CHAPTER 0
A Precalculus Review

C H A P T E R 0
A Precalculus Review

Section 0.1 The Real Number Line and Order

Solutions to Even-Numbered Exercises

2. Since $-3678 = -\frac{3678}{1}$, it is rational.

4. $3\sqrt{2} - 1$ is irrational because $\sqrt{2}$ is irrational.

6. $\frac{22}{7}$ is rational.

8. $0.8177\overline{8177}$ is rational since it has a repeating decimal expansion.

10. $2e$ is irrational since e is irrational.

12. $x + 1 < \dfrac{2x}{3}$

$3x + 3 < 2x$

$x + 3 < 0$

$x < -3$

(a) No, if $x = 0$, then x is not less than -3.

(b) No, if $x = 4$, then x is not less than -3.

(c) Yes, if $x = -4$, then x is less than -3.

(d) No, if $x = -3$, then x is not less than -3.

14. $-1 < \dfrac{3 - x}{2} \le 1$

$-2 < 3 - x \le 2$

$-5 < -x \le -1$

$5 > x \ge 1$ or $1 \le x < 5$

(a) No, if $x = 0$, then x is not greater than or equal to 1.

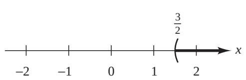
(b) Yes, if $x = \sqrt{5}$, then $1 \le x < 5$.

(c) Yes, if $x = 1$, then $1 \le x < 5$.

(d) No, if $x = 5$, then x is not less than 5.

16. $2x > 3$

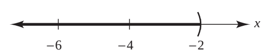
$x > \frac{3}{2}$



18. $2x + 7 < 3$

$2x < -4$

$x < -2$

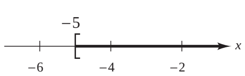


20. $x - 4 \le 2x + 1$

$-x - 4 \le 1$

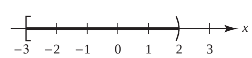
$-x \le 5$

$x \ge -5$



22. $0 \le x + 3 < 5$

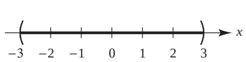
$-3 \le x < 2$



24. $-1 < -\dfrac{x}{3} < 1$

$-3 < -x < 3$

$3 > x > -3$

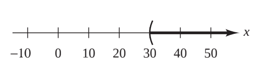


26. $\dfrac{x}{2} - \dfrac{x}{3} > 5$

$6\left(\dfrac{x}{2} - \dfrac{x}{3}\right) > 6(5)$

$3x - 2x > 30$

$x > 30$

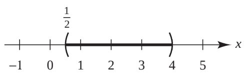


28. $2x^2 + 1 < 9x - 3$

$2x^2 - 9x + 4 < 0$

$(2x - 1)(x - 4) < 0$

Therefore, the solution is $\frac{1}{2} < x < 4$.



30. $A = 20$ and $r = 220 - A = 200$.
Let T be the target heart rate. Then

$$(0.60)(200) \le T \le (0.90)(200)$$

$$120 \le T \le 180$$

32. Revenue: $R = 3.50x$

Cost: $C = 1.75x + 170$

Profit: $P = R - C$

$= 3.50x - (1.75x + 170) = 1.75x - 170$

Thus, $40 \le 1.75x - 170 \le 250$

$210 \le 1.75x \le 420$

$120 \le x \le 240$ dozen doughnuts per day.

34. Let $x =$ length of the side of the square. Then, the area of the square is x^2, and we have

$x^2 \ge 500$

$x \ge \sqrt{500}$

$x \ge 10\sqrt{5} \approx 22.36$ meters.

36. (a) True. Since $a < b$, $a - 4 < b - 4$.

(b) False. Since $a < b$, $-a > -b$ and $4 - a > 4 - b$.

(c) True. Since $a < b$, $-3a > -3b$.

(d) True. Since $a < b$, $\frac{1}{4}a < \frac{1}{4}b$.

Section 0.2 Absolute Value and Distance on the Real Number Line

2. (a) The directed distance from a to b is $-75 - (-126) = 51$.

(b) The directed distance from b to a is $-126 - (-75) = -51$.

(c) The distance between a and b is $|-75 - (-126)| = 51$.

4. (a) The directed distance from a to b is $4.25 - (-2.05) = 6.3$.

