MA 222 **Final Exam Practice Problems**

The Formula Page may be used. It will be attached to the final exam.

1. Find $f'(\pi/2)$ if $f(x) = \dfrac{\sin(2x)}{x}$.
   A. $-2/\pi$ B. $-4/\pi$ C. $2/\pi$ D. π E. $\pi/8$

2. If $y = \ln(\sec x)$, then $\dfrac{dy}{dx} =$.
   A. $\cos x$ B. $\ln(\sec x \tan x)$ C. $\sin x$ D. $\tan x$ E. $\sec x$

3. Express as a single logarithm: $\ln x^3 - \ln \sqrt{x}$.
   A. $\ln(x^3 - \sqrt{x})$ B. $\ln(\frac{5}{2}x)$ C. $\ln(x^6)$ D. $\ln(3x - \frac{x}{2})$ E. $\ln(x^{\frac{5}{2}})$

4. If $y = e^{x^2}$ calculate y'.
   A. $2xe^{x^2}$ B. e^{2x} C. $x^2e^{x^2-1}$ D. $2xe^{2x}$ E. e^{x^2}

5. If $y = \ln \sqrt{x^2+1}$ calculate y'.
   A. $\dfrac{1}{\sqrt{x^2+1}}$ B. $\dfrac{2x}{\sqrt{x^2+1}}$ C. $\dfrac{x}{x^2+1}$ D. $\dfrac{1}{2(x^2+1)}$ E. None of these.

6. Find an equation for the tangent line to the curve $e^y + x^2 = 2$ at the point $(1, 0)$.
   A. $y = x - 1$ B. $y = 2x - 2$ C. $y = -2x + 2$ D. $y = -x + 1$ E. $y = -2x - 2$

7. Find the maximum value of the function $f(x) = x^2 \ln(2/x)$.
   A. 1 B. e^2 C. $2e$ D. 2 E. $2/e$

8. Which of the following best describes the function $y = \ln x - x$?
   A. There is a relative minimum at $x = 1$ and the curve is concave down for all $x > 0$.
   B. There is a relative maximum at $x = 1$ and the curve is concave down for all $x > 0$.
   C. There is a relative maximum at $x = 1$, the curve is concave down for $0 < x < 1$, and concave up for $x > 1$.
   D. There is a relative minimum at $x = 1$, the curve is concave down for $0 < x < 1$, and concave up for $x > 1$.
   E. None of these.

9. The velocity of an object falling through a resisting medium is given by $v = 100(1 - e^{-0.001t})$. Find the acceleration when $t = 100$. Give your answer correct to two decimal places.
   A. 0.09 B. 9.52 C. 90.48 D. 0.38 E. 1.14

10. Find y' if $y = x \cos 2x$.
    A. $-x\sin 2x + \cos 2x$ B. $-2x \sin 2x + \cos 2x$ C. $x \sin 2x + \cos 2x$ D. $2x \sin 2x + \cos 2x$
    E. $-2\sin 2x + \cos 2x$

11. Evaluate $\displaystyle\int \frac{x\,dx}{\sqrt{1-x^2}}$.
    A. $x \ln \mid 1 - x^2 \mid + C$ B. $2\sqrt{1-x^2} + C$ C. $-\frac{1}{2} \ln \mid 1 - x^2 \mid + C$ D. $-\sqrt{1-x^2} + C$ E. None of these.

