Math 460: Homework # 9. Due November 8.

1. (Use Geometer's Sketchpad.) In Figure 1, find an equation relating $\angle 1$, arc ABC and arc DEF.

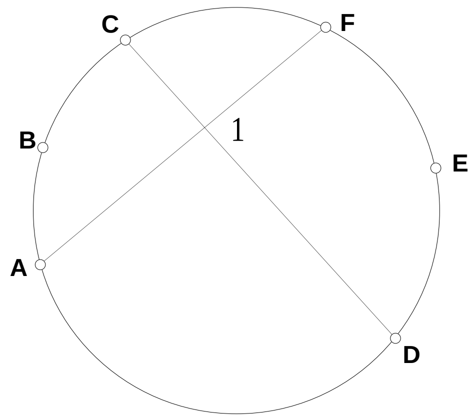

Figure 1

2. (Use Geometer's Sketchpad.) For most quadrilaterals, the four angle bisectors are not concurrent.

   (a) Find an equation that the sides of the quadrilateral have to satisfy if the four angle bisectors are concurrent.

   (b) Make a quadrilateral in which all four sides have different lengths and the four angle bisectors are concurrent. Verify that the equation you found in (a) holds for this example.

3. (In this problem we prove the fact that you discovered in Problem 1 of Assignment 8.) Given a triangle ABC with orthocenter H, prove that C is the orthocenter of ABH.

4. (In this problem we begin to prove the fact that you discovered in Problem 1 of Assignment 7.) See Figure 2. Given: the lines that look straight are straight. To prove:
$$\frac{XB}{XC}\frac{YC}{YA}\frac{ZA}{ZB} = 1$$
(Hint: apply Theorem 28 to $\triangle ABC$ five times. Do not draw in any extra lines.)

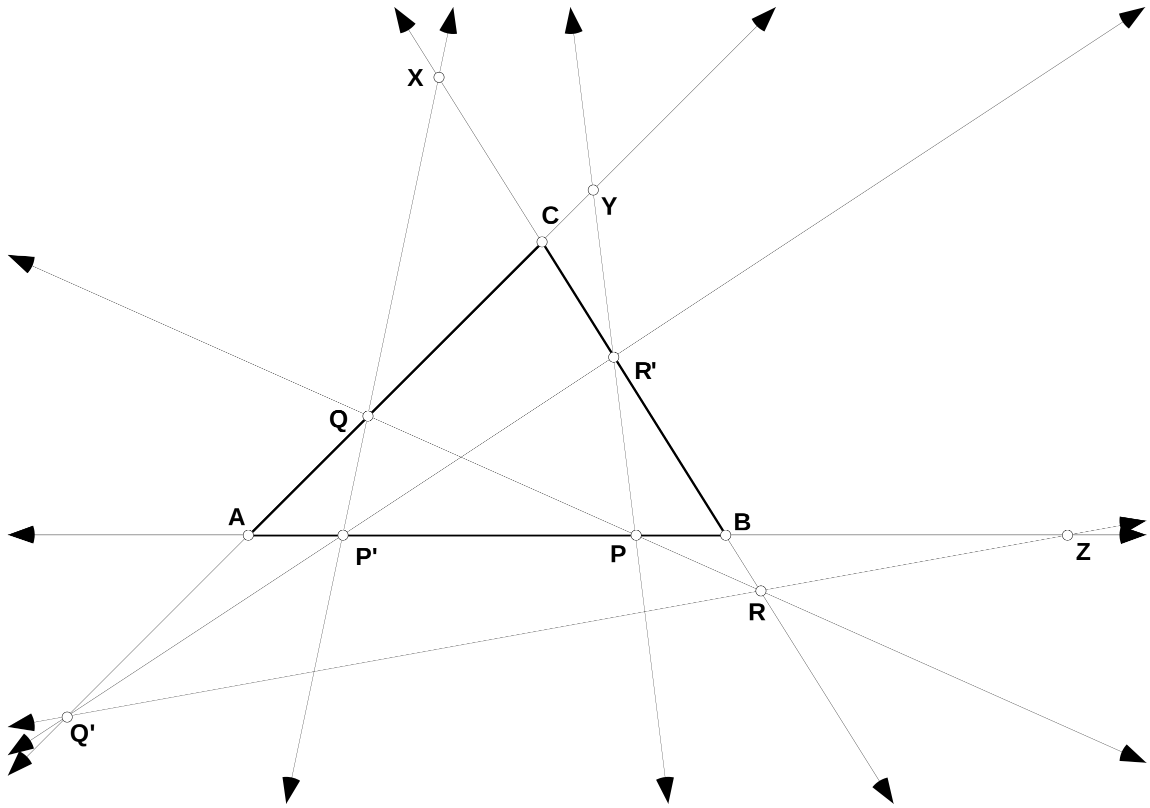

Figure 2

5. Prove the case of Theorem 40 which is illustrated in Figure 41 in the course notes. (Hint: use Theorem 39 as one ingredient.)

6. (In this problem we continue to prove what you discovered in Problem 2 of Assignment 6.) See Figure 3. Given: G is the centroid of $\triangle ABC$ and U, V, W, X, Y and Z are the centroids of the six "little triangles." To prove: the area of $\triangle DEF$ is 4/9 of the area of $\triangle ABC$. (You may use anything that you have already proved about this picture in Problem 4 of Assignment 7 and Problem 4 of Assignment 8. In particular, the lines in Figure 3 that look concurrent are concurrent.)

