

Math 460: Comments on Final Exam

There were a lot of subtle points in the exam, and this sheet discusses some of them. I hope they will help you understand some of the things we tried to accomplish in this course. Have a good holiday and 2008.

1. In problem 1, we have that a rectangle is a quadrilateral with four right angles. So what we need to show is that the opposite sides are parallel. We do this by showing that alternate interior angles are equal (or similar observations).
2. Answers: *ii, iii, iv*.
3. There is more than one way to do this, but I would use Props 11 and 45. Using Prop 11, we can construct a right angle at one of the endpoints of the segment AB of length one. Then 45 allows us to construct a parallelogram with the same area as the triangle, where the parallelogram has base 1 and one right angle.
4. This was a tricky one, and most seemed to miss the point. A common answer is to say that one constructs two lines cut by a transversal such that alternate interior angles are equal. It is true that by Prop. 27 we would then know that the lines are parallel, but I don't see how that shows that one **can** construct a system of three lines as just described.

I would use Prop. 11 twice. Notice that Euclid **does** show (there and in other places) that one can construct 90 degree angles. So doing this twice, we get a system of lines with alternate interior angles equal.

Question: if Euclid bothers to show us how to make a right angle, why should we consider it 'obvious' that one can arrange alternate interior angles being equal.

Postulate 5 plays no role in this. It says that the only possibility for parallel lines is that alternate interior angles be equal, but doesn't guarantee that we can create such a system of lines.

5. This is discussed in the course notes. In terms of signed distances the product is to be ± 1 , and if it is -1 we have the situation of Ceva's theorem.

Note: many students took a picture of Menelaus's theorem and, for example, shifted the point C' to the other side of line AB . But for the expression to be -1 , you have to choose the location of the new C' carefully.

Many of you did B without mentioning that A', B', C' could be as in Ceva's theorem.

6. Most of you got this. We take the perpendicular line and compare it to any other line from P to ℓ . We then get a right triangle, and by the Pythagorean theorem, the hypotenuse is the longest side, so certainly longer than the perpendicular line segment.

Euclid does this a few places, for example Proposition 11.

7. See notes from class. You need to use Prop 16 or maybe 17. If you used the fifth postulate, you lost about 15 of the 20 points. *This was something we discussed very frequently, and were quizzed on it more than once.* If you did not understand this, think about it; you should be clear on this when you leave this course.

8. This was also the subject of a lot of class talk. Let $P \in \ell_2$ be where ℓ intersects ℓ_2 . What you have to prove boils down to that only one line goes through P parallel to ℓ_3 , and that requires some work, and is no means an obvious consequence of the fifth postulate. If there is only one line through P parallel to ℓ_3 , then it has to be ℓ_2 , and so ℓ has to meet ℓ_3 .

But if we take a transversal ℓ^* from P through ℓ_3 , we can use Prop. 29 to show that any parallel to ℓ_3 through P would have to have an angle with ℓ^* equal to one of the angles that ℓ^* makes when meeting ℓ_3 .

9. Proof by contradiction; this was a homework problem and done in class. If A, B, C are both on the circle and ℓ , we show that lines OA, OB, OC are parallel, since we would have alternate interior angles being 90° . Then these lines could not meet at the center of the circle.
10. I allowed you to use a Theorem from the notes (Theorem 40?), but it was also routine via similar triangles.
11. This is in the notes.