12. Evaluate $\displaystyle\int \frac{3x+1}{x^2+x}\,dx$
    A. $6 \ln|x+5| \ln|x+1| + C$ B. $3\ln(x^2) + \ln|x| + C$ C. $3\ln|x^2+x| + C$ D. $\ln|x| - \ln|x+1| + C$
    E. $\ln|x+2| \ln|x+1| + C$

13. Evaluate $\displaystyle\int_1^3 \sqrt{x} \ln x\,dx$. (Give your answer correct to 2 decimal places.)
    A. 1.94 B. 1.50 C. -0.21 D. 1.01 E. 1.27

14. Find the area of the region bounded by the graph of $y = \sin 2x$, the x-axis, and the lines $x = 0$ and $x = \frac{\pi}{2}$.
A. 2 B. 1 C. 0 D. $\dfrac{1}{2}$ E. $\dfrac{3}{4}$

15. Find the first three non-zero terms of the Maclaurin series of $f(x) = \sqrt{1+3x}$.
A. $f(x) = 1 + \frac{3}{2}x - \frac{9}{4}x^2$ B. $f(x) = 1 + \frac{1}{2}\sqrt{1+3x} - \frac{1}{8}(1+3x)$ C. $f(x) = 1 + \frac{1}{2}x - \frac{1}{8}x^2$
D. $f(x) = 1 + \frac{3}{2}\sqrt{1+3x} - \frac{9}{8}(1+3x)$ E. $f(x) = 1 + \frac{3}{2}x - \frac{9}{8}x^2$

16. Using the Maclaurin series $\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5 - \ldots$, find the minimum number of terms required to calculate $\ln(1.3)$ so that the error is ≤ 0.001.
A. 2 B. 3 C. 4 D. 5 E. 6

17. Find the first three non-zero terms in the Taylor series for $f(x) = \sin 2x$ in powers of $(x - \frac{\pi}{8})$.
A. $f(x) = \sqrt{2}[\frac{1}{2} + (x - \frac{\pi}{8}) - (x - \frac{\pi}{8})^2]$ B. $f(x) = 2(x - \frac{\pi}{8}) - \frac{3}{2}(x - \frac{\pi}{8})^2 + \frac{4}{15}(x - \frac{\pi}{8})^5$
C. $f(x) = (x - \frac{\pi}{8}) - \frac{1}{3!}(x - \frac{\pi}{8})^3 + \frac{1}{5!}(x - \frac{\pi}{8})^5$ D. $f(x) = \sqrt{2}[\frac{1}{2} + \frac{1}{2}(x - \frac{\pi}{8}) - \frac{1}{4}(x - \frac{\pi}{8})^2]$
E. None of these.

18. Approximate $\displaystyle\int_0^{0.3} \cos\sqrt{x}dx$ using three terms of the appropriate Maclaurin series. (Give your answer correct to 4 decimal places.)
A. 0.8538 B. 0.2779 C. 0.9553 D, 0.2955 E. 0.1863

19. If f is a periodic function of period 2π and

$$f(x) = \begin{cases} 0 & \text{for } -\pi \leq x < 0 \\ 1 & \text{for } 0 \leq x \leq \frac{\pi}{2} \\ 0 & \text{for } \frac{\pi}{2} < x \leq \pi \end{cases}$$

calculate the first three non-zero terms of the Fourier series for $f(x)$. (That is, the first three non-zero terms in the series: $a_0 + a_1 \cos x + b_1 \sin x + a_2 \cos 2x + b_2 \sin 2x + \cdots$)
A. $\frac{\pi}{4} + \cos x + \sin x$ B. $\frac{1}{4} + \frac{1}{\pi}\cos x - \frac{1}{\pi}\sin x$. C. $\frac{1}{4} - \frac{\sqrt{2}}{\pi}\cos x + \frac{1}{\pi}\cos 2x$
D. $\frac{1}{4} + \frac{1}{\pi}\cos x + \frac{1}{\pi}\sin x$ E. None of these.

20. Find the general solution of the differential equation $y^2dx + (x+1)^2dy = 0$.
A. $\frac{1}{3}(x+1)^3 + \frac{1}{3}y^3 = C$ B. $\dfrac{1}{x+1} + \dfrac{1}{y} = C$ C. $\ln|x+1| + \ln|y| = C$
D. $2(x+1) + 2y = C$ E. $x + \dfrac{1}{y} = C$

21. Find the particular solution of the differential equation $y' + \dfrac{1}{x}y = x^2$ where $y = 2$ when $x = 1$.
A. $y = \dfrac{x^4}{4} + \dfrac{7}{4}$ B. $y = \dfrac{x^3}{3} + \dfrac{5}{3x}$ C. $y = \dfrac{x^3}{4} + \dfrac{7}{4x}$ D. $y = \dfrac{x^3}{4} + \dfrac{7}{4}$ E. None of these.

