MA 221 **Final Exam Practice Problems**

The Formula Page may be used. It will be attached to the final exam.

1. Calculate $\dfrac{dy}{dx}$ if $y = \cos(1-2x)$.
   A. $-\sin(1-2x)$ B. $-2\sin(1-2x)$ C. $2\sin(1-2x)$ D. $\sin(1-2x)$ E. $-2\cos(1-2x)$

2. Find y' if $y = x\tan^2 x$.
   A. $2x\tan x + \tan^2 x$ B. $2x\tan x\sec^2 x$ C. $x\sec^2 x + \tan^2 x$ D. $2x\tan x\sec^2 x + \tan^2 x$
   E. None of these.

3. If $\sin\theta = -0.5473$ and $\pi < \theta < \frac{3\pi}{2}$, find θ in radians. Give your answer correct to 4 decimal places.
   A. -0.5791 B. 3.7207 C. 3.1511 D. 2.5625 E. 1.761

4. Find the slope of the line perpendicular to the line containing the points $(-2,4)$ and $(6,-3)$.
   A. $-8/7$ B. $-7/8$ C. $1/4$ D. $8/7$ E. $-1/2$

5. Give the equation, in slope-intercept form, of the line which is parallel to the line $2y-6x-5=0$ and passes through the point $(-1,3)$.
   A. $y = 3x+10$ B. $y = \frac{1}{3}x - 9$ C. $y = \frac{1}{3}x + 2$ D. $y = -3x$ E. $y = 3x+6$

6. If $f(x) = \dfrac{x}{x^2+1}$, find $\dfrac{1}{f(3)}$.
   A. 3/10 B. 3/16 C. 16/3 D. 10/3 E. 1/3

7. If $f(x) = \dfrac{2}{x}$ then $\dfrac{f(x+\Delta x) - f(x)}{\Delta x} =$
   A. $\dfrac{-2}{x^2}$ B. $\dfrac{2}{x+\Delta x} - \dfrac{2}{x}$ C. $\dfrac{-2}{x(x+\Delta x)}$ D. $\dfrac{2}{x(x+\Delta x)}$ E. $\dfrac{-2}{x^2+\Delta x}$

8. $\lim\limits_{x\to 1} \dfrac{x^2+4x-5}{x^2-1} =$
   A. 3 B. 1 C. ∞ D. 0 E. 5

9. If the tangent line to the graph of $y = f(x)$ at (2,3) has equation $x - y + 1 = 0$, then $f'(2) =$
   A. 1 B. 3/2 C. 2/3 D. $-2/3$ E. $-3/2$

10. The derivative of $\dfrac{x^2+1}{x+5}$ is
    A. $\dfrac{(x+5)(2x) - (x^2+1)}{(x+5)^2}$ B. $2x$ C. $\dfrac{(x+5)(2x)}{(x^2+1)^2}$ D. $\dfrac{(x^2+1)+(x+5)(2x)}{(x+5)^2}$ E. $\dfrac{(x^2+1)-(x+5)(2x)}{(x+5)^2}$

11. If $y = (3-x^2)^3$ then $y'' =$
    A. $-6x(3-x^2)^2$ B. $24x^2(3-x^2) - 6(3-x^2)^2$ C. $6(3-x^2)$ D. $24x^2(3-x^2)$ E. None of these.

12. The line tangent to the graph of $f(x) = x - \dfrac{1}{x}$ at $(2, 3/2)$ has slope
    A. 3/4 B. 3/2 C. 0 D. l/4 E. 5/4

13. A point moves along the x-axis in such a way that its distance, s, from the origin at time $t \geq 1$ is given by $s = (t^3 - t)^{3/2}$. Find the velocity of the point when $t = 2$. Give your answer correct to two decimal places.
    A. 3.67 B. 3.83 C. 40.42 D. 36.74 E. 21.08

14. Differentiate $y = \sin(x^2)$. $\dfrac{dy}{dx} =$
    A. $\cos(x^2)$ B. $2x\cos(x^2)$ C. $-cos(x^2)$ D. $\cos(2x)$ E. $-2x\cos(x^2)$

15. Find y' if $y = x\tan x$.
A. $\sec^2 x$ B. $x\sec^2 x$ C. $x\tan 1 + \tan x$ D. $1 + \sec^2 x$ E. $x\sec^2 x + \tan x$

16. Find $f'(x)$ if $f(x) = \dfrac{x}{\cos(4x)}$. $f'(x) =$
A. $\dfrac{\cos(4x) + 4x\sin(4x)}{\cos^2(4x)}$ B. $\dfrac{-1}{4\sin(4x)}$ C. $\dfrac{\cos(4x) - x\sin(4x)}{\cos^2(4x)}$ D. $\dfrac{\cos(4x) + x\sin(4x)}{\cos^2(4x)}$
E. $\dfrac{-4x\sin(4x)}{\cos^2(4x)}$

17. The maximum value of $f(x) = x^3(40 - x)^2$ on the closed interval, $0 \le x \le 40$, occurs at $x =$
A. 20 B. 15 C. 35 D. 18 E. 24

18. Which of the following best describes the graph of $y = 4x^3 - 3x^4$? The graph has a
A. relative maximum point and two points of inflection.
B. relative maximum point, a relative minimum point and one point of inflection.
C. a relative minimum point and two points of inflection.
D. a relative minimum point, two relative maximum points and two points of inflection.
E. None of these.

