MA 26100M Exam 1 October 8, 2008

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Signature

Instructions

1. SHOW YOUR WORK IN EACH PROBLEM in order to receive credit.
2. This exam contains 9 problems. Problems 1-5 are multiple choice and are worth 9 points each. Problems ~~7~~ 6-9 are not multiple choice and are worth14 points each as indicated.
3. Work only in the space provided, or on the backside of the pages. Also circle your choice for each problem in this booklet.
4. No books, notes or calculator, please.

X

1. C
2. A
3. B
4. D
5. E

(9 pt) 1. The area of the triangle with vertices at P(2,1,5), Q(-1,3,4), R(3,0,6) is:

$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \vec{i} & \vec{j} & k \\ -3 & 2 & -1 \\ 1 & -1 & 1 \end{vmatrix} = \vec{i}(2-1) - \vec{j}(-3+1) + \vec{k}(3-2)$

$= \vec{i} + 2\vec{j} + \vec{k}$

area $= \frac{1}{2} |\vec{PQ} \times \vec{PR}| = \frac{1}{2}\sqrt{1+4+1}$

$= \sqrt{6}/2$

A. $\sqrt{3}/2$

B. 3

C. $\sqrt{6}/2$

D. $\sqrt{6}$

E. $\sqrt{14}/2$

(9 pt) 2. Find symmetric equations for the line in the plane $x + 4y + z = 2$ containing the point $(2, 1, -4)$ and parallel to the line $x = 1 + 3t, y = -1 - t, z = 2 + t$:

line direction $= \langle 3, -1, 1 \rangle$

$\frac{x-2}{3} = \frac{y-1}{-1} = \frac{z+4}{1}$

A. $\frac{x-2}{3} = \frac{y-1}{-1} = z + 4$

B. $x + 2 = \frac{y+1}{4} = z - 4$

C. $x - 2 = \frac{y-1}{-1} = \frac{z+4}{3}$

D. $x - 2 = \frac{y-1}{4} = z + 4$

E. $\frac{x+2}{3} = \frac{y+1}{-1} = z - 4$

(9 pt) 3. A particle travels with acceleration $\vec{a}(t) = 2t\vec{i} + \cos t\vec{j} + e^t\vec{k}$ and initial velocity $\vec{v}(0) = \vec{j} + \vec{k}$. What is the velocity at time $t = \pi$?

A. $\pi^2\vec{i} + e^\pi\vec{k}$

B. $\pi^2\vec{i} + \vec{j} + e^\pi\vec{k}$

C. $\pi^2\vec{i} + \vec{2}j + e^\pi\vec{k}$

D. $\pi^2\vec{i} + \vec{j} + (1 + e^\pi)\vec{k}$

E. $\pi^2\vec{i} + 2\vec{j} + (1 + e^\pi)\vec{k}$

$v' = \langle 2t, \cos t, e^t \rangle$

$\Rightarrow v(t) = \langle t^2, \sin t, e^t \rangle + \langle c_1, c_2, c_3 \rangle$

$\Rightarrow \langle 0, 1, 1 \rangle = v(0) = \langle 0, 0, 1 \rangle + \langle c_1, c_2, c_3 \rangle$

$= \langle c_1, c_2, c_3 + 1 \rangle$

So $c_1 = 0$, $c_2 = 1$, $c_3 = 0$

$\Rightarrow v(t) = \langle t^2, 1 + \sin t, e^t \rangle$

$v(\pi) = \langle \pi^2, 1 + \cancel{\sin \pi}^{0}, e^\pi \rangle$

$= \langle \pi^2, 1, e^\pi \rangle$

(9pt) 4. The tangent plane at $x = 3, y = 2$ to the graph of $z = \sqrt{x^2 + 4y^2}$ $= f(x,y)$ intersects the xy plane in the line:

