1. Which of the following three statements are always true.

(1) If $f(x,y)$ has a local maximum on the circle $x^2+y^2=1$ at $(1,0)$, then $\nabla f(1,0)=\mathbf{0}$.

(2) Let $f(1,4)=1$ and $\nabla f(1,4)=\langle 2,3\rangle$. Then the tangent plane to f has equation $z=1+2(x-1)+3(y-4)$.

(3) Let $f(x,y)$ be differentiable at $(2,4)$ and suppose f increases most rapidly in the direction $\langle 1/\sqrt{5}, 2/\sqrt{5}\rangle$. Let $\mathbf{u}=\langle -2/\sqrt{5}, 1/\sqrt{5}\rangle$. Then $D_{\mathbf{u}}f(2,4)=0$.

A. Only (2) and (3) are true.

B. Only (1) and (2) are true.

C. Only (1) is true.

D. Only (2) is true.

E. Only (3) is true.

2. Find an equation for the tangent plane to $e^{x+y+z}=(z+1)^2$, at $(0,0,0)$.

A. $x+y=0$

B. $x+y+z=0$

C. $x+y-2z=0$

D. $x+y-z=0$

E. $x-y-z=0$

3. Find α, $0 \le \alpha \le \pi$, so that the directional derivative of $f(x, y) = (x + y)^2$ at $(1/4, 1/4)$, in the direction $\mathbf{u} = \langle \cos\alpha, \sin\alpha \rangle$, is -1.

A. $\alpha = 0$.

B. $\alpha = \pi/2$.

C. $\alpha = \pi/3$.

D. $\alpha = \pi/4$.

E. $\alpha = \pi$.

4. Classify the critical points $(1, 4)$, and $(1, -2)$, of the function

$$f(x, y) = x^2 - 2x + y^3 - 3y^2 - 24y.$$

A. $(1, 4)$ saddle point
$(1, -2)$ local minimum

B. $(1, 4)$ local minimum
$(1, -2)$ saddle point

C. $(1, 4)$ local maximum
$(1, -2)$ saddle point

D. $(1, 4)$ saddle point
$(1, -2)$ local maximum

E. $(1, 4)$ local maximum
$(1, -2)$ local minimum

5. The minimum value of $f(x,y,z) = x + y + z$ on the surface $x^2 + y^2 + z^2 = 1$, is

A. $-\sqrt{3}$.

B. $\frac{1}{3}$

C. -1

D. $-\sqrt{2}$

E. -3

6. Evaluate $\iint_R 2y\, dA$ where R is the plane region bounded by the curves

$y = \sqrt{2 - x^2}$, $y = x$, and $x = 0$.

A. 1/3

B. 4/3

C. 7/3

D. 11/5

E. 14/5

7. Find the values of a and b such that

$$\int_1^2 \int_{e^x}^{e^2} f(x,y)\,dydx = \int_e^a \int_1^b f(x,y)\,dxdy.$$

A. $a = e^2, \quad b = \ln y.$

B. $a = e^2, \quad b = e^y.$

C. $a = 2e, \quad b = 2.$

D. $a = 2e, \quad b = \ln y.$

E. $a = e^2, \quad b = \dfrac{\ln y}{2}.$

8. A lamina with density $\rho(x,y) = y$, occupies the triangle with vertices $(0,0)$, $(2,0)$, and $(1,1)$. Find the mass of the lamina.

A. 2/5

B. 1/3

C. 4

D. 3/7

E. 2/9

9. Use polar coordinates to evaluate the double integral

$$\iint_R (x^2+y^2)^{3/2}\, dA,$$

where R is the part of the unit disk $x^2+y^2 \le 1$, such that $x \ge 0$, $y \ge 0$, and $y \le x$.

A. $\pi/8$

B. $\pi/10$

C. $\pi/16$

D. $\pi/20$

E. $\pi/32$

10. Find the surface area of the portion of the paraboloid $z = x^2+y^2$ below the plane $z=2$.

A. $\dfrac{\pi}{3}$

B. $\dfrac{4\pi}{9}$

C. $\dfrac{13\pi}{3}$

D. $\dfrac{8\pi}{3}$

E. $\dfrac{7\pi}{9}$

11. Which iterated integral gives the volume of the solid in the first octant bounded by the coordinate planes, the plane $y + z = 2$, and the cylinder $x = 4 - y^2$.

A. $\int_0^2 \int_{4-y^2}^4 \int_{2-y}^2 dzdxdy$

B. $\int_0^4 \int_0^{2-y} \int_{2-y}^{4-y^2} dzdxdy$

C. $\int_0^2 \int_0^{2-y} \int_0^{4-y^2} dzdxdy$

D. $\int_0^2 \int_1^{4-y^2} \int_y^{2-y} dzdxdy$

E. $\int_0^2 \int_0^{4-y^2} \int_0^{2-y} dzdxdy$

12. Evaluate $\int_1^e \int_1^e \int_1^e \frac{1}{xyz} dxdydz$.

A. e

B. e^3

C. 1

D. e^{-1}

E. e^{-3}