Computer Project # 2

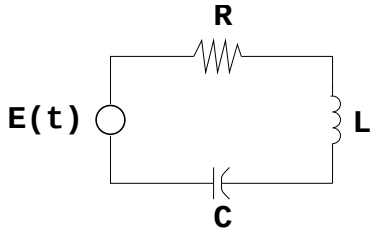
RLC-Circuits

Goal: Investigate the charge on a capacitor in an RLC circuit with varying voltage.

Tools needed: **ode45**, **plot**.

Description: If $Q(t) =$ charge on a capacitor at time t in an RLC circuit (with R, L and C being the resistance, inductance and capacitance, respectively, and $E(t) =$ applied voltage), then Kirchoff's Laws give the following 2^{nd} order differential equation for $Q(t)$:

$$(*) \quad L\,Q''(t) + R\,Q'(t) + \frac{1}{C}\,Q(t) = E(t)$$



Questions: Assume $L = 1$, $C = \frac{1}{5}$, $R = 4$ and $E(t) = 10\cos\omega t$.

(1) Use **ode45** (and plot routines) to plot the solution of $(*)$ with $Q(0) = 0$ and $Q'(0) = 0$ over the interval $0 \le t \le 80$ for $\omega = 0,\ 0.5,\ 1,\ 2,\ 4,\ 8,\ 16$.

(2) Let $A(\omega) =$ maximum of $|Q(t)|$ over the interval $30 \le t \le 80$ (this approximates the amplitude of the steady-state solution). Experiment with various values of ω and discuss what appears to happens to $A(\omega)$ as $\omega \to \infty$ and as $\omega \to 0$. Also, interpret your findings in terms of an equivalent spring-mass system.

Remark: There is an analogy between spring-mass systems and RLC circuits given by :

Spring-Mass System	RLC Circuit
$\boldsymbol{mu'' + cu' + ku = F(t)}$	$\boldsymbol{LQ'' + RQ' + \frac{1}{C}\,Q = E(t)}$
$u =$ Displacement	$Q =$ Charge
$u' =$ Velocity	$Q' = I =$ Current
$m =$ Mass	$L =$ Inductance
$c =$ Damping constant	$R =$ Resistance
$k =$ Spring constant	$1/C =$ (Capacitance)$^{-1}$
$F(t) =$ External force	$E(t) =$ Voltage