A. $3x + 2y = -37$

B. $3x + 2y = 13$

C. $3x + 4y = -8$

D. $3x + 8y = 0$

E. $3x + 8y = 5$

tang. plane: $z = f(3,2) + f_x(3,2)(x-3) + f_y(3,2)(y-2)$

$= \sqrt{9+16} + \frac{1}{\cancel{2}}(x^2+4y^2)^{-1/2} \cdot \cancel{2}x \Big|_{(3,2)} (x-3)$

$+ \frac{1}{2}(x^2+4y^2)^{-1/2} \cdot 8y \Big|_{(3,2)} (y-2)$

$= 5 + \frac{3}{5}(x-3) + \frac{8}{5}(y-2)$

xy plane $\Rightarrow z = 0$ So

$0 = 5 + \frac{3}{5}(x-3) + \frac{8}{5}(y-2)$

$\Rightarrow 0 = 25 + 3(x-3) + 8(y-2) = 25 + 3x - 9 + 8y - 16$

$= 3x + 8y + 0$

$= 3x + 8y$

(9pt) 5. The temperature, T, in a plate is given by $T = xy^2$. At the point (4,3), find a vector $\vec{v}$ with the property that the directional derivative of T in the direction $\vec{v}$ is zero.

$T = xy^2$

$\nabla T\big|_{(4,3)} = \langle T_x, T_y \rangle\big|_{(4,3)} = \langle y^2, 2xy \rangle\big|_{(4,3)}$

$= \langle 9, 24 \rangle$

A. $\vec{v} = 9\vec{i} + 24\vec{j}$

B. $\vec{v} = 36\vec{i} + 36\vec{j}$

C. $\vec{v} = 36\vec{i} - 36\vec{j}$

D. $\vec{v} = 9i - 24\vec{j}$

E. $\vec{v} = -24i + 9\vec{j}$

want $\frac{\vec{V}}{|\vec{V}|} \cdot \langle 9, 24 \rangle = 0$

so $\vec{V} \cdot \langle 9, 24 \rangle = 0$

Take $\vec{V} = \langle -24, 9 \rangle$

(14 pt) 6. Let $z = y^2 - x^2$. (a) Compute the $x = k$, $y = k$, and $z = k$ traces and IDENTIFY THEM. (b) Use this information to SKETCH and IDENTIFY the graph of $z = y^2 - x^2$.

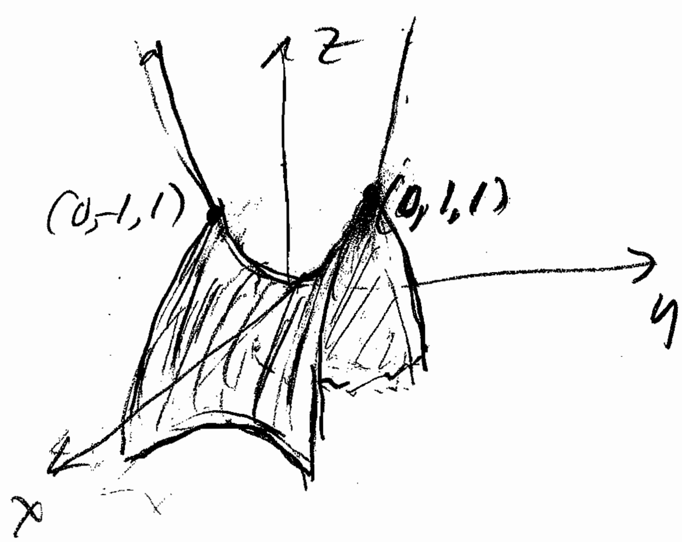
6a) $\underline{x=k}$: $z = y^2 - k^2$: parabola

$\underline{y=k}$: $z = k^2 - x^2$: parabola

$\underline{z=k}$: $k = y^2 - x^2$ $\begin{cases} \text{hyperbola if } k \neq 0 \\ \text{2 lines } (y=x, y=-x) \text{ if } k=0. \end{cases}$

6b) : $x=0$ trace is $z = y^2$

when $y=k$: $z = k^2 - x^2$ (parabolas pointing down)