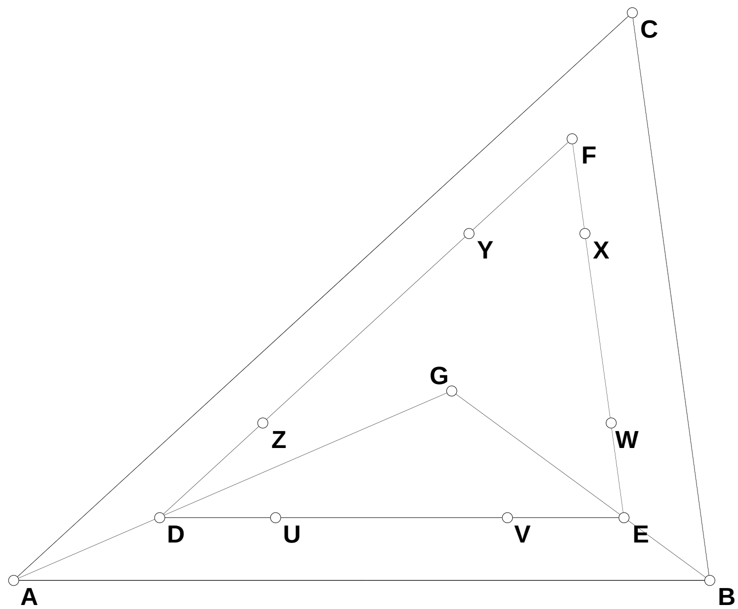

Figure 3

7. Euclid's proof of Proposition 9 uses Proposition 8 as one ingredient. Find a proof of Proposition 9 which uses only facts from Euclid that come *before* Proposition 6 (and nothing from the course notes).

8. (See Figure 4.) Given: AB is parallel to DE, AC is parallel to DF, and BC is parallel to EF. To prove: the lines AD, BE and CF are concurrent. (Hint: use a proof similar to Theorem 26.)

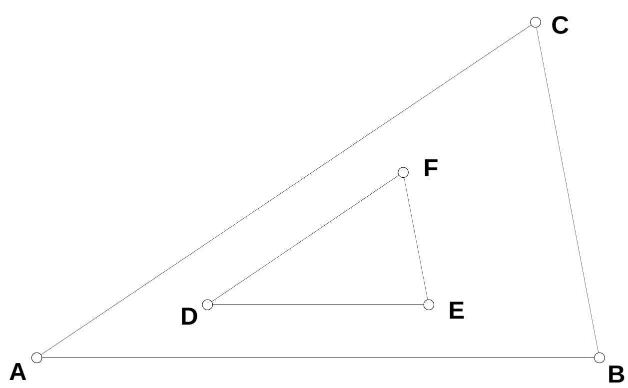

Figure 4

9. (See Figure 5.) Given: Angles ABC and AHG are right angles, and $ABDE$ and $ACFG$ are squares. To prove: $BC = 2HI$.

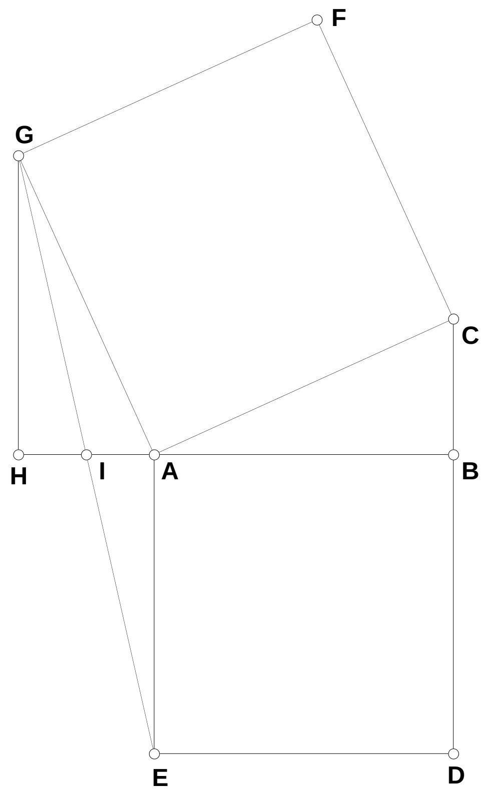

Figure 5

10. (This is a problem about three-dimensional geometry. For problems like this you are allowed to use anything in the course notes up to Theorem 5 except BF 5 and Theorem 2.) See Figure 6. Given: $\angle PMN = \angle PNM$ and $\angle MPQ = \angle NPQ$. To prove: $\angle PMQ = \angle PNQ$..

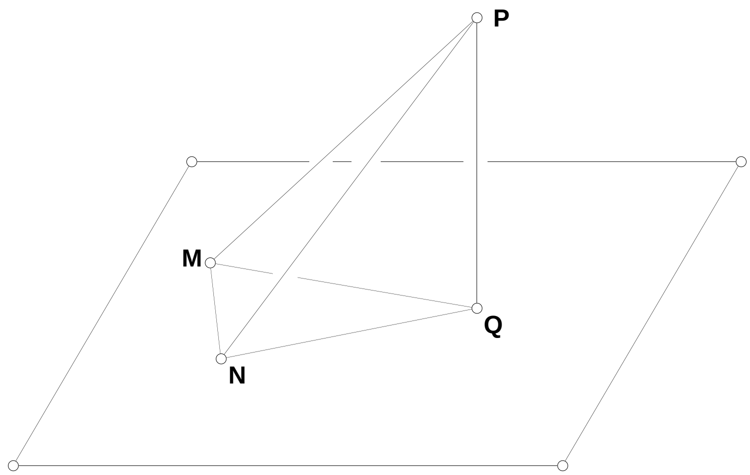

Figure 6