[10pts] 1. Find the domain D of the function. (Write answer in interval notation.)

$$f(x)=\frac{x-1}{x^2+5x-6}$$

$$f(x)=\frac{x-1}{(x+6)(x-1)}$$

$$(x+6)(x-1)=0 \Rightarrow x=-6, x=1$$

So the domain of $f(x)$ is all real # except -6 and 1.

Domain D by interval is :

$$D=(-\infty,\quad -6)\cup(-6,\quad 1)\cup(1,\quad \infty)$$

[8 pts] 2. Find all solutions of $\cos(2t)=-1$

$$2t=\pi+2\pi n$$
$$\Rightarrow t=\frac{\pi}{2}+\pi n$$

$$t=\frac{\pi}{2}+\pi n$$

[16 pts] 3. Find the following limit. If the limit doesn't exist, write 'DNE'

(a) $\lim_{x\to 4}\frac{16-x^2}{4-x}=\lim_{x\to 4}\frac{(4+x)(4-x)}{4-x}=\lim_{x\to 4}(4+x)=8$

8

(b) $\lim_{x\to 3}\frac{x+3}{x-3}=\lim_{x\to 3}\frac{6}{x-3}$

DNE

[9 pts] 4. Find the value of c, such that function

$$f(x)=\begin{cases}\frac{c-2}{x}, & if \quad x\leq -1 \\ x+2c, & if \quad x>-1\end{cases}$$

is continuous at $x=-1$.

Solution: from continuity at x=-1, we only need

$$\frac{c-2}{-1}=-1+2c$$
$$\Rightarrow -c+2=-1+2c$$
$$\Rightarrow 3=3c$$
$$\Rightarrow c=1$$

c=1

[8 pts]5. Let $f(x)=\frac{1}{x-1}$, find $f'(x)$ by using $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\frac{1}{x+h-1}-\frac{1}{x-1}}{h}$$

$$=\lim_{h\to 0}\frac{\frac{1}{x+h-1}\cdot\frac{x-1}{x-1}-\frac{1}{x-1}\cdot\frac{x+h-1}{x+h-1}}{h}$$

$$=\lim_{h\to 0}\frac{\frac{(x-1)-(x+h-1)}{(x+h-1)(x-1)}}{h}$$

$$=\lim_{h\to 0}\frac{\frac{x-1-x-h+1}{(x+h-1)(x-1)}}{h}$$

$$=\lim_{h\to 0}\frac{-h}{(x+h-1)(x-1)}\bullet\frac{1}{h}$$

$$=\lim_{h\to 0}\frac{-1}{(x+h-1)(x-1)}=\frac{-1}{(x-1)^2}$$

$$f'(x)=\frac{-1}{(x-1)^2}$$

[8 pts]6. Find an equation of the tangent line

to the graph of $f(x)=x^2+1$ at the point $(1, \quad 2)$.

From $f'(x)=2x$, so the slope of tangent line at (1,2) is $f'(1)=2\bullet 1=2$

Then equation of the tangent line at (1,2) is

$y-2=2(x-1) \quad \Rightarrow y-2=2x-2\Rightarrow y=2x$

Equation is:

$y=2x$

7. Find the derivative of functions by using the rules of differentiation.

[8pts] (a) $f(x)=\frac{3}{x^3}-\frac{x^3}{3}=3x^{-3}-\frac{1}{3}x^3$

$$f'(x)=\frac{d}{dx}(3x^{-3})-\frac{d}{dx}(\frac{1}{3}x^3)$$

$$=3\bullet(-3)x^{-4}-\frac{1}{3}\bullet 3x^2$$

$$=-9x^{-4}-x^2$$

$f'(x)=-9x^{-4}-x^2$

[8pts] (b) $g(x)=\frac{2}{\sqrt{x}}+\sin x=2x^{-\frac{1}{2}}+\sin x$

$$g'(x)=\frac{d}{dx}(2x^{-\frac{1}{2}})+\frac{d}{dx}\sin x$$

$$=2\bullet(-\frac{1}{2})x^{-\frac{3}{2}}+\cos x$$

$$=-x^{-\frac{3}{2}}+\cos x$$

$g'(x)=-x^{-\frac{3}{2}}+\cos x$

8. A hot-air balloon rises vertically from the ground so that its height after t sec is $h(t)=\frac{1}{3}t^3+3t$ ft $(0\le t\le 10)$.

[5 pts]

(a) What is the average velocity of the balloon between t=1 and t=3 ?

$$\frac{h(3)-h(1)}{3-1}=\frac{(\frac{1}{3}3^3+3\cdot 3)-(\frac{1}{3}+3)}{2}=\frac{15-\frac{1}{3}}{2}=\frac{22}{3}$$

$\frac{22}{3}$ ft/sec

[5 pts]

(b) What is the instantaneous velocity of the balloon at the end of 3 sec ?

$v(t)=h'(t)=t^2+3$

So $v(3)=3^2+3=12$

12 ft/sec

9. Growth Rate. The population of a city grows from an initial size of $10{,}000$ to an amount P, given by $P(t) = 10{,}000 + 50t^2$, where t is in years.

[5pts](a) Find the growth rate of P with respect to t.

$$P'(t) = \frac{d}{dt}10000 + \frac{d}{dt}(50t^2) = 0 + 50 \bullet 2t = 100t$$

100t

[5pts](b) Find the number of people in the city after 20 years (at $t = 20$).

$$P(20) = 10000 + 50(20)^2 = 30000$$

30000

[5pts](c) Find the growth rate at $t = 20$.

$$P'(20) = 100 \bullet 20 = 2000$$

2000