22. Find the particular solution of the differential equation $y'' + y' - 6y = 0$ where $y' = 0$ and $y = -1$ when $x = 0$.
A. $y = -\frac{1}{5}(2e^{-3x} + 3e^{2x})$ B. $y = -\frac{1}{5}(2e^{3x} + 3e^{-2x})$ C. $y = -\frac{1}{2}(e^{-3x} + e^{2x})$
D. $y = -\frac{1}{2}(e^{3x} + e^{-2x})$ E. None of these.

23. Find the general solution of the differential equation $D^2y - Dy + y = 0$.
A. $y = c_1e^{(1+\sqrt{3})x/2} + c_2e^{(1-\sqrt{3})x/2}$ B. $y = e^x[c_1\sin(\sqrt{3}x/2) + c_2\cos(\sqrt{3}x/2)]$
C. $y = e^x[c_1\sin(\sqrt{3}x) + c_2\cos(\sqrt{3}x)]$ D. $y = e^{x/2}[c_1\sin(\sqrt{3}x/2) + c_2\cos(\sqrt{3}x/2)]$

E. None of these.

24. Find the equation of the orthogonal trajectories of the curves $y = cx^5$.
A. $15cx^3y = 1$ B. $x^2 + 5y^2 = c$ C. $y = \dfrac{1}{15x^3} + c$ D. $\frac{1}{5}\ln|y| + \ln|x| = c$ E. $5cyx^4 = -1$.

25. Find the equation of the curve for which the slope at any point (x, y) is $x + y$ and which passes through the point $(0, 1)$.
A. $y = 2e^{-x} - x - 1$ B. $y = \frac{1}{2}e^x + \frac{1}{2}x^2$ C. $y = -x + 1$ D. $y = 2e^x - x - 1$ E. $y = e^x + x$

26. An object moves with simple harmonic motion according to the equation $\frac{d^2x}{dt^2} + 64x = 0$. Find the displacement x as a function of t if $x = 4$ and $\frac{dx}{dt} = 3$ when $t = 0$.
A. $x = 4\sin 8t + \frac{3}{8}\cos 8t$ B. $x = 3\sin 8t + 4\cos 8t$ C. $x = \frac{3}{64}\sin 64t + 4\cos 64t$
D. $x = \frac{3}{8}\sin 8t + 4\cos 8t$ E. $x = 8\sin 8t + 4\cos 8t$

27. Find the general solution of the differential equation $D^2y + 8Dy + 16y = 0$.
A. $y = c_1e^{-4x} + c_2xe^{-4x}$ B. $y = c_1e^{4x} + c_2xe^{4x}$ C. $y = c_1e^{-4x} + c_2e^{-4x}$ D. $y = c_1\sin 4x + c_2\cos 4x$
E. $y = c_1e^{4x} + c_2e^{-4x}$

28. Calculate the Laplace transform of $2e^{-3t}\sin 4t$.
A. $\dfrac{2}{(s-3)^2+16}$ B. $\dfrac{8}{(s+3)^2+16}$ C. $\dfrac{8}{(s-3)^2+16}$ D. $\dfrac{8}{(s+3)(s^2+16)}$ E. $\dfrac{2}{(s+3)^2+16}$

29. Calculate the inverse Laplace transform of $\dfrac{2s}{s^2+3s-4}$.
A. $\frac{1}{10}(4e^{4t} - e^t)$ B. $\frac{2}{5}(4e^{-4t} + e^t)$ C. $\frac{1}{10}(4e^{4t} + e^{-t})$ D. $\frac{2}{5}(4e^{4t} + e^{-t})$ E. None of these