(b) The directed distance from b to a is $-2.05 - 4.25 = -6.3$.

(c) The distance between a and b is $|4.25 - (-2.05)| = 6.3$.

6. (a) The directed distance from a to b is $\frac{61}{15} - \left(\frac{-18}{5}\right) = \frac{61}{15} + \frac{54}{15} = \frac{115}{15} = \frac{23}{3}$.

(b) The directed distance from b to a is $\frac{-18}{5} - \frac{61}{15} = \frac{-23}{3}$.

(c) The distance between a and b is $\left|\frac{61}{15} - \left(-\frac{18}{5}\right)\right| = \frac{23}{3}$

8. $|x| < 3$

10. $|x| \ge 3$

12. $|x + 4| < 3$

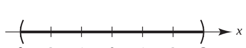
14. $|x - 22| > 2$

16. $|x - 3| > 6$

18. $|y - c| < h$

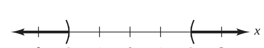
20. $-6 < 2x < 6$

$-3 < x < 3$



22. $5x < -10$ or $5x > 10$

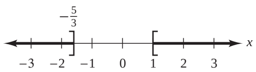
$x < -2$ $\quad x > 2$



24. $3x + 1 \le -4$ or $3x + 1 \ge 4$

$3x \le -5$ $\quad$ $3x \ge 3$

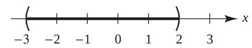
$x \le -\frac{5}{3}$ $\quad$ $x \ge 1$



26. $-5 < 2x + 1 < 5$

$-6 < 2x < 4$

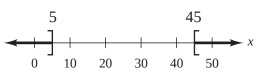
$-3 < x < 2$



28. $25 - x \le -20$ or $25 - x \ge 20$

$-x \le -45$ $\quad$ $-x \ge -5$

$x \ge 45$ $\quad$ $x \le 5$



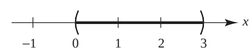
30. $-1 < 1 - \dfrac{2x}{3} < 1$

$-2 < -\dfrac{2x}{3} < 0$

$-6 < -2x < 0$

$3 > x > 0$

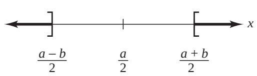
$0 < x < 3$



32. $2x - a \le -b$ or $2x - a \ge b$

$2x \le a - b$ $\quad$ $2x \ge a + b$

$x \le \dfrac{a-b}{2}$ $\quad$ $x \ge \dfrac{a+b}{2}$

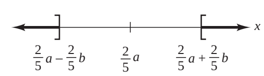


34. $a - \dfrac{5x}{2} < -b$ or $a - \dfrac{5x}{2} > b$

$\dfrac{-5x}{2} < -a - b$ $\quad$ $\dfrac{-5x}{2} > b - a$

$\dfrac{5x}{2} > a + b$ $\quad$ $\dfrac{5x}{2} < a - b$

$x > \dfrac{2}{5}(a + b)$ $\quad$ $x < \dfrac{2}{5}(a - b)$



36. Midpoint $= \dfrac{8.6 + 11.4}{2} = 10$

38. Midpoint $= \dfrac{-4.6 + (-1.3)}{2} = -2.95$

40. Midpoint $= \dfrac{\frac{5}{6} + \frac{5}{2}}{2} = \frac{5}{3}$

42. $|p - 33\tfrac{1}{8}| \le 2$

44. $\left|\dfrac{w - 57.5}{7.5}\right| \le 1$

$-1 \le \dfrac{w - 57.5}{7.5} \le 1$

$-7.5 \le w - 57.5 \le 7.5$

$50 \le w \le 65$

46. $|x - 20| \le 0.75$

$-0.75 \le x - 20 \le 0.75$

$19.25 \le x \le 20.75$

48. (a) $|I - 15{,}000| \le 500 \Longrightarrow 14{,}500 \le I \le 15{,}500$

$|I - 15{,}000| \le 0.05(15{,}000)$

$|I - 15{,}000| \le 750 \Longrightarrow 14{,}250 \le I \le 15{,}750$

(b) The given expense is not at variance with the budget restrictions.

50. (a) $|T - 7500| \le 500 \Longrightarrow 7000 \le T \le 8000$

$|T - 7500| \le 0.05(7500)$

$|T - 7500| \le 375 \Longrightarrow 7125 \le T \le 7875$

(b) The given expense is at variance with the budget restrictions.

