MA 511 ASSIGNMENT SHEET Summer 2008

Text: G. Strong *Linear Algebra and its Applications*, Fourth Edition

We hope to cover most of the text, and in particular give full attention to some interesting applications. This sheet will be updated throughout the semester, and I may make some remarks on several of the homework problems.

The course will move fast, and it is important to come to **every** *class.* The book is written in a very informal way, and unless you read it very critically you will have difficulty understanding what the author is saying; my job is partly to help you in this. The author has good summaries in the text, but you might slide over them — read carefully.

Some of the homework problems have answers/solutions in the back. There are far too many problems for us to penetrate a good percentage, but there are lots of opportunities for you to work out problems on your own.

I plan at least one major exam and either one or two more big exams or else several quizzes, likely every Friday; a program of many quizzes may help the class keep to date.

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We will learn soon that matrix multiplication is not commutative. So we will usually think of vectors as row vectors, and so usually write $A\mathbf{x}$ for the action of A on the vector $\mathbf{x}$. That means that rows and columns will play different roles.

1.1-1.3 *Systems of Linear Equations.* This introduces the basic framework and reinforces the value of a geometric viewpoint, as well as simply computing. From high school we learn that n 'linear' equations in n unknowns 'has' one and only one solution, but in fact this is not always true — it depends on how you count! We view such a system as either n linear equations with real numbers unknown or as a single equation in n-dimensional vectors. We introduce Gaussian elimination and address the *efficiency* of this algorithm. **Problems:** p. 9: 2, 3 4; p. 15: 6, 11, 18.

1.4 *Matrices and their algebra.* Matrices are an efficient may to express systems of equations in a way to which humans can relate. Elementary matrices provide an algebraic way to interpret Gaussian elimination. **Problems:** p. 26: 2, 3(a), 7, 20, 24

1.5 *Triangular factorization.* Decomposition $A = LU$ or (more symmetrically) $A = LDU$ (p. 36) *if there are no row exchanges necessary.* Otherwise, need to apply principle to PA instead of A, where P is a permutation matrix, and $P^{-1} = P^T$. **Problems:** p. 39: 1, 5, 6, 8, 13.

1.6 *Inverses, symmetric matrices.* **Problems:** p. 52: 5, 6, 11 (a, b), 13.

Review: p. 65: 12, 19, 22.

2.1 *Vector spaces (subspaces).* Closed under + and scalar multiplication. This is where you should be clear on the definition: some strange objects can be vector spaces. Contrast *subspace* and *subset*. Two important subspaces arise in solving systems of linear equations: the nullspace and the column space; be sure that you can make clear sentences about solving linear equations in terms of these (sub)spaces. **Problems:** p. 73: 2, 3, 7 (a, b, c).

2.2 $A\mathbf{x} = \mathbf{b}$ *in the general case.* Echelon, row(-reduced) echelon form, pivot and free variables. Note procedures outlined informally on pp. 80 and 83. **Problems:** p. 85: 2, 4, 6, 11.

2.3 *Linear independence, basis, dimension.* **Problems:** p. 98: 1, 6, 18, 19.

2.4 *Fundamental (sub)spaces* of an $(m \times n)$ matrix A. Column space (dim r), nullspace (dim $n-r$), row space (sol space of A^T), left nullspace (nullspace of A^T). (The first two are in $\mathbb{R}^m$; the other two in $\mathbb{R}^n$.) These are related to the echelon forms U and (reduced echelon) R of A. The *fundamental theorem of linear algebra* is on p. 106. We learn about left/right inverses. **Problems:** p. 110: 3, 4, 7, 11; p. 137: 2, 5.

2.6 *Linear Transformations.* Definition: $T(c\mathbf{x}+d\mathbf{y}) = cT(\mathbf{x})+dT(\mathbf{y})$. Examples come from matrix algebra and also from 'function spaces,' operations such as $f \to f',\ f \to \int_0^x f(t)\,dt$. Determined by action on a basis (however, $(x+y)^2 \neq x^2+y^2!$). Special matrices: P (projection), Q (rotation), H (reflection). **Problems:** p. 133: 4, 6, 7; p. 137: 29. 31.

3.1 **Lengths, Angles, Orthogonality** Orthogonal complements. Fundamental theorem of orthogonality (p. 144). Reinterpret fundamental theorem of linear algebra. **Problems:** p. 148: 2, 3, 11, 14, 19.

3.2 *Cosine!* is more important than sin . Projection onto a subspace (high-school math helps here). Projection: $P^2 = P$. Note: sometimes I write (x, y) instead of xy^T (which the book uses). **Problems:** p. 157: 3, 5, 10, 12, 17.

3.3 *Least squares.* Find 'best' solution to $A\mathbf{x} = \mathbf{b}$ with b confined to a subspace S. If $e(\subset S)$ is this solution so that $e = A\hat{x}$, then e is perpendicular to S. The number of applications of this section is a course in itself, we just skim the surface. **Problems:** p. 170: 3, 4 (think about why calculus is relevant here!), 22, 23, 31.