Math 460: Homework # 11. Due Tuesday November 27

1. (Use Geometer's Sketchpad.) Construct a triangle ABC and its incenter I. Let ℓ, m and n be the perpendiculars from I to the three sides AB, AC and BC respectively. Let D be the intersection of ℓ with AB, let E be the intersecton of m with AC, and let F be the intersection of n with BC. (D, E and F are called the "feet" of the perpendiculars from I to the three sides.) Now hide the lines ℓ, m and n. Finally, draw in the lines AF, BE and CD. What do you notice about these three lines?

2. (In this problem we prove the fact that you discovered in Problem 1 of Assignment 10.) In Figure 1, prove that $\angle 1 = \frac{1}{2}\text{arc } DEF - \frac{1}{2}\text{arc } ABC$.

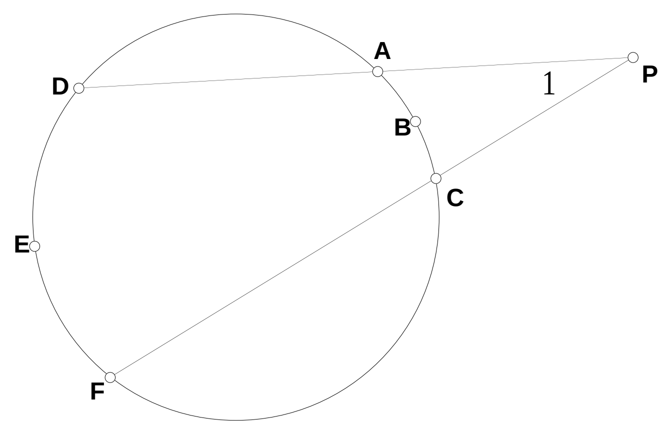

Figure 1

3. (In this problem we prove the fact that you discovered in Problem 2 of Assignment 10.) See Figure 2. Given: nothing. To prove:

$$\frac{BC}{BE}\frac{DE}{CD} = \frac{\sin(\angle BAC)\sin(\angle DAE)}{\sin(\angle BAE)\sin(\angle CAD)}$$

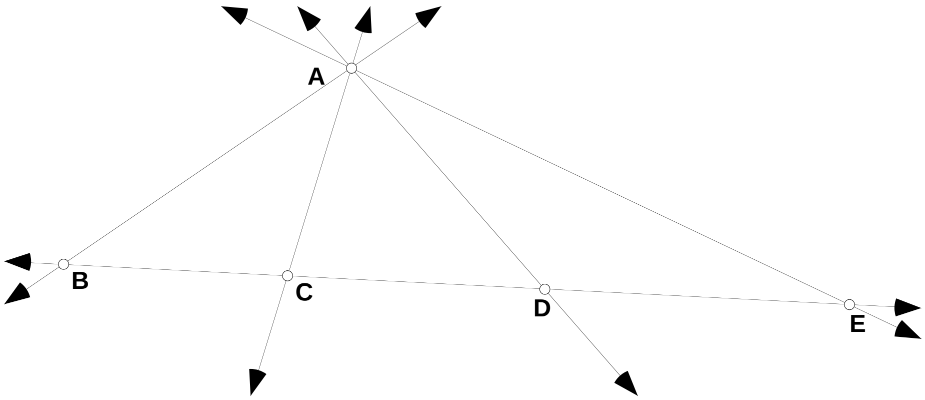

Figure 2

4. (In this problem we finish—at last—the proof of the fact that you discovered in Problem 2 of Assignment 6.) See Figure 3. Given: G is the centroid of $\triangle ABC$ and U, V, W, X, Y and Z are the centroids of the six "little triangles." To prove: the area of hexagon $UVWXYZ$ is exactly 13/36 of the area of $\triangle ABC$. (This is pretty easy if you combine facts that you proved in earlier homework.)

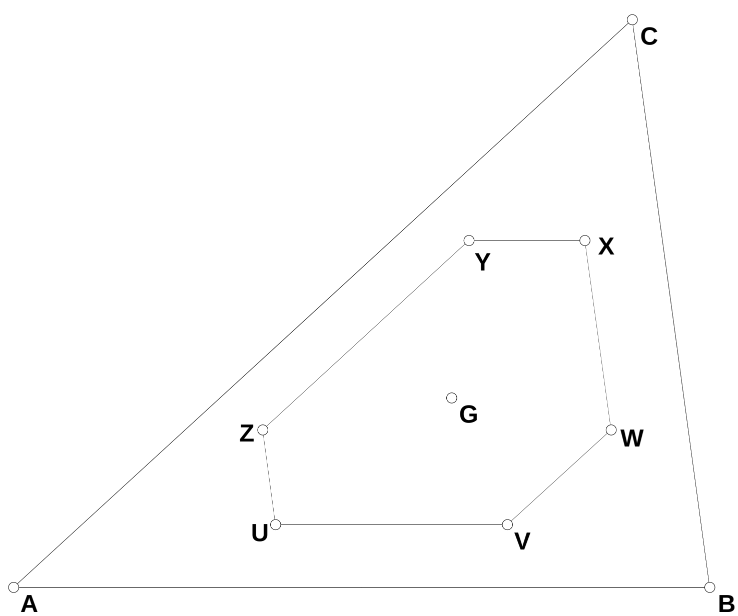

Figure 3

5. Euclid's proof of Proposition 23 uses Proposition 8 as one ingredient. Find a new proof of Proposition 23 that doesn't use Proposition 8. For this purpose you are

allowed to use anything that comes before Proposition 8, and also Proposition 11, since we proved Proposition 11 without using Proposition 8.

