are greater than the remaining one.

Proposition 21.

If on one of the sides of a triangle, from its extremities, there be constructed two straight lines meeting within the triangle, the straight lines so constructed will be less than the remaining two sides of the triangle, but will contain a greater

angle.

Proposition 22.

Out of three straight lines, which are equal to three given straight lines, to construct a triangle: thus it is necessary that two of the straight lines taken together in any manner should be greater than the remaining one. [I.20]

Proposition 23.

On a given straight line and at a point on it to construct a rectilineal angle equal to a given rectilineal angle.

Proposition 24.

If two triangles have the two sides equal to two sides respectively, but have the one of the angles contained by the equal straight lines greater than the other, they will also have the base greater than the base.

Proposition 25.

If two triangles have the two sides equal to two sides respectively, but have the base greater than the base, they will also have the one of the angles contained by the equal straight lines greater that the other.

Proposition 26.

If two triangles have the two angles equal to two angles respectively, and one side equal to one side, namely, either the side adjoining the equal angles, of that subtending one of the equal angles, they will also have the remaining sides equal to the remaining sides and the remaining angle to the remaining angle. (ASA and AAS)

Proposition 27.

If a straight line falling on two straight lines make the alternate angles equal to one another, the straight lines will be parallel to one another.

Proposition 28.

If a straight line falling on two straight lines make the exterior angle equal to the interior and opposite angle on the same side, or the interior angles on the same side equal to two right angles, the straight lines will be parallel to one another.

Proposition 29.

A straight line falling on parallel straight lines makes the alternate angles equal to one another, the exterior angle equal to the interior and opposite angle, and the interior angles on the same side equal to two right angles.

Proposition 30.

Straight lines parallel to the same straight line are also parallel to one another.

Proposition 31.

Through a given point to draw a straight line parallel to a given straight line.

Proposition 32.

In any triangle, if one of the sides be produced, the exterior angle is equal to the two interior and opposite angles, and the three interior angles of the triangle are equal to two right angles.

Proposition 33.

The straight lines joining equal and parallel straight lines (at the extremities which are) in the same directions (respectively) are themselves also equal and parallel.

Proposition 34.

In parallelogrammic areas the opposite sides and angles are equal to one another, and the diameter bisects the areas.

Proposition 35.

Parallelograms which are on the same base and in the same parallels are equal to one another.

Proposition 36.

Parallelograms which are on equal bases and in the same parallels are equal to one another.

Proposition 37.

Triangles which are on the same base and in the same parallels are equal to one another.

Proposition 38.

Triangles which are on equal bases and in the same parallels are equal to one another.

Proposition 39.

Equal triangles which are on the same base and on the same side are also in the same parallels.

Proposition 40.

Equal triangles which are on equal bases and on the same side are also in the same parallels.

Proposition 41.

If a parallelogram have the same base with a triangle and be in the same parallels, the parallelogram is double of the triangle.

Proposition 42.

To construct, in a given rectilineal angle, a parallelogram equal to a given triangle.

Proposition 43.

In any parallelogram the complements of the parallelograms about the diameter are equal to one another.

Proposition 44.

To a given straight line to apply, in a given rectilineal angle, a parallelogram equal to a given triangle.

Proposition 45.

To construct, in a given rectilineal angle, a parallelogram equal to a given rectilineal figure.

Proposition 46.

On a given straight line to describe a square.

Proposition 47.

In right-angled triangles the square on the side subtending the right angle is equal to the squares on the sides containing the right angle.

Proposition 48.

If in a triangle the square on one of the sides be equal to the squares on the remaining two sides of the triangle, the angle contained by the remaining two sides of the triangle is right.