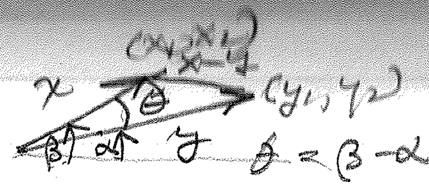



Tues 6/24 §3.1 Recall so far goal

Solve $y = Ax$

For what $y$? ($R(A)$ or col space)

How unique? ($N(A)$)

Orthogonality

$\|x\|$ (or $|x|$) $= x^T x$ ($x x^T$ no good)

$\perp$? $|x|^2 + |y|^2 = |x-y|^2$

So $x^T y = 0$ (we write this as $(x, y)$ sometimes)

Let $V$ be a subspace. Then $\{y \mid (x,y) = x^T y = 0$ for all in $V\}$ is a subspace (check!) and is called $V^\perp$ ($V$-perp). ($= y^T x$ — order immaterial!)

Not (yet) in book!

$*$ $x^T y = |x||y| \cos\theta$ (high school)

So (p 143) if $x^T y < 0$ the angle between the vectors is greater than $90°$.

Note! If $x_1, \ldots, x_n$ are nonzero and orthogonal, then they are lin ind.

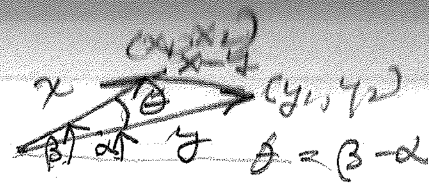
(Exercise in checking definition of linearly independent)

Orthogonal subspaces. Let's do a line and plane.

$L$: $\{t(x_1, x_2, x_3)\}$ $(1,4,5)$

$P$. $P$ has dim 2 so need 2 lin ind vectors

(if $x_1, x_2, x_3 \neq 0$) Try $(-x_2, x_1, 0)$ and $(0, -x_3, x_2)$.




Tues 6/24 §3.1 Recall so far goal

Solve $y = Ax$

For what $y$? ($R(A)$ or Col space)

How unique? $N(A)$)

Orthognality

$\|x\|$ (or $|x|$) $= x^T x$ ($x x^T$ no good)

$\perp$? $|x|^2 + |y|^2 = |x-y|^2$

So $x^T y = 0$ (we write this as $(x, y)$ sometimes)

Let $V$ be a subspace. Then $\{y ;\ (x, y) = x^T y = 0$ for all in $V\}$ is a subspace (check!) and is called $V^{\perp}$ ($V$-perp), ($\stackrel{!}{=} y^T x$ – order immaterial!)

Not (yet) in book!

Cunil $\times$ $x^T y = |x||y| \cos\theta$ (high school)

So (p 143) if $x^T y < 0$ the angle between the vectors is greater than $90^\circ$.

Note! If $x_1, \ldots x_n$ are non zero and orthogonal, then they are lin ind.

(Exercise in checking definition of linearly independent)

$L$

Orthogonal subspaces. Let's do a line and plane.

$L$ : $\{t\,(x_1, x_2, x_3)\}$ $(1, 4, 5)$

P. P has dim 2 so need 2 lin. ind vectors


(if $x_1 x_2 x_3 \neq 0$) Try $(-x_2, x_1, 0)$ and $(0, -x_3, x_2)$.

Def: $\mathcal{U}$ and $V$ are orthogonal if
$u^T v = 0$ for all $u \in \mathcal{U}$, $v \in V$.

Ex $\mathcal{U} = te_1$, $V = te_2$. Orthogonal in $\mathbb{R}^4$.
Find a vector orthogonal to both!
try $(0,0,4,3) = v_3$
Find something $\perp$ to $e_1, e_2, v_3$?
$(0,0,-3,4) = v_4$
So $e_1, e_2, v_3, v_4$ are another basis of $\mathbb{R}^4$.
Homework so far! #2, 3 p 149

Finish section

Fundamental subspaces of $A$

| | | |
|---|---|---|
| $R$ | Row space | Column space $C(A)$ [Range] |
| $N$ | Null space | Left Nullspace ($x^TA$ or $A^Tx = 0$) $(N(A^T))$ |

$(R^\perp) = N$, $N^\perp = R$, $(C(A))^\perp = N(A^T)$, $(N(A^T))^\perp = C(A)$
So $A$ sends Row space to column space.
Proofs on p 145

Alternative formulation (p 146)
$Ax = b \iff y^T b = 0$ (or $b^T y$) whenever $y^T A = 0$

Proof $A\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = b$ so $b \in C(A)$
So $b$ is $\perp$(!) to $N(A^T)$. But that means when
$A^T y = 0$ we have $b^T y = 0$
Argument is reversable.