19. Which of the following best describes the graph of $f(x) = \dfrac{x^2 + 1}{x^2 - 1}$?
A. Vertical asymptotes $x = 1, x = -1$, and symmetric to the x-axis.
B. Vertical asymptotes $x = 1, x = -1$ but not symmetric to either the x or the y-axis.
C. Vertical asymptotes $x = 1, x = -1$ and symmetric to the y-axis.
D. No vertical asymptotes and symmetric to the y-axis.
E. Vertical asymptote $x = 1$ and symmetric to the x-axis.

20. Find the area of the region bounded by the curves $x^2 + 4y = 0$ and $x^2 - 4y - 8 = 0$.
A. 2/3 B. 16/3 C. 6 D. 4/3 E. 10/3

21. What is the area of the largest rectangle with sides parallel to the axes which can be inscribed in the first quadrant under the parabola $y = 4 - x^2$? (Give your answer correct to 2 decimal places.)
A. 1.15 B. 1.33 C. 3.08 D. 4.00 E. 2.67

22. A box with square base and no top is to have a volume of 108 in.^{3}. What is the smallest possible surface area of such a box.
A. 32 in.^{2} B. 24 in.^{2} C. 256 in.^{2} D. 108 in.^{2} E. 56 in.^{2}

23. Find the value of dy and Δy for $y = 2x^3 - 4x, x = 2$ and $dx = \Delta x = 0.1$. Give your answer correct to two decimal places.
A. $dy = 2, \Delta y = 2.12$ B. $dy = 2, \Delta y = 2.46$ C. $dy = 2.12, \Delta y = 2$ D. $dy = 2.46, \Delta y = 2$
E. None of these.

24. Calculate $\lim\limits_{x\to\infty} \dfrac{2 + 3x - 2x^3}{3 - 4x + x^3}$.
A. 2 B. ∞ C. 2/3 D. $-3/4$ E. -2

25. Evaluate $\displaystyle\int \sqrt{2x+1}\,dx$
A. $\frac{2}{3}(2x+1)^{3/2} + C$ B. $\frac{1}{3}(2x+1)^{3/2} + C$ C. $(2x+1)^{-1/2} + C$ D. $2(2x+1)^{1/2} + C$
E. None of these.

26. Evaluate $\int_1^2 (6\sqrt{x} - \frac{1}{2\sqrt{x}})dx$. Give your answer correct to 2 decimal places.
A. 9.90 B. 6.90 C. 5.66 D. 7.35 E. None of these

27. An object is thrown vertically downward from the top of a building 200 ft high with an initial velocity of 40 ft/sec. Find its velocity when it hits the ground. ($s = -16t^2 - 40t + 200$.)
A. -40 ft/sec B. -200 ft/sec C. -120 ft/sec D. -80 ft/sec E. None of these.

28. Calculate the area bounded by the parabola $y = x^2$ and the line $y = x + 2$.
A. 9/2 B. 10/3 C. 7/6 D. 15/2 E. None of these.

29. Calculate the volume generated by revolving the area bounded by $y = \sqrt{x}$, the x-axis and $x = 4$ about the y-axis. (Express your answer as a definite integral.)
A. $\pi \int_0^4 xdx$ B. $\pi \int_0^4 \sqrt{x}dx$ C. $2\pi \int_0^4 (4-x)\sqrt{x}dx$ D. $2\pi \int_0^4 x^2dx$ E. $2\pi \int_0^4 x^{3/2}dx$

30. Find the function, y, satisfying the following conditions: $\frac{dy}{dx} = 3x^2 - 1$, and the graph of y passes through the point $(1, 3)$.
A. $y = x^3 - x + 3$ B. $y = x^3 - x$ C. $y = 6x - 3$ D. $y = 6x$ E. $y = 3x^3 - x + 1$

31. Calculate the volume generated by revolving the area bounded by $y = \sqrt{x}$, the y-axis, and the line $y = 2$, about the x-axis. (Express your answer as a definite integral.)
A. $\pi \int_0^4 (2 - \sqrt{x})^2 dx$ B. $\pi \int_0^4 (4 - x)dx$ C. $2\pi \int_0^4 (2 - \sqrt{x})dx$ D. $2\pi \int_0^4 (4\sqrt{x} - x)dx$
E. $\pi \int_0^4 xdx$

32. If $f'(x) = 4x - 3$ and $f(0) = 7$ calculate $f(2)$.
A. 5 B. 2 C. 7 D. 9 E. 3

33. Calculate the centroid of a quarter circle of radius r.
A. $\overline{x} = \frac{r}{3\pi}, \overline{y} = \frac{r}{3\pi}$ B. $\overline{x} = \frac{4r}{3}, \overline{y} = \frac{4r}{3}$ C. $\overline{x} = \frac{4r}{\pi}, \overline{y} = 0$ D. $\overline{x} = \frac{4r}{\pi}, \overline{y} = \frac{4r}{\pi}$
E. $\overline{x} = \frac{4r}{3\pi}, \overline{y} = \frac{4r}{3\pi}$