Section 0.3 Exponents and Radicals

2. $\dfrac{(6)^2}{2} = \dfrac{36}{2} = 18$

4. $7(4)^{-2} = \dfrac{7}{(4)^2} = \dfrac{7}{16}$

6. $3 - 4(3)^{-2} = 3 - \dfrac{4}{(3)^2} = \dfrac{3}{1} - \dfrac{4}{9} = \dfrac{27}{9} - \dfrac{4}{9} = \dfrac{23}{9}$

8. $5(-3)^3 = 5(-27) = -135$

10. $\dfrac{1}{(-4)^{-3}} = \dfrac{1}{1/(-4)^3} = \dfrac{1}{1/(-64)} = (1)\left(\dfrac{-64}{1}\right) = -64$

12. $\sqrt{(1/9)^3} = \left(\sqrt{1/9}\right)^3 = \left(\dfrac{1}{3}\right)^3 = \dfrac{1}{27}$

14. $16^{-3/4} = \dfrac{1}{16^{3/4}} = \dfrac{1}{\left(\sqrt[4]{16}\right)^3} = \dfrac{1}{(2)^3} = \dfrac{1}{8}$

16. $(10^{2/3})^3 = 10^2 = 100$

18. $\dfrac{10{,}000}{1.075^{120}} \approx 1.7021$

20. $\sqrt[6]{325} \approx 2.6221$

22. $z^{-3}(3z^4) = 3z^{-3+4} = 3z, \quad z \neq 0$

24. $(4x^3)^2 = (4)^2 x^{(3)(2)} = 16x^6$

26. $\dfrac{x^{-3}}{\sqrt{x}} = \dfrac{1}{x^3 x^{1/2}} = \dfrac{1}{x^{7/2}}$

28. $\left(\dfrac{12s^2}{9s}\right)^3 = \left(\dfrac{4s}{3}\right)^3 = \dfrac{64s^3}{27}, \quad s \neq 0$

30. $\left(\sqrt[3]{x^2}\right)^3 = (x^{2/3})^3 = x^2$

32. (a) $\sqrt[3]{\dfrac{16}{27}} = \dfrac{\sqrt[3]{8}\,\sqrt[3]{2}}{\sqrt[3]{27}} = \dfrac{2\sqrt[3]{2}}{3}$

(b) $\sqrt[3]{\dfrac{24}{125}} = \dfrac{\sqrt[3]{8}\,\sqrt[3]{3}}{\sqrt[3]{125}} = \dfrac{2\sqrt[3]{3}}{5}$

34. (a) $\sqrt[4]{(3x^2y^3)^4} = |3x^2y^3| = 3x^2|y|^3$

(b) $\sqrt[3]{54x^7} = \sqrt[3]{27x^6}\,\sqrt[3]{2x} = 3x^2\,\sqrt[3]{2x}$

36. (a) $\sqrt[4]{32xy^5z^{-8}} = \dfrac{2y}{z^2}\,\sqrt[4]{2xy}$

(b) $\sqrt{90(2x - 3y)^6} = 3|2x - 3y|^3\sqrt{10}$

38. $8x^4 - 6x^2 = 2x^2(4x^2 - 3)$

40.
$$5x^{3/2} - x^{-3/2} = x^{-3/2}(5x^3 - 1) = \frac{5x^3 - 1}{x^{3/2}}$$

42.
$$\begin{aligned} 2x(x-1)^{5/2} - 4(x-1)^{3/2} &= 2(x-1)^{3/2}(x(x-1) - 2) \\ &= 2(x-1)^{3/2}(x^2 - x - 2) \\ &= 2(x-1)^{3/2}(x-2)(x+1) \end{aligned}$$

44.
$$\begin{aligned} \frac{(x-4)(2x-1)^3 - (2x-1)^4}{(x-4)^2} &= \frac{(2x-1)^3}{(x-4)^2}((x-4) - (2x-1)) \\ &= \frac{(2x-1)^3}{(x-4)^2}(-x-3) \\ &= -\frac{(x+3)(2x-1)^3}{(x-4)^2} \\ &= \frac{(x+3)(1-2x)^3}{(x-4)^2} \end{aligned}$$

46. $(x^4+2)^3(x+3)^{-1/2} + 4x^3(x^4+2)^2(x+3)^{1/2} = (x^4+2)^2(x+3)^{-1/2}((x^4+2) + 4x^3(x+3))$

$$= (x^4+2)^2(x+3)^{-1/2}(5x^4+12x^3+2)$$

$$= \frac{(x^4+2)^2(5x^4+12x^3+2)}{(x+3)^{1/2}}$$

48. $\sqrt{5-2x}$ is defined when $x \le \frac{5}{2}$.

