MATH 290B **EXAM 3** **Fall 2007**

Name: ____

Student ID number: _________________________________

Instructions:

1. Please fill in the above information. There are 7 problems.
2. You must show sufficient work to justify all answers. Correct answers with insufficient work will not receive full credit. Partial credit may be obtained provided sufficient work is shown.
3. No books, notes or papers may be used.
4. Only non-programmable, non-graphing calculator may be used.
5. The exam is self-explanatory. Please do not ask the instructor to interpret any of the exam questions.
6. Write your final answer in the box provided.
7. Good luck!

Problem #	Max possible	Your score
1	10	
2	18	
3	16	
4	10	
5	15	
6	15	
7	16	
Total	100	

1. [10pts] Find the equation of tangent line of the curve $x^3 + y^3 = 9$ at point $(1,2)$.

$$\frac{d}{dx}(x^3 + y^3) = \frac{d}{dx}9 \Rightarrow 3x^2 + 3y^2\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-x^2}{y^2}$$

$\frac{dy}{dx}|(1,2) = \frac{-1}{4}$,which is the slope of tangent line

$$y - 2 = \frac{-1}{4}(x - 1)$$

2.[18 pts] Find the derivative of following functions:

(a)[9pts] $f(x) = x^3 e^{-x}$

$$f'(x) = 3x^2 e^{-x} + x^3 e^{-x}(-1) = e^{-x}(3x^2 - x^3)$$

$$f'(x) = e^{-x}x^2(3 - x)$$

(b)[9pts] $g(x) = \ln\frac{x^2 + 1}{x} = \ln(x^2 + 1) - \ln x$

$$g'(x) = \frac{2x}{x^2 + 1} - \frac{1}{x} = \frac{x^2 - 1}{(x^2 + 1)x}$$

$$g'(x) = \frac{x^2 - 1}{\left(x^2 + 1\right)x}$$

3. (a)[10pts] Find the linearization of $f(x)=\sqrt{x}$ at $a=100$.

$$f'(x)=\frac{1}{2}x^{-\frac{1}{2}},\quad f'(100)=\frac{1}{2}100^{\frac{-1}{2}}=\frac{1}{20}$$

$$L(x)=f(100)+f'(100)(x-100)=\sqrt{100}+\frac{1}{20}(x-100)=\frac{1}{20}x+5$$

$$L(x)=\frac{1}{20}x+5$$

(b) [6pts] Approximate $\sqrt{98.9}$ by using linearization.

$$\sqrt{98.9}\approx L(98.9)=\frac{1}{20}98.9+5=9.945$$

$$\sqrt{98.9}\approx 9.945$$

4. [10pts] Solve t from the equation:

$$\frac{400}{5+2e^{5t}}=50$$

$$400=50(5+2e^{5t})\Rightarrow 8=5+2e^{5t}\Rightarrow e^{5t}=\frac{3}{2}\Rightarrow t=\frac{\ln 1.5}{5}=0.0811$$

$$t=0.0811$$

5. [15 pts] An apple orchard has an average yield of 40 bushels of apples/tree if tree density is 10 trees/acre. For each unit increase in tree density, the yield decreases by 1 bushel. How many trees should be planted in order to maximize the yield?

$Y(x) = (10 + x)(40 - x) = 400 + 30x - x^2$

$Y'(x) = 30 - 2x = 0 \Rightarrow c.p \quad is \quad x = 15$

$Y''(x) = -2 < 0$, so Y get max at x=15

25 trees

6. [15 Pts] If an open box is made from a square tin sheet (6 in. by 6 in.) by cutting out identical squares from each corner and bending up the resulting flaps, what dimensions will yield a box of maximum volume?

Volume $V(x) = (6 - 2x)(6 - 2x)x = 36x - 24x^2 + 4x^3$

Domain of V is $0 \le x \le 3$

$V'(x) = 36 - 48x + 8x^2 = 0 \Rightarrow cp \quad are \quad x = 3, x = 1$

$V(0) = 0$

$V(3) = 0$

$V(1) = 16$

So dimensions are 4x4x1 yields the max of volume

4x4x1

7. [16 pts] Limited Population Growth. A lake is stocked with 200 fish of a new variety. The size of the lake, the availability of food, and the number of other fish restrict growth in the lake to a limiting value of 400. The population of fish in the lake after time t, in months, is given by

$$P(t) = \frac{400}{1 + e^{-0.5t}}$$

(a) [6pts] Find the population after 4 months (at $t = 4$).

$P(4) = 352$

352

(b) [10pts] Find the growth rate $P'(t)$

$$P'(t) = \frac{200e^{-.5t}}{(1 + e^{-.5t})^2}$$

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