

1. If  $\vec{a} = \vec{i} + \vec{j} + \vec{k}$  and  $\vec{b} = 2\vec{i} - \vec{k}$ , find the vector projection of  $\vec{b}$  onto  $\vec{a}$ ,  $\text{proj}_{\vec{a}} \vec{b}$ .  
 A.  $\frac{1}{3}\vec{a}$       B.  $\frac{1}{\sqrt{3}}\vec{a}$       C.  $\frac{1}{\sqrt{5}}\vec{a}$       D.  $\frac{1}{\sqrt{3}}\vec{b}$       E.  $\frac{1}{3}\vec{b}$
2. Find the angle between the vectors  $\vec{a} = -\vec{i} + 2\vec{j}$  and  $\vec{b} = \vec{i} + 3\vec{j}$ .  
 A.  $\frac{3\pi}{4}$       B.  $\frac{\pi}{4}$       C.  $\frac{2\pi}{3}$       D.  $\frac{5\pi}{6}$       E.  $\frac{11\pi}{12}$
3. Find the area of the triangle with vertices at the points  $(1, 0, 2)$ ,  $(2, 4, -3)$  and  $(1, 2, 1)$ .  
 A.  $\frac{1}{2}\sqrt{41}$       B.  $\sqrt{41}$       C.  $\sqrt{10}$       D.  $\frac{\sqrt{2}}{2}\sqrt{21}$       E.  $\frac{41}{2}$
4. If  $\vec{a} = \vec{i} - \vec{j}$  and  $\vec{b} = 2\vec{j} - \vec{k}$ , find a unit vector orthogonal to both  $\vec{a}$  and  $\vec{b}$ .  
 A.  $\frac{1}{\sqrt{2}}(\vec{i} + \vec{j})$       B.  $\frac{1}{\sqrt{6}}(\vec{i} + \vec{j} + 2\vec{k})$       C.  $\frac{1}{\sqrt{5}}(\vec{j} + 2\vec{k})$       D.  $\vec{i} + \vec{k}$       E.  $\frac{1}{\sqrt{5}}(\vec{i} + 2\vec{k})$
5. The radius of the sphere  $x^2 + y^2 + z^2 + 2x + 4y - 6z = 3$  is  
 A.  $3 + \sqrt{13}$       B.  $\sqrt{13}$       C.  $\sqrt{65}$       D.  $3 + \sqrt{56}$       E.  $\sqrt{17}$
6. The area of the region enclosed by the curves  $y = x^2 + 1$  and  $y = 2x + 9$  is given by  
 A.  $\int_{-2}^4 (x^2 + 1 - 2x - 9)dx$       B.  $\int_{-2}^4 (2x + 9 - x^2 - 1)dx$       C.  $\int_{-2}^2 (2x + 9 - x^2 - 1)dx$   
 D.  $\int_{-4}^2 (2x + 9 - x^2 - 1)dx$       E.  $\int_{-4}^2 (x^2 + 1 - 2x - 9)dx$
7. The volume of the solid obtained by rotating about the  $x$ -axis the region in the first quadrant bounded by the graphs of  $y = 1 - x^2$ ,  $y = 2x$ , and  $x = 0$  is given by  
 A.  $\int_0^{\sqrt{2}-1} (1 - x^2 - 2x)dx$       B.  $\int_0^{\sqrt{2}-1} \pi(1 - x^2 + 2x)dx$       C.  $\int_{-\sqrt{2}-1}^0 \pi[(1 - x^2)^2 - (2x)^2]dx$   
 D.  $\int_0^{\sqrt{2}-1} \pi[(1 - x^2)^2 - (2x)^2]dx$       E.  $\int_0^{\sqrt{2}-1} [(2x)^2 - (1 + x^2)^2]dx$
8. Let  $R$  be the region bounded by the curves  $y = x^2$  and  $y = 2x$ . Using the method of cylindrical shells, the volume of the solid generated by rotating  $R$  about the  $x$ -axis, is given by  
 A.  $\int_0^2 \pi(2x - x^2)dx$       B.  $\int_0^2 2\pi(2x - x^2)^2 dx$       C.  $\int_0^2 \pi x^2(x^2 - \frac{1}{2}x)dy$       D.  $\int_0^4 \pi y^2(\frac{1}{2}y - \sqrt{y})dy$   
 E.  $\int_0^4 2\pi y(\sqrt{y} - \frac{1}{2}y)dy$
9. A right circular conical tank of height 20 ft. and base radius 5 ft. has its vertex at the bottom, and its axis vertical. If the tank is full of water at 62.5 lb./cu. ft., the work required to pump all the water over the top is: (Take the  $y$ -axis upwards along the axis of the tank and the origin at its vertex).  
 A.  $62.5\pi \int_0^{20} (20 - y)(\frac{y}{4})^2 dy$       B.  $62.5\pi \int_0^{2\pi} y(\frac{y}{4})^2 dy$       C.  $62.5\pi \int_0^{20} (20 - y)^2(\frac{y}{4}) dy$   
 D.  $62.5\pi \int_0^{20} (20 - y)(\frac{y}{2})^2 dy$       E.  $62.5\pi \int_0^{20} (20 - y)(2y)^2 dy$
10. A force of 9 lb. is required to stretch a spring from its natural length of 6 in. to a length of 8 in. Find the work required to stretch it from its natural length to 10 in.  
 A. 1 ft.-lb.      B. 1.5 ft.-lb.      C. 2 ft.-lb.      D. 3 ft.-lb.      E. 4 ft.-lb.
11.  $\int_0^1 xe^{3x} dx =$   
 A.  $\frac{2}{9}e^3$       B.  $\frac{1}{9} + \frac{2}{9}e^3$       C. 1      D.  $\frac{1}{9}$       E.  $\frac{1}{9}e^3 - 1$