Therefore, the domain is $\left(-\infty, \frac{5}{2}\right]$.

50. $\sqrt{4x^2+1}$ is defined for all real numbers.

Therefore, the domain is $(-\infty, \infty)$.

52. $\dfrac{1}{\sqrt[3]{x+4}}$ is defined when $x \ne -4$.

Therefore, the domain is $(-\infty, -4) \cup (-4, \infty)$.

54. $\dfrac{\sqrt{x-1}}{x+1}$ is defined when $x \ge 1$.

Therefore, the domain is $[1, \infty)$.

56. $\dfrac{1}{\sqrt{2x+3}}$ is defined when $x > -\dfrac{3}{2}$, and $\sqrt{6-4x}$ is defined when $x \le \dfrac{3}{2}$.

Therefore, the domain of $\dfrac{1}{\sqrt{2x+3}} + \sqrt{6-4x}$ is $-\dfrac{3}{2} < x \le \dfrac{3}{2}$ or $\left(-\dfrac{3}{2}, \dfrac{3}{2}\right]$.

58. $A = 7000\left(1 + \dfrac{0.05}{365}\right)^{1000} \approx \$8027.61$

60. $A = 8000\left(1 + \dfrac{0.07}{12}\right)^{180} \approx \$22{,}791.57$

62. $A = P(1+r) + P(1+r)^2 + P(1+r)^3 + \cdots + P(1+r)^n = P(1+r)[1 + (1+r) + (1+r)^2 + \cdots + (1+r)^{n-1}]$

Section 0.4 Factoring Polynomials

2. Since $a = 8$, $b = -2$, and $c = -1$, we have $x = \dfrac{2 \pm \sqrt{4+32}}{16} = \dfrac{2 \pm 6}{16}$.

Thus, $x = \dfrac{2+6}{16} = \dfrac{1}{2}$ or $x = \dfrac{2-6}{16} = -\dfrac{1}{4}$.

4. Since $a = 9$, $b = 12$, and $c = 4$, we have $x = \dfrac{-12 \pm \sqrt{144-144}}{18} = \dfrac{-12 \pm 0}{18} = -\dfrac{2}{3}$.

6. Since $a = 1$, $b = 6$, and $c = -1$, we have $x = \dfrac{-6 \pm \sqrt{36+4}}{2} = \dfrac{-6 \pm 2\sqrt{10}}{2} = -3 \pm \sqrt{10}$.

8. Since $a = 3$, $b = -8$, and $c = -4$, we have $x = \dfrac{8 \pm \sqrt{64 - 4(3)(-4)}}{6} = \dfrac{8 \pm 4\sqrt{7}}{6} = \dfrac{4}{3} \pm \dfrac{2}{3}\sqrt{7}$.

10. $x^2 + 10x + 25 = (x+5)^2$

12. $9x^2 - 12x + 4 = (3x-2)^2$

14. $2x^2 - x - 1 = (2x+1)(x-1)$

16. $x^2 - xy - 2y^2 = (x-2y)(x+y)$

18. $a^2b^2 - 2abc + c^2 = (ab-c)^2$

20. $x^4 - 16 = (x^2+4)(x^2-4) = (x^2+4)(x+2)(x-2)$

22. $y^3 - 64 = y^3 - 4^3 = (y-4)(y^2+4y+16)$

24. $z^3 + 125 = z^3 + 5^3 = (z+5)(z^2-5z+25)$

26. $(x-a)^3 + b^3 = [(x-a)+b][(x-a)^2 - (x-a)b + b^2]$
$= (x-a+b)[x^2 - 2xa + a^2 - xb + ab + b^2]$
$= (x-a+b)[x^2 - x(2a+b) + (a^2 + ab + b^2)]$

