

MA 222 - EXAM 2 FORMULAS

Taylor Series: $f(x) = f(c) + f'(c)(x - c) + \frac{f''(c)}{2!}(x - c)^2 + \dots + \frac{f^{(n)}(c)}{n!}(x - c)^n + \dots$

Maclaurin Series: $f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots + \frac{f^{(n)}(0)}{n!}x^n + \dots$

Examples:

$$\begin{aligned} e^x &= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots \quad (\text{for all } x), & \sin x &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \quad (\text{for all } x), \\ \ln(1+x) &= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \quad (-1 < x \leq 1), & \cos x &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots \quad (\text{for all } x) \end{aligned}$$

Fourier Series: If f is periodic with period $2p$,

$$\begin{aligned} f(t) &= \frac{a_0}{2} + a_1 \cos \frac{\pi t}{p} + a_2 \cos \frac{2\pi t}{p} + \dots + a_n \cos \frac{n\pi t}{p} + \dots \\ &\quad + b_1 \sin \frac{\pi t}{p} + b_2 \sin \frac{2\pi t}{p} + \dots + b_n \sin \frac{n\pi t}{p} + \dots \end{aligned}$$

where

$$a_0 = \frac{1}{p} \int_{-p}^p f(t) dt, \quad a_n = \frac{1}{p} \int_{-p}^p f(t) \cos \frac{n\pi t}{p} dt \quad (n \neq 0), \quad b_n = \frac{1}{p} \int_{-p}^p f(t) \sin \frac{n\pi t}{p} dt$$