12.  $\int_0^{\pi/2} \cos^3 x dx =$

- A.  $\frac{\pi}{2} - \frac{1}{3}$       B.  $\frac{\pi}{2} + \frac{1}{3}$       C. 0      D.  $-\frac{2}{3}$       E.  $\frac{2}{3}$

13. For the integral  $\int (1-x^2)^{3/2} dx$ , (i) choose a trigonometric substitution to simplify the integral and (ii) give the resulting integral

- A. (i)  $x = \sec \theta$ , (ii)  $\int \tan^3 \theta d\theta$       B. (i)  $x = \sec \theta$ , (ii)  $\int \tan^4 \theta \sec \theta d\theta$   
 C. (i)  $x = \sec \theta$ , (ii)  $\int \tan^3 \theta \sec^2 \theta d\theta$       D. (i)  $x = \sin \theta$ , (ii)  $\int \cos^3 \theta d\theta$       E. (i)  $x = \sin \theta$ , (ii)  $\int \cos^4 \theta d\theta$

14. Give the form of the partial fraction decomposition of  $\frac{3x+2}{(x^2+1)(x-1)^2}$ .

- A.  $\frac{A}{x^2+1} + \frac{B}{(x-1)^2}$       B.  $\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-1} + \frac{D}{(x-1)^2}$       C.  $\frac{Ax+B}{x^2+1} + \frac{C}{x-1} + \frac{D}{(x-1)^2}$   
 D.  $\frac{A}{x^2+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$       E.  $\frac{Ax+B}{x^2+1} + \frac{C}{(x-1)^2}$

15.  $\int_1^2 \frac{1}{x^3+x} dx =$

- A.  $\ln 2 + \ln \frac{2}{5}$       B.  $\ln 2 + \frac{1}{2} \ln \frac{2}{5}$       C.  $\ln 2 + \tan^{-1} 5 - \tan^{-1} 2$       D.  $2 + \frac{1}{2} \tan^{-1} 5 - \frac{1}{2} \tan^{-1} 2$       E.  $\ln \frac{2}{5}$

16. Indicate convergence or divergence for each of the following improper integrals:

(I)  $\int_2^{\infty} \frac{1}{(x-1)^2} dx$       (II)  $\int_0^2 \frac{1}{(x-1)^2} dx$

- A. (I) converges; (II) diverges      B. (I) converges; (II) converges  
 C. (I) diverges; (II) converges      D. (I) diverges; (II) diverges

17. The length of the graph of  $y = x^{3/2}$  for  $0 \leq x \leq 1$  is

- A.  $\frac{2}{3} \sqrt{\frac{5}{2}}$       B.  $\frac{4}{27} \sqrt{13}$       C.  $\frac{4}{9} \sqrt{\frac{5}{2}}$       D.  $\frac{8}{27} \left( \left( \frac{13}{4} \right)^{3/2} - 1 \right)$       E.  $\frac{4}{9} \left( \sqrt{\frac{5}{2}} - 1 \right)$

18. If  $R$  is the semicircular region bounded by the  $x$  axis and  $y = \sqrt{4-x^2}$ ,  $-2 < x < 2$ , the centroid  $(\bar{x}, \bar{y})$  of  $R$  is