28. $x^3 - x^2 - x + 1 = x^2(x-1) - (x-1)$
$= (x-1)(x^2-1)$
$= (x-1)(x+1)(x-1)$
$= (x+1)(x-1)^2$

30. $x^3 - 5x^2 - 5x + 25 = x^2(x-5) - 5(x-5)$
$= (x-5)(x^2-5)$

32. $x^3 - 7x^2 - 4x + 28 = x^2(x-7) - 4(x-7)$
$= (x-7)(x^2-4)$
$= (x-7)(x+2)(x-2)$

34. $2x^4 - 49x^2 - 25 = (2x^2+1)(x^2-25)$
$= (x-5)(x+5)(2x^2+1)$

36. $2x^2 - 3x = 0$
$x(2x-3) = 0$
$x = 0, \frac{3}{2}$

38. $x^2 - 25 = 0$
$(x+5)(x-5) = 0$
$x = -5, 5$

40. $x^2 - 8 = 0$
$\left(x + \sqrt{8}\right)\left(x - \sqrt{8}\right) = 0$
$\left(x + 2\sqrt{2}\right)\left(x - 2\sqrt{2}\right) = 0$
$x = \pm 2\sqrt{2}$

42. $(x+1)^2 - 8 = 0$
$(x+1)^2 = 8$
$x + 1 = \pm 2\sqrt{2}$
$x = -1 \pm 2\sqrt{2}$

44. $x^2 + 5x + 6 = 0$
$(x+3)(x+2) = 0$
$x = -3, -2$

46. $x^2 + x - 20 = 0$
$(x+5)(x-4) = 0$
$x = -5, 4$

48. $x^3 - 216 = 0$
$x^3 = 216$
$x = \sqrt[3]{216} = 6$

50. $x^4 - 625 = 0$
$x^4 = 625$
$x = \pm\sqrt[4]{625} = \pm 5$

52. $2x^3 + x^2 + 6x + 3 = 0$
$x^2(2x+1) + 3(2x+1) = 0$
$(2x+1)(x^2+3) = 0$
$x = -\frac{1}{2}$

[**Note:** $x^2 + 3 = 0$ has no real roots.]

54. Since $\sqrt{4 - x^2} = \sqrt{(2+x)(2-x)}$, the roots are $x = \pm 2$. By testing points inside and outside the interval $[-2, 2]$, we find that the expression is defined when $-2 \le x \le 2$. Thus, the domain is $[-2, 2]$.

56. Since $\sqrt{x^2 - 8x + 15} = \sqrt{(x-3)(x-5)}$, the roots are $x = 3$ and $x = 5$. By testing the intervals $(-\infty, 3)$, $(3, 5)$, and $(5, \infty)$, we find that the expression is defined when $x \le 3$ or $x \ge 5$. Thus, the domain is $(-\infty, 3] \cup [5, \infty)$.

58.
$$\begin{array}{r|rrrr} -1 & 1 & -2 & -1 & 2 \\ & & -1 & 3 & -2 \\ \hline & 1 & -3 & 2 & 0 \end{array}$$

Therefore, the factorization is

$$x^3 - 2x^2 - x + 2 = (x+1)(x^2 - 3x + 2)$$
$$= (x+1)(x-1)(x-2).$$

60. $\begin{array}{r|rrrrr} 4 & 1 & -16 & 96 & -256 & 256 \\ & & 4 & -48 & 192 & -256 \\ \hline & 1 & -12 & 48 & -64 & 0 \end{array}$

Therefore, $x^4 - 16x^3 + 96x^2 - 256x + 256 = (x - 4)(x^3 - 12x^2 + 48x - 64)$.

62. Possible rational zeros: $\pm6, \pm3, \pm2, \pm1$

Using synthetic division for $x = -1$, we have

$\begin{array}{r|rrrr} -1 & 1 & 0 & -7 & -6 \\ & & -1 & 1 & 6 \\ \hline & 1 & -1 & -6 & 0 \end{array}$

Therefore,

$$\begin{aligned} x^3 - 7x - 6 &= 0 \\ (x + 1)(x^2 - x - 6) &= 0 \\ (x + 1)(x - 3)(x + 2) &= 0 \\ x &= -1, 3, -2 \end{aligned}$$

64. Possible rational roots: $\pm1, \pm2, \pm3, \pm6$

Using synthetic division for $x = 2$, we have the following.