(14 pt) 7. Suppose $z = \sin(x^2y)$. If $x(t) = t^3 + 1$ and $y(t) = 5t^2 - 2$, use the chain rule to compute

$$\frac{dz}{dt}(1)$$

note: $x(1) = 1+1 = 2$
$y(1) = 5-2 = 3$

$$\left.\frac{dz}{dt}\right|_1 = \left.\frac{\partial z}{\partial x}\right|_{(2,3)} \cdot \left.\frac{dx}{dt}\right|_1 + \left.\frac{\partial z}{\partial y}\right|_{(2,3)} \cdot \left.\frac{dy}{dt}\right|_1$$

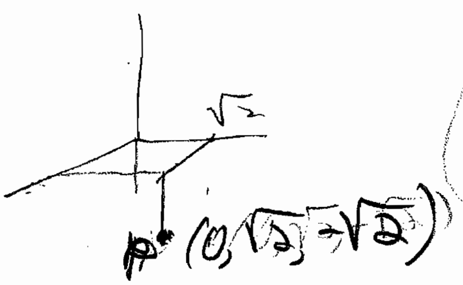
$$= 2xy \cdot \cos(x^2y)\Big|_{(2,3)} \cdot 3t^2\Big|_1$$

$$+ x^2 \cdot \cos(x^2y)\Big|_{(2,3)} \cdot 10t\Big|_1$$

$$= 12 \cdot (\cos 12) \cdot 3 + 4 \cdot (\cos 12) \cdot 10$$

$$= 76 \cdot (\cos 12)$$

(14 pt) 8. A point P has rectangular coordinates $(0, \sqrt{2}, -\sqrt{2})$. What are its spherical coordinates, (ρ, ϕ, θ)?

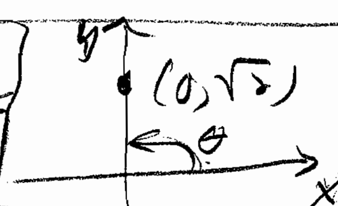


$$\rho = \sqrt{x^2+y^2+z^2} = \sqrt{0+2+2} = \sqrt{4} = 2.$$

$(x, y) = (0, \sqrt{2})$

so $r = \sqrt{0+2} = \sqrt{2}$

and $\boxed{\theta = \frac{\pi}{2}}$ (polar coords for (x,y).)



ϕ comes from: $z = \rho\cos\phi$ & $0 \le \phi \le \pi$

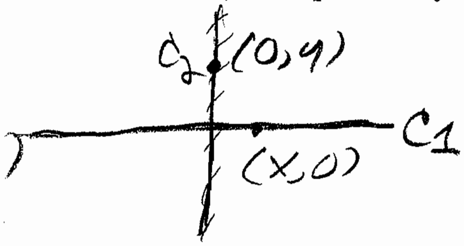
ie $-\sqrt{2} = 2\cos\phi$

ie $-\frac{1}{\sqrt{2}} = \cos\phi$

so $\phi = \frac{3\pi}{4}$

(14 pt) 9. Find the limit if it exists, or explain why it doesn't exist:

$$\lim_{(x,y)\to(0,0)} \underbrace{\frac{y^2+xy}{x^2+5y^2}}_{\equiv f(x,y)}.$$



Let C_1 be the curve formed by $(x,0)$

$$\lim_{x\to0} f(x,0) = \lim_{x\to0} \underbrace{\frac{0^2+0}{x^2+0}}_{=0} = \lim_{x\to0} 0 = 0$$

Let C_2 be the curve formed by $(0,y)$.

$$\lim_{y\to0} f(0,y) = \lim_{y\to0} \underbrace{\frac{y^2+0}{0+5y^2}}_{=\frac{1}{5}} = \lim_{y\to0} \frac{1}{5} = \frac{1}{5}$$

So the limit does not exist.

(because the limit as $(x,y)\to(0,0)$ along C_1 $\neq$ limit as $(x,y)\to(0,0)$ along C_2)