

1. Which of the following three statements are always true.

- (1) If $f(x, y)$ has a local maximum on the circle $x^2 + y^2 = 1$ at $(1, 0)$, then $\nabla f(1, 0) = \mathbf{0}$.
- (2) Let $f(1, 4) = 1$ and $\nabla f(1, 4) = \langle 2, 3 \rangle$. Then the tangent plane to f has equation $z = 1 + 2(x - 1) + 3(y - 4)$.
- (3) Let $f(x, y)$ be differentiable at $(2, 4)$ and suppose f increases most rapidly in the direction $\langle 1/\sqrt{5}, 2/\sqrt{5} \rangle$. Let $\mathbf{u} = \langle -2/\sqrt{5}, 1/\sqrt{5} \rangle$. Then $D_{\mathbf{u}}f(2, 4) = 0$.

- A. Only (1) and (2) are true.
- B. Only (2) and (3) are true.
- C. Only (1) is true.
- D. Only (2) is true.
- E. Only (3) is true.

2. Find an equation for the tangent plane to $e^{x+y+z} = (z+1)^2$, at $(0, 0, 0)$.

- A. $x + y = 0$
- B. $x + y + z = 0$
- C. $x + y - 2z = 0$
- D. $x - y - z = 0$
- E. $x + y - z = 0$

3. Find α , $0 \leq \alpha \leq \pi$, so that the directional derivative of $f(x, y) = (x + y)^2$ at $(1/4, 1/4)$, in the direction $\mathbf{u} = \langle \cos \alpha, \sin \alpha \rangle$, is -1 .

- A. $\alpha = \pi$.
- B. $\alpha = \pi/2$.
- C. $\alpha = \pi/3$.
- D. $\alpha = \pi/4$.
- E. $\alpha = 0$.

4. Classify the critical points $(1, 4)$, and $(1, -2)$, of the function

$$f(x, y) = x^2 - 2x + y^3 - 3y^2 - 24y.$$

- A. $(1, 4)$ saddle point
 $(1, -2)$ local minimum
- B. $(1, 4)$ local maximum
 $(1, -2)$ local minimum
- C. $(1, 4)$ local maximum
 $(1, -2)$ saddle point
- D. $(1, 4)$ saddle point
 $(1, -2)$ local maximum
- E. $(1, 4)$ local minimum
 $(1, -2)$ saddle point

5. The minimum value of $f(x, y, z) = x + y + z$ on the surface $x^2 + y^2 + z^2 = 1$, is

- A. -1
- B. $\frac{1}{3}$
- C. $-\sqrt{3}$.
- D. $-\sqrt{2}$
- E. -3

6. Evaluate $\iint_R 2y \, dA$ where R is the plane region bounded by the curves

$$y = \sqrt{2 - x^2}, \quad y = x, \quad \text{and} \quad x = 0.$$

- A. $1/3$
- B. $4/3$
- C. $7/3$
- D. $11/5$
- E. $14/5$

7. Find the values of a and b such that

$$\int_1^2 \int_{e^x}^{e^2} f(x, y) \, dy \, dx = \int_e^a \int_1^b f(x, y) \, dx \, dy.$$

- A. $a = e^2, \quad b = \frac{\ln y}{2}.$
- B. $a = e^2, \quad b = e^y.$
- C. $a = 2e, \quad b = 2.$
- D. $a = 2e, \quad b = \ln y.$
- E. $a = e^2, \quad b = \ln y.$

8. A lamina with density $\rho(x, y) = y$, occupies the triangle with vertices $(0, 0)$, $(2, 0)$, and $(1, 1)$. Find the mass of the lamina.

- A. $1/3$
- B. $2/5$
- C. 4
- D. $3/7$
- E. $2/9$

9. Use polar coordinates to evaluate the double integral

$$\iint_R (x^2 + y^2)^{3/2} dA,$$

where R is the part of the unit disk $x^2 + y^2 \leq 1$, such that $x \geq 0$, $y \geq 0$, and $y \leq x$.

- A. $\pi/8$
- B. $\pi/20$
- C. $\pi/16$
- D. $\pi/10$
- E. $\pi/32$

10. Find the surface area of the portion of the paraboloid $z = x^2 + y^2$ below the plane $z = 2$.

- A. $\frac{\pi}{3}$
- B. $\frac{4\pi}{9}$
- C. $\frac{13\pi}{3}$
- D. $\frac{8\pi}{3}$
- E. $\frac{7\pi}{9}$

11. Which iterated integral gives the volume of the solid in the first octant bounded by the coordinate planes, the plane $y + z = 2$, and the cylinder $x = 4 - y^2$.

A. $\int_0^2 \int_{4-y^2}^4 \int_{2-y}^2 dz dx dy$

B. $\int_0^4 \int_0^{2-y} \int_{2-y}^{4-y^2} dz dx dy$

C. $\int_0^2 \int_1^{4-y^2} \int_y^{2-y} dz dx dy$

D. $\int_0^2 \int_0^{4-y^2} \int_0^{2-y} dz dx dy$

E. $\int_0^2 \int_0^{2-y} \int_0^{4-y^2} dz dx dy$

12. Evaluate $\int_1^e \int_1^e \int_1^e \frac{1}{xyz} dx dy dz$.

A. 1

B. e^3

C. e

D. e^{-1}

E. e^{-3}