$\begin{array}{r|rrrr} 2 & 1 & 2 & -5 & -6 \\ & & 2 & 8 & 6 \\ \hline & 1 & 4 & 3 & 0 \end{array}$

Therefore, we have

$$\begin{aligned} x^3 + 2x^2 - 5x - 6 &= 0 \\ (x - 2)(x^2 + 4x + 3) &= 0 \\ (x - 2)(x + 1)(x + 3) &= 0 \\ x &= 2, -1, -3. \end{aligned}$$

66. Possible rational roots: $\pm1, \pm2, \pm4, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}, \pm\frac{1}{9}, \pm\frac{1}{18}, \pm\frac{2}{3}, \pm\frac{2}{9}, \pm\frac{4}{3}, \pm\frac{4}{9}$

Using synthetic division for $x = \frac{1}{2}$, we have the following.

$\begin{array}{r|rrrr} \frac{1}{2} & 18 & -9 & -8 & 4 \\ & & 9 & 0 & -4 \\ \hline & 18 & 0 & -8 & 0 \end{array}$

Therefore, we have

$$\begin{aligned} 18x^3 - 9x^2 - 8x + 4 &= 0 \\ \left(x - \tfrac{1}{2}\right)(18x^2 - 8) &= 0 \\ \left(x - \tfrac{1}{2}\right)(2)(9x^2 - 4) &= 0 \\ (2x - 1)(3x + 2)(3x - 2) &= 0 \\ x &= \tfrac{1}{2}, \pm\tfrac{2}{3}. \end{aligned}$$

68. Possible rational zeros: $\pm6, \pm3, \pm\frac{3}{2}, \pm2, \pm1, \pm\frac{1}{2}$

Using synthetic division for $x = 3$, we have

$\begin{array}{r|rrrr} 3 & 2 & -1 & -13 & -6 \\ & & 6 & 15 & 6 \\ \hline & 2 & 5 & 2 & 0 \end{array}$

Therefore,

$$\begin{aligned} 2x^3 - x^2 - 13x - 6 &= 0 \\ (x - 3)(2x^2 + 5x + 2) &= 0 \\ (x - 3)(2x + 1)(x + 2) &= 0 \\ x &= 3, -\tfrac{1}{2}, -2 \end{aligned}$$

70.
$$\begin{aligned} -200x^2 + 2000x - 3800 &> 1000 \\ 0 &> 200x^2 - 2000x + 4800 \\ 0 &> 200(x^2 - 10x + 24) \\ 0 &> 200(x - 4)(x - 6) \end{aligned}$$

The roots are $x = 4, 6$. By testing points inside and outside the interval $[4, 6]$, we find that the expression is defined when $4 < x < 6$.

72.
$$\begin{aligned} 1300 &= 1200(1 + r)^2 \\ \tfrac{13}{12} &= (1 + r)^2 \\ 1 + r &= \sqrt{\tfrac{13}{12}} \\ r &= \sqrt{\tfrac{13}{12}} - 1 \approx 0.0408 \quad \text{or} \quad 4.08\% \end{aligned}$$

Section 0.5 Fractions and Rationalization

2. $\dfrac{2x - 1}{x + 3} + \dfrac{1 - x}{x + 3} = \dfrac{x}{x + 3}$

4. $\dfrac{5x + 10}{2x - 1} - \dfrac{2x + 10}{2x - 1} = \dfrac{5x + 10 - (2x + 10)}{2x - 1} = \dfrac{3x}{2x - 1}$

6. $\dfrac{x}{x^2 + x - 2} - \dfrac{1}{x + 2} = \dfrac{x}{(x + 2)(x - 1)} - \dfrac{1}{x + 2} = \dfrac{x - (x - 1)}{(x + 2)(x - 1)} = \dfrac{1}{(x + 2)(x - 1)}$

8. $\dfrac{x}{2-x}+\dfrac{2}{x-2}=\dfrac{-x}{x-2}+\dfrac{2}{x-2}=\dfrac{-x+2}{x-2}=\dfrac{-(x-2)}{x-2}=-1$