30. Calculate the Laplace transform of the expression: $y'' - 3y' + 2y$, where $y = f(x)$, $f(0) = -1$ and $f'(0) = 2$.
A. $(s^2 - 3s + 2)L(f)$ B. $s^2L(f) + s - 2$ C. $(s^2 - 3s + 2)L(f) + s - 1$ D. $(s^2 - 3s + 2)L(f) + s + 1$
E. $(s^2 - 3s + 2)L(f) + s - 5$

31. Find the Laplace transform of the solution of the differential equation: $y' + 2y = e^{-2t}$; $y(0) = 2$.
A. $\dfrac{1}{(s+2)^2}$ B. $2 + \dfrac{1}{s+2}$ C. $\dfrac{2}{s+2} + \dfrac{1}{(s+2)^2}$ D. $\dfrac{2}{s-2} + \dfrac{1}{(s-2)^2}$ E. $\dfrac{1}{(s-2)^2}$

32. Use Laplace transforms to solve the differential equation: $y'' + 9y = 3t$; $y(0) = 1, y'(0) = -1$.
A. $y = \frac{1}{3}t - \frac{4}{9}\sin 3t + \cos 3t$ B. $y = \frac{1}{9}t - \frac{10}{27}\sin 3t + \cos 3t$
C. $y = 4\cos 3t - \frac{1}{3}\sin 3t$ D. $y = \cos 3t - \frac{1}{3}\sin 3t$ E. None of these.

33. Use Laplace transforms to solve the differential equation $D^2y - 2Dy + y = e^t$; $y(0) = 0, y'(0) = 0$.
A. $y = 2t^2e^t$ B. $y = \frac{1}{2}t^2e^{-t}$ C. $y = \frac{1}{2}t^2e^t$ D. $y = t^2e^{-t}$ E. $y = 2te^{-t}$

34. If $f(s) = \dfrac{s}{(s-1)^2(s+2)}$, which of the following is the partial fraction expansion of $f(s)$? (A, B and C are constants.)
A. $\dfrac{A}{s-1} + \dfrac{B}{s-1} + \dfrac{C}{s+2}$ B. $\dfrac{A}{(s-1)^2} + \dfrac{B}{s+2}$ C. $\dfrac{As}{s-1} + \dfrac{Bs}{(s-1)^2} + \dfrac{Cs}{s+2}$
D. $\dfrac{A}{s-1} + \dfrac{B}{(s-1)^2} + \dfrac{C}{s+2}$ E. $\dfrac{A}{s-1} + \dfrac{B}{s+2}$

35. A body whose temperature is 30°C is placed in a room whose temperature is 5°C. After two minutes the temperature of the object has dropped to 27°C. How long will it take for the

temperature to drop tp 15°C.
A. 9.35 min. B. 12.5 min. C. 14.34 min. D. 8.62 min. E. 17.33 min.

36. If the current in an AC circuit is given by $i = \cos t + \sin t$, then the first maximum of the current after $t = 0$ is
A. 2 A B. $\frac{1}{\sqrt{2}}$ A C. 1 A D. $\sqrt{2}$ A E. $\frac{1}{2}$ A

37. A certain radioactive substance decays according to the law $N = 6e^{-2t}$, where N (in kilograms) is the amount present and t is the time in years. Find the time rate of change of N with respect to t when $t = 2$, rounded to the nearest hundredth.
A. -0.22 B. -0.02 C. 0.02 D. 0.22 E. -0.012

Answers

1. B; 2. D; 3. E; 4. A; 5. C; 6. C; 7. E; 8. B; 9. A; 10. B; 11. D; 12. E; 13. A; 14. B 15. E; 16. C; 17. A; 18. B; 19. D; 20. B; 21. C; 22. A; 23. D; 24. B; 25. D; 26. D; 27. A; 28. B; 29. B; 30. E; 31. C; 32. A; 33. C; 34. D; 35. C; 36. D; 37. A