Math 460: Homework # 4. Due September 24

Note: Problems 1, 2 and 4 all use Geometer's Sketchpad.

1. (Use Geometer's Sketchpad.) Construct a quadrilateral $ABCD$ and let M, N, P and Q be the midpoints of the sides. Find the areas of the quadrilaterals $ABCD$, $MNPQ$ and of the four small triangles at the corners of $ABCD$ (use "Polygon interior" from the "Construct" menu and then "Area" from the "Measure" menu). Find an equation relating the areas of the four triangles. Then find an equation relating the areas of the two quadrilaterals. Print out a picture with the calculations that demonstrate the equations you found. You do not have to prove anything for this problem.

2. (Use Geometer's Sketchpad.) Draw a triangle ABC and a point P in the interior of ABC. Draw the lines (not just the line segments) connecting P to each of the vertices, and let D, E, and F be the points where these lines meet AB, AC and BC, respectively. Find an equation relating the three ratios $\frac{DA}{DB}$, $\frac{EC}{EA}$ and $\frac{FB}{FC}$. Print out a copy of your picture, including the measurements and calculations.

3. (See Figure 1.) Give the proof of Theorem 22 for Case (iii). Given: M and N are the midpoints of AB and AC, $MX \perp AB$, $NX \perp AC$, and X is on BC. To prove: X is on the perpendicular bisector of BC.

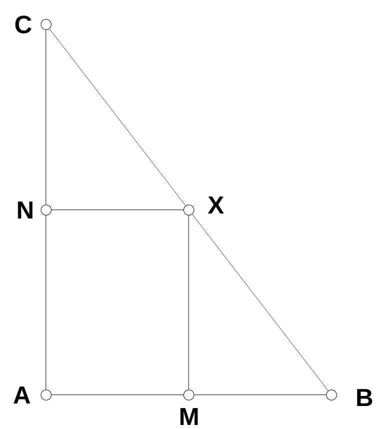

Figure 1

4. Prove Theorem 23. (Hint: Use ideas similar to those of the proof of Theorem 22, but note that there is only one case.) Make a picture to illustrate your proof with Geometer's Sketchpad.

5. Let ABC be an equilateral triangle and let P be a point which is outside the triangle but in the interior of $\angle ABC$. Let a, b, and c be the distances from P to $\overleftrightarrow{AB}$, $\overleftrightarrow{AC}$ and $\overleftrightarrow{BC}$ respectively. Let h be the height of triangle ABC. To prove: $a - b + c = h$.

6. (See Figure 2.) Given: $AD = BC$, $AC = BD$, $AK = BN$. To prove: $KG = NH$.

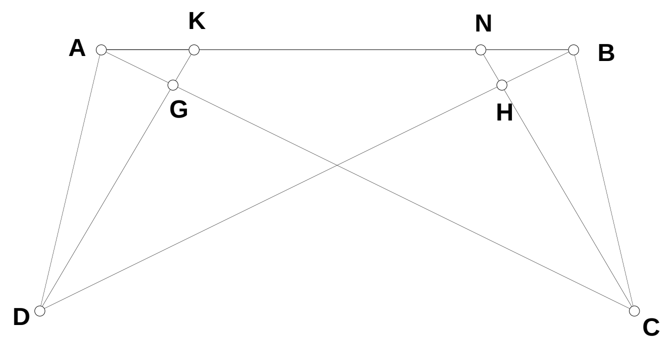

Figure 2

7. (See Figure 3.) Given: The lines that look straight are straight, $\angle D = \angle 1$, $KM = TM = CM$. To prove: $AD = BC$.

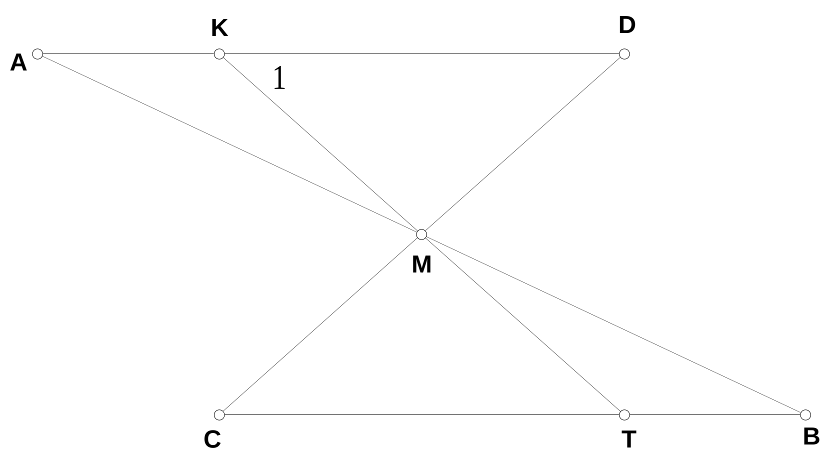

Figure 3

8. (See Figure 4.) Given: $ABCD$ is a parallelogram, $\angle 1 = \angle 2$, $\angle 3 = \angle 4$, EF is parallel to AD. To prove: $AF = FB$.

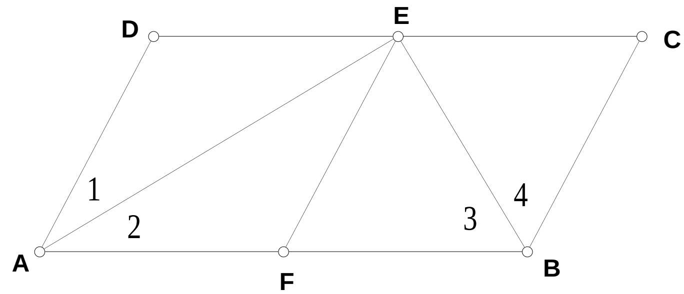

Figure 4

9. (See Figure 5.) Given: O is the center of the circle. To prove: $\angle AOC = 2\angle ABC$.

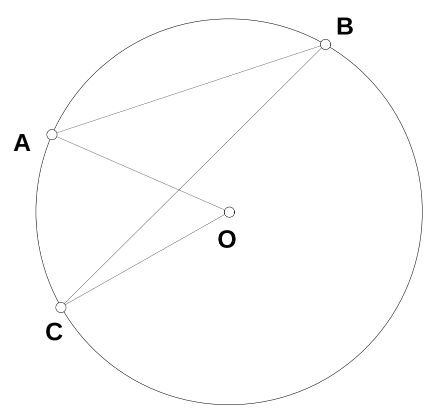

Figure 5

10. (See Figure 6.) Given that AD is parallel to BE and BE is parallel to CF, prove that $\dfrac{AB}{AC} = \dfrac{DE}{DF}$.

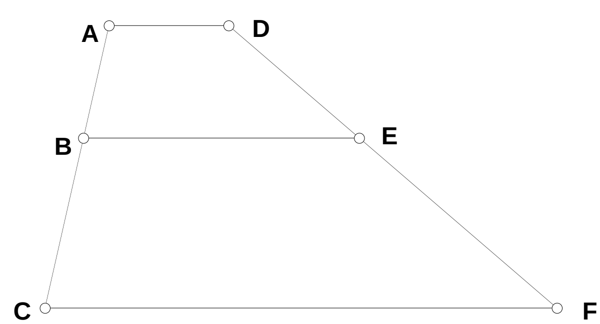

Figure 6