10. $$\begin{aligned}\frac{A}{x-5}+\frac{B}{x+5}+\frac{C}{(x+5)^2}&=\frac{A(x+5)^2+B(x-5)(x+5)+C(x-5)}{(x-5)(x+5)^2}\\&=\frac{A(x^2+10x+25)+B(x^2-25)+Cx-5C}{(x-5)(x+5)^2}\\&=\frac{(A+B)x^2+(10A+C)x+5(5A-5B-C)}{(x-5)(x+5)^2}\end{aligned}$$

12. $$\begin{aligned}\frac{Ax+B}{x^2+2}+\frac{C}{x-4}&=\frac{(Ax+B)(x-4)+C(x^2+2)}{(x-4)(x^2+2)}\\&=\frac{Ax^2-4Ax+Bx-4B+Cx^2+2C}{(x-4)(x^2+2)}\\&=\frac{(A+C)x^2-(4A-B)x-2(2B-C)}{(x-4)(x^2+2)}\end{aligned}$$

14. $$\begin{aligned}\frac{2}{x+1}+\frac{1-x}{x^2-2x+3}&=\frac{2(x^2-2x+3)+(1-x)(x+1)}{(x+1)(x^2-2x+3)}\\&=\frac{2x^2-4x+6+x-x^2+1-x}{(x+1)(x^2-2x+3)}\\&=\frac{x^2-4x+7}{(x+1)(x^2-2x+3)}\end{aligned}$$

16. $$\begin{aligned}\frac{x-1}{x^2+5x+4}+\frac{2}{x^2-x-2}+\frac{10}{x^2+2x-8}&=\frac{x-1}{(x+1)(x+4)}+\frac{2}{(x+1)(x-2)}+\frac{10}{(x-2)(x+4)}\\&=\frac{(x-1)(x-2)+2(x+4)+10(x+1)}{(x+1)(x-2)(x+4)}\\&=\frac{x^2-3x+2+2x+8+10x+10}{(x+1)(x-2)(x+4)}\\&=\frac{x^2+9x+20}{(x+1)(x-2)(x+4)}\\&=\frac{(x+4)(x+5)}{(x+1)(x-2)(x+4)}\\&=\frac{x+5}{(x+1)(x-2)},\quad x\neq -4\end{aligned}$$

18. $$\begin{aligned}2\sqrt{x}(x-2)+\frac{(x-2)^2}{2\sqrt{x}}&=\frac{2\sqrt{x}(x-2)}{1}\cdot\frac{2\sqrt{x}}{2\sqrt{x}}+\frac{(x-2)^2}{2\sqrt{x}}\\&=\frac{4x(x-2)+(x-2)^2}{2\sqrt{x}}=\frac{(x-2)[4x+(x-2)]}{2\sqrt{x}}=\frac{(x-2)(5x-2)}{2\sqrt{x}}\end{aligned}$$

20. $$\begin{aligned}-\frac{\sqrt{x^2+1}}{x^2}+\frac{1}{\sqrt{x^2+1}}&=-\frac{\sqrt{x^2+1}}{x^2}\cdot\frac{\sqrt{x^2+1}}{\sqrt{x^2+1}}+\frac{1}{\sqrt{x^2+1}}\cdot\frac{x^2}{x^2}\\&=\frac{-(x^2+1)+x^2}{x^2\sqrt{x^2+1}}=-\frac{1}{x^2\sqrt{x^2+1}}\end{aligned}$$

22. $$\begin{aligned}\left(\sqrt{x^3+1}-\frac{3x^3}{2\sqrt{x^3+1}}\right)\div(x^3+1)&=\left(\frac{\sqrt{x^3+1}\left(2\sqrt{x^3+1}\right)-3x^3}{2\sqrt{x^3+1}}\right)\frac{1}{x^3+1}\\&=\frac{2(x^3+1)-3x^3}{2(x^3+1)^{1/2}(x^3+1)}=\frac{2-x^3}{2(x^3+1)^{3/2}}\end{aligned}$$

24. $\dfrac{x(x+1)^{-1/2}-(x+1)^{1/2}}{x^2}=\dfrac{(x+1)^{-1/2}[x-(x+1)]}{x^2}=\dfrac{-1}{x^2\sqrt{x+1}}$

26. $\dfrac{\dfrac{2x^2}{3(x^2-1)^{2/3}}-(x^2-1)^{1/3}}{x^2}=\dfrac{2x^2-(x^2-1)^{1/3}(3)(x^2-1)^{2/3}}{3(x^2-1)^{2/3}}\cdot\dfrac{1}{x^2}$