- A.  $(0, \frac{8}{3\pi})$       B.  $(\frac{8}{3\pi}, 0)$       C.  $(0, 1)$       D.  $(1, 0)$       E.  $(0, 0)$

19. Evaluate the limit  $\lim_{n \rightarrow \infty} \left[ 1 + \frac{(-1)^n}{n} \right]$ .

- A. 0      B. 1      C. -1      D. 2      E. limit does not exist

20. Evaluate the limit  $\lim_{n \rightarrow \infty} \left( \sqrt[n]{n} + \frac{1}{n!} \right)$ .

- A. 0      B. 1      C.  $e$       D.  $1/e$       E. limit does not exist

21. If  $s = \sum_{n=0}^{\infty} \frac{2^{n+1}}{3^n}$ , then  $s =$

- A. 3      B. 6      C. 9      D. 2      E.  $4/3$

22. If  $L = \sum_{n=1}^{\infty} \frac{1}{2^n} + \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n}$ , then  $L =$

- A.  $1/3$       B.  $2/3$       C. 1      D.  $4/3$       E.  $5/3$

23. Find all values of  $p$  for which  $\sum_{n=1}^{\infty} \frac{1}{(n^2+1)^p}$  converges.

- A.  $p > 1$       B.  $p \leq 1$       C.  $p \geq 1$       D.  $p > \frac{1}{2}$       E.  $p \leq \frac{1}{2}$

24.  $\sum_{n=1}^{\infty} \left( 1 + \frac{1}{n} \right)^p$  converges for

- A.  $p \leq 1$       B.  $p > 1$       C.  $p < 0$       D.  $p > 0$       E. no values of  $p$

25. Which of the following series converge conditionally?

$$(i) \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \quad (ii) \sum_{n=2}^{\infty} \frac{(-1)^n n}{\ln n} \quad (iii) \sum_{n=1}^{\infty} \frac{(-1)^n n}{e^n}$$

A. only (ii)    B. only (i) and (iii)    C. only (i) and (ii)    D. all three    E. none of them

26. Which of the following series converge?

$$(i) \sum_{n=1}^{\infty} \frac{(-1)^n}{n^4} \quad (ii) \sum_{n=1}^{\infty} \frac{n!}{1 \cdot 3 \cdot 5 \cdots (2n-1)} \quad (iii) \sum_{n=1}^{\infty} \frac{4}{3} \left(\frac{1}{2}\right)^n$$

A. only (ii)    B. only (i) and (iii)    C. only (i) and (ii)    D. all three    E. none of them

27. The interval of convergence for the power series  $\sum_{n=2}^{\infty} \frac{3^n x^n}{n \ln n}$  is

A.  $-\frac{1}{3} \leq x < \frac{1}{3}$     B.  $-\frac{1}{3} < x \leq \frac{1}{3}$     C.  $0 \leq x \leq \frac{1}{3}$     D.  $-1 \leq x \leq 2$     E.  $-1 < x < 1$

28. Find the interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{nx^n}{2^n}$$

A.  $-\frac{1}{2} < x < \frac{1}{2}$     B.  $-2 < x < 2$     C.  $-2 \leq x \leq 2$     D.  $-2 < x \leq 2$     E.  $-\infty < x < \infty$

29. The fourth term of the Maclaurin series for  $\frac{x^2+3}{x-1}$  is

A.  $-x^3$     B.  $3x^3$     C.  $-3x^3$     D.  $-4x^3$     E.  $4x^3$

30. The first three nonzero terms of the Maclaurin series for  $f(x) = (1-x^2)\sin x$  are

A.  $x - \frac{5}{6}x^3 + \frac{31}{150}x^5$     B.  $1 - \frac{3}{2}x^2 + \frac{13}{24}x^4$     C.  $x - \frac{7}{6}x^3 + \frac{31}{150}x^5$     D.  $x^2 - \frac{7}{6}x^3 + \frac{1}{25}x^5$     E.  $x - \frac{7}{6}x^3 + \frac{21}{120}x^5$

31. The fourth term of the Taylor series for  $f(x) = \ln x$  centered at  $a = 2$  is

A.  $\frac{1}{6}(x-2)^3$     B.  $\frac{1}{12}(x-2)^3$     C.  $\frac{1}{24}(x-2)^3$     D.  $-\frac{1}{3}(x-2)^3$     E.  $-(x-2)^3$

32. Using Maclaurin series and the Estimation Theorem for alternating series, we can obtain the approximation

$$\int_0^{0.1} e^{-x^2} dx \approx 0.1 - \frac{1}{3000}, \text{ with error } \leq c$$