6. The course notes has a proof of the $\Longleftarrow$ direction of Theorem 41, using the Pythagorean Theorem. Give a new proof, using only the definition of tangent and facts from Euclid (but not the Pythagorean Theorem). Hint: proof by contradiction.

7. Suppose we are given a circle $\mathcal{C}$ with center O and a line l. Prove that l cannot have three points that are on $\mathcal{C}$. (Hint: use proof by contradiction.)

8. See Figure 4. Given: PC is tangent to the circle. To prove: $PA \cdot PB = PC^2$. (Hint: the proof is similar to one of the cases of Theorem 40. But Theorem 40 itself does not apply to this problem.)

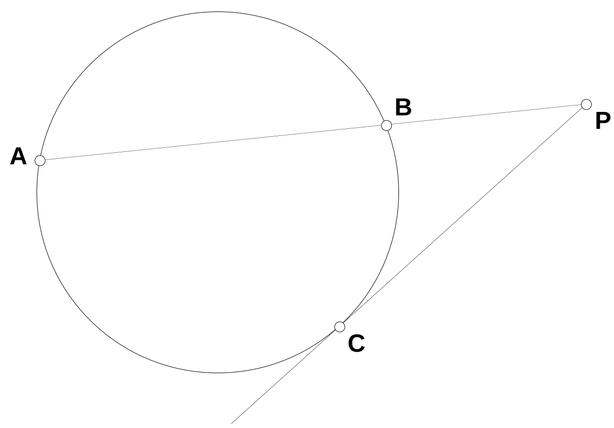

Figure 4

9. See Figure 5. Given: the lines that look straight are straight. To prove: BC is parallel to FG.

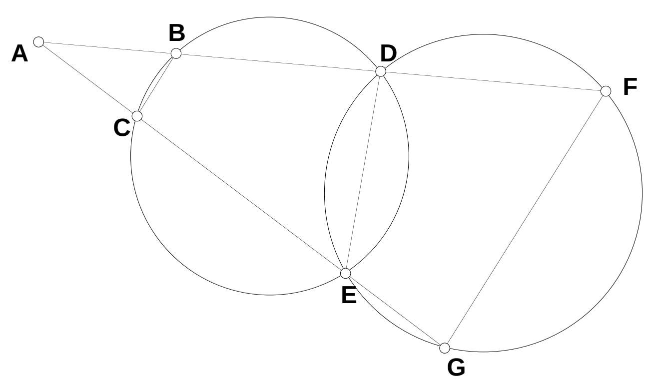

Figure 5

10. See Figure 6. Given: A, B, P and Q are in the indicated plane, RPS and AQB are straight lines, $RS \perp PA$, $RS \perp PB$, $SP = RP$. To prove: $RS \perp PQ$.

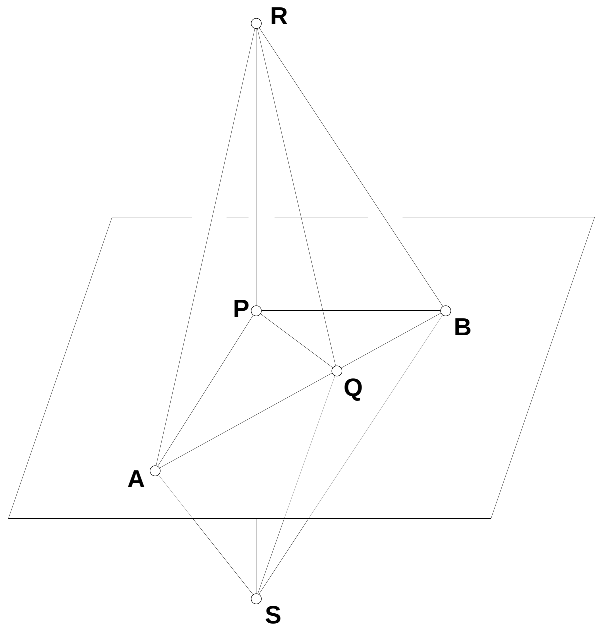

Figure 6

Note: by doing this problem you will be proving a theorem in 3-dimensional geometry:

Theorem. Let Π be a plane, let P be a point on the plane Π, and let ℓ be any line through P. If ℓ is perpendicular to two different lines through P that lie in the plane Π, then ℓ is perpendicular to *every* line through P that lies in the plane Π.