Name: Solutions **Math 511** **June 12, 2001**

(*Work will be graded on the basis of clarity as well as accuracy.*)

(15) 1. True-False. Write T or F and give a reason in each case If the answer is F, it would be convincing to give an example.

(*a*) A square matrix A may be factored

$$A = LU$$

with $L(U)$ lower (upper) triangular. False, You may need to change rows.
Ex (from class): $\begin{bmatrix} 0 & 3 \\ 2 & 4 \end{bmatrix}$

(*b*) Three linear equations in two unknowns can never have a solution~~s~~.
False

$$\begin{aligned} 3x + 2y &= 4 \\ x - y &= 3 \\ 4x + y &= 7 \end{aligned}$$

this row is the sum of the first two

(*c*) If matrices A and B are both 3×3 and invertible, then BA is invertible.
True $(BA)^{-1} = A^{-1}B^{-1}$

(15) 2. Write down the inverse of the 4×4 matrix $E_{31}(-5)$.

$$E_{31}(-5) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -5 & 0 & 1 \end{bmatrix}$$

(25) 3. Use the Gauss elimination method to solve the system $Ax = b$:

$$\begin{bmatrix} 1 & 1 & 3 \\ 2 & 4 & -4 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} h \\ i \\ j \end{bmatrix}$$

.

In doing this, show which elementary matrix you are using at each stage. You should end up with a system $Ux = b'$, and so you can write the components of b' in terms of the components $(h\ i\ j)^T$ of the original b. Be sure to write down the solution vector x (whose components will involve those of b).

$$\begin{bmatrix} 1&0&0\\0&1&0\\-1&0&1 \end{bmatrix}\begin{bmatrix} 1&0&0\\-2&1&0\\0&0&1 \end{bmatrix}\begin{bmatrix} 1&1&3\\2&4&-4\\1&2&-1 \end{bmatrix}$$

$$\begin{bmatrix} 1&0&0\\0&1&0\\-1&0&1 \end{bmatrix}\begin{bmatrix} 1&1&3\\0&2&-10\\1&2&-1 \end{bmatrix}$$

$$\begin{pmatrix} h \\ i \\ j \end{pmatrix} \to \begin{pmatrix} h \\ i-2h \\ j \end{pmatrix} \to \begin{pmatrix} h \\ i-2h \\ j-h \end{pmatrix} \to \begin{pmatrix} h \\ i-2h \\ j-h-\frac{1}{2}(i-2h) \end{pmatrix}$$

$$\begin{bmatrix} 1&0&0\\0&1&0\\0&-\frac{1}{2}&1 \end{bmatrix}\begin{bmatrix} 1&1&3\\0&2&-10\\0&1&-4 \end{bmatrix}$$

$$\begin{bmatrix} 1&1&3\\0&2&-10\\0&0&1 \end{bmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} h \\ i-2h \\ -\frac{i}{2}+j \end{pmatrix}$$

$z = (j - \frac{i}{2})$

$y = \frac{1}{2}[i - 2h + 10z] = \frac{1}{2}[i - 2h + 10(j - \frac{i}{2})]$

$= \frac{1}{2}(-2h + 10j - 4i)$

$= -h - 2i + 5j$

$x = h - y - 3z$

$= h - (-h - 2i + 5j) - 3(j - i/2)$

$= 2h + \frac{7}{2}i - 8j$