34. Calculate the x-coordinate of the centroid, $\overline{x}$, of the area given in problem 26, if the area is 16/3 square units.
A. $\overline{x} = 2$ B. $\overline{x} = 3/2$ C. $\overline{x} = 9/5$ D. $\overline{x} = 12/5$ E. $\overline{x} = 1/5$

35. Find the work done in pumping the water out of the top of a cylindrical tank 5 ft in radius and 10 ft high, if the tank is initially half full of water, which weighs 62.4 lb/ft^3.
A. 93,750π ft-lb B. 58,500π ft-lb C. 7,800π ft-lb D. 15,600π ft-lb E. None of these.

36. A spring of natural length 12 ft. requires a force of 6 lb. to stretch it 2 ft. Find the work done in stretching it 6 ft. ($F = kx$)
A. 54 ft-lb B. 108 ft-lb C. 6 ft-lb D. 36 ft-lb E. 24 ft-lb

37. A vertical rectangular floodgate on a dam is 5 ft long and 4 ft deep. Find the force on the floodgate if its upper edge is 3 ft below the surface. (The weight of water is 62.4 lb/ft^3.) Give your answer correct to the nearest integer.
A. 7644 lb B. 3900 lb C. 1248 lb D. 6240 lb E. 2100 lb

38. A horizontal tank with vertical circular ends is filled with oil. If the radius of each end is 2 m, find the force on one end of the tank. (Assume w is the weight of the oil.) Express your answer as a definite integral. (Hint: Assume that the origin is at the center of one of the circular ends.)
A. $2w\int_0^2 y\sqrt{4-y^2}dy$ B. $w\int_{-2}^2 \sqrt{4-y^2}dy$ C. $2w\int_{-2}^2 (2-y)\sqrt{4-y^2}dy$ D. $2w\int_{-2}^2 (2-y)dy$
E. None of these.

39. Grant and Stadium Streets are straight and perpendicular to each other. A black 1997 Porsche 911 is going on Grant Street toward the intersection of the two streets at 60 mph (miles per hour), and a red 1993 Volkswagen Golf is going on Stadium Street toward the same intersection at a rate of 40 mph. At what rate is the distance between the two cars decreasing when the Porsche is 1/2 mile from the intersection and the Golf is 3/8 mile from it?
A. 40 mph B. 56 mph C. 60 mph D. 72 mph E. 32 mph

40. The line perpendicular to the graph of $f(x) = x - \dfrac{1}{x}$ at $(2, \frac{3}{2})$ has slope
A. 3/4 B. 3/2 C. 0 D. l/4 E. $-4/5$

41. Find the center C and radius r of the circle whose equation is
$$x^2 + y^2 - 10x + 6y + 30 = 0$$
A. $C = (-5, 3)$; $r = 2$ B. $C = (-5, 3)$; $r = 4$ C. $C = (5, -3)$; $r = 8$ D. $C = (5, -3)$; $r = 4$
E. $C = (5, -3)$; $r = 2$

42. If $y^3 + x^2 = 9$ and $\frac{dx}{dt} = 5$, find $\frac{dy}{dt}$ when $x = 1$.
A. $-5/6$ B. 2/3 C. -10 D. 1/3 E. 10/3

43. Water is flowing into a tank which is in the shape of a right circular cylinder standing on its circular base. If the water is flowing in at a rate of 80 cu. ft. per min. and the radius of the base of the tank is 4 ft., how fast is the water rising when the water is 10 ft. deep?
A. $\dfrac{\pi}{5}$ ft/min B. 5π ft/min C. $\dfrac{50}{\pi}$ ft/min D. $\dfrac{5}{\pi}$ ft/min E. 50π ft/min

44. Find the mean value of $f(x) = x^{1/3}$ on the interval $[0, 2]$.
A. $\dfrac{3}{2^{2/3}}$ B. $\dfrac{3}{4}$ C. $\dfrac{1}{2^{4/3}}$ D. $\dfrac{3}{2^{1/3}}$ E. $\dfrac{3}{2^{5/3}}$

45. Find the root mean square of $f(x) = \sqrt{x}(1-x)$ on the interval $[0, 2]$.
A. $1/\sqrt{3}$ B. 2/3 C. $2/\sqrt{3}$ D. $\sqrt{3}/2$ E. $1/\sqrt{6}$

Answers

1. C; 2. D; 3. B; 4. D; 5. E; 6. D; 7. C; 8. A; 9. A; 10. A; 11. B; 12. E; 13. C; 14. B; 15. E; 16. A; 17. E; 18. A; 19. C; 20. B; 21. C; 22. D; 23. A; 24. E; 25. B; 26. B; 27. C; 28. A; 29. E; 30. A; 31. B; 32. D; 33. E; 34. D; 35. B; 36. A; 37. D; 38. C; 39. D; 40. E; 41. E; 42. A; 43. D; 44. E; 45. A.