$=\dfrac{2x^2-3(x^2-1)}{3x^2(x^2-1)^{2/3}}=\dfrac{3-x^2}{3x^2(x^2-1)^{2/3}}$

28. $\dfrac{x}{(x-5)^{1/2}}+2(x-5)^{1/2}=\dfrac{x+2(x-5)}{(x-5)^{1/2}}$

$=\dfrac{3x-10}{(x-5)^{1/2}}$

30. $\dfrac{-x}{2(3+x^2)^{3/2}}+\dfrac{3}{(3+x^2)^{1/2}}=\dfrac{-x+3(2)(3+x^2)}{2(3+x^2)^{3/2}}$

$=\dfrac{6x^2-x+18}{2(3+x^2)^{3/2}}$

32. $\dfrac{5}{\sqrt{10}}=\dfrac{5}{\sqrt{10}}\cdot\dfrac{\sqrt{10}}{\sqrt{10}}=\dfrac{5\sqrt{10}}{10}=\dfrac{\sqrt{10}}{2}$

34. $\dfrac{4y}{\sqrt{y+8}}=\dfrac{4y}{\sqrt{y+8}}\cdot\dfrac{\sqrt{y+8}}{\sqrt{y+8}}=\dfrac{4y\sqrt{y+8}}{y+8}$

36. $\dfrac{10(x+2)}{\sqrt{x^2-x-6}}=\dfrac{10(x+2)}{\sqrt{x^2-x-6}}\cdot\dfrac{\sqrt{x^2-x-6}}{\sqrt{x^2-x-6}}$

$=\dfrac{10(x+2)\sqrt{x^2-x-6}}{x^2-x-6}$

$=\dfrac{10(x+2)\sqrt{x^2-x-6}}{(x+2)(x-3)}$

$=\dfrac{10\sqrt{x^2-x-6}}{x-3}$

38. $\dfrac{13}{6+\sqrt{10}}=\dfrac{13}{6+\sqrt{10}}\cdot\dfrac{6-\sqrt{10}}{6-\sqrt{10}}$

$=\dfrac{13\left(6-\sqrt{10}\right)}{36-10}$

$=\dfrac{6-\sqrt{10}}{2}$

40. $\dfrac{x}{\sqrt{2}+\sqrt{3}}=\dfrac{x}{\sqrt{2}+\sqrt{3}}\cdot\dfrac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}$

$=\dfrac{x\left(\sqrt{2}-\sqrt{3}\right)}{2-3}$

$=x\left(\sqrt{3}-\sqrt{2}\right)$

42. $\dfrac{\sqrt{15}+3}{12}=\dfrac{\sqrt{15}+3}{12}\cdot\dfrac{\sqrt{15}-3}{\sqrt{15}-3}$

$=\dfrac{15-9}{12\left(\sqrt{15}-3\right)}$

$=\dfrac{1}{2\left(\sqrt{15}-3\right)}$

44. $\dfrac{10}{\sqrt{x}+\sqrt{x+5}}=\dfrac{10}{\sqrt{x}+\sqrt{x+5}}\cdot\dfrac{\sqrt{x}-\sqrt{x+5}}{\sqrt{x}-\sqrt{x+5}}$

$=\dfrac{10\left(\sqrt{x}-\sqrt{x+5}\right)}{x-(x+5)}$

$=-2\left(\sqrt{x}-\sqrt{x+5}\right)$

$=2\left(\sqrt{x+5}-\sqrt{x}\right),\quad x\geq 0$

46. $\dfrac{\dfrac{\sqrt{x^2+1}}{x^2}-\dfrac{1}{x\sqrt{x^2+1}}}{x^2+1}=\dfrac{(x^2+1)-x}{x^2(x^2+1)^{3/2}}$

$=\dfrac{x^2-x+1}{x^2(x^2+1)^{3/2}}$

48. (a) $C=6x+\dfrac{900{,}000}{x}$

$=\dfrac{6x^2+900{,}000}{x}$

$=\dfrac{6(x^2+150{,}000)}{x}$

(b) When $x=240$,

$$C=6(240)+\frac{900{,}000}{240}=\$5190.00.$$