The value of  $c$  is

A.  $10^{-5}$     B.  $10^{-6}$     C.  $\frac{1}{2} 10^{-6}$     D.  $\frac{10^{-7}}{7}$     E.  $\frac{10^{-5}}{2}$

33. The parametric equations of a curve  $C$  are

$$x = 2 \cos t, \quad y = 3 \sin t \quad 0 \leq t \leq \frac{\pi}{2}$$

The curve  $C$  is

A. a quarter of a circle    B. an ellipse    C. a half of an ellipse    D. a half of a circle    E. a quarter of an ellipse

34. The length of the parametric curve

$$x = \frac{1}{2}t^2, \quad y = 2 + \frac{1}{3}t^3, \quad 0 \leq t \leq \sqrt{3}$$

is

A.  $\frac{21}{4}$     B.  $\frac{7}{2}$     C.  $\frac{7}{3}$     D.  $\frac{14}{3}$     E.  $\frac{8}{3}$

35. A point  $P$  has polar coordinates  $(2, \frac{2\pi}{3})$ . The Cartesian coordinates of  $P$  are

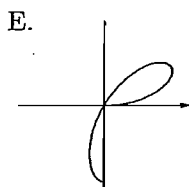
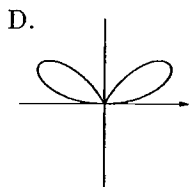
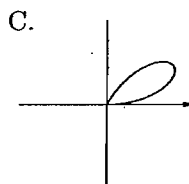
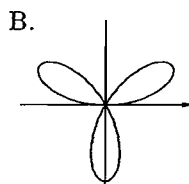
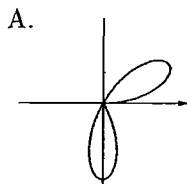
A.  $(-1, \sqrt{3})$     B.  $(-2, \sqrt{3})$     C.  $(1, \frac{\sqrt{3}}{2})$     D.  $(-1, \frac{1}{2})$     E.  $(1, -\frac{1}{2})$

36. A point  $P$  has polar coordinates  $(3, \frac{\pi}{4})$ . Which of the following are also polar coordinates of  $P$ ?

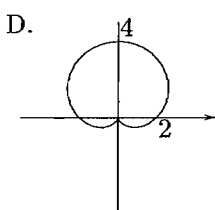
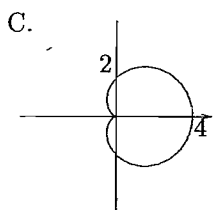
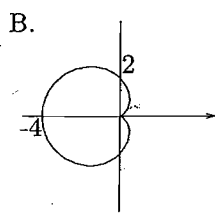
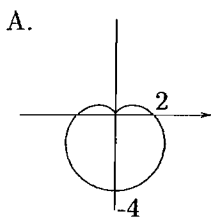
I.  $(-3, -\frac{\pi}{4})$  II.  $(-3, \frac{5\pi}{4})$  III.  $(3, -\frac{7\pi}{4})$  IV.  $(3, -\frac{5\pi}{4})$

A. I and II only B. I and III only C. I and IV only D. II and III only E. II and IV only

37. Which of the following looks most like the graph of the polar curve  $r = \sin(3\theta)$ , for  $0 \leq \theta \leq \frac{\pi}{2}$ ?



38. Which of the following looks most like the graph of the polar curve  $r = 2 \cos \theta - 2$ ?



39. The rectangular coordinates of the center  $C$  and radius  $a$  of the circle  $r = -2 \sin \theta$  are

A.  $C(0, -\frac{1}{2}), a = 1$  B.  $C(0, 1), a = 2$  C.  $C(-1, 0), a = 2$  D.  $C(0, -1), a = 1$  E.  $C(-\frac{1}{2}, 0), a = 1$

40. Write the complex number  $1 - \sqrt{3}i$  in polar form with argument between  $0$  and  $2\pi$ .

A.  $4(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})$  B.  $2(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3})$  C.  $4(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4})$   
 D.  $2(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3})$  E.  $2(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6})$

Answers: 1-A, 2-B, 3-A, 4-B, 5-E, 6-B, 7-D, 8-E, 9-A, 10-D, 11-B, 12-E, 13-E, 14-C, 15-B, 16-A, 17-D, 18-A, 19-B, 20-B, 21-B, 22-E, 23-D, 24-E, 25-E, 26-D, 27-A, 28-B, 29-D, 30-E, 31-C, 32-B, 33-E, 34-C, 35-A, 36-D, 37-E, 38-C, 39-D, 40-D