P 149 #12 Orthog complement of row space

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 4 \end{bmatrix}$$

dim $R(A)$ is 2! And it is in 3 dim.

So by fundamental theorem, there is a vector in the nullspace: $N(A)$ has 1 dim

This gives a system of 2 equations in 3 unknowns

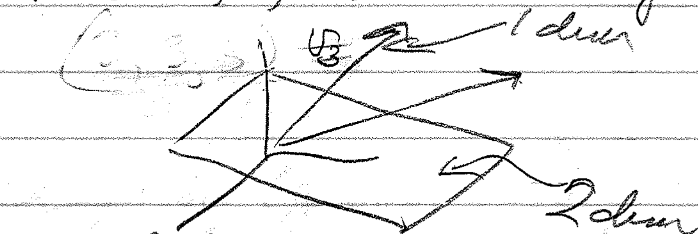
$$\begin{pmatrix} 1 & 0 & 2 \\ 1 & 1 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Gauss form $\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \end{pmatrix}\begin{pmatrix} \ \end{pmatrix} = \begin{pmatrix} \ \end{pmatrix}$

pivots

So bases for $N(A)$ is $\begin{pmatrix} -2 \\ -2 \\ 1 \end{pmatrix}$, it is $\perp$ to R.

Let's take $(3,3,3)$. Think geometrically



Find s so that

$(3,3,3) - s(-2,-2,1)$ is $\perp$ N

So $(3+2s, 3+2s, 3-s)\cdot(-2,-2,1) = 0$

This gives equation

$-9+s = 0$ or $s = 1$

So $(3,3,3)-(-2,-2,-1) = (5,5,4)$ is in

The row space

$s(1,0,2) + t(1,1,4) = (5,5,4)$

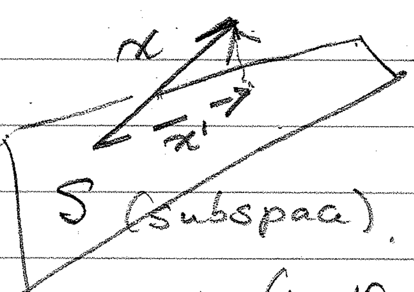
2 equations in two unknowns (s,t). Get

$s = 8$, $t = -3$

§3.2 Cosines, projections This leads to important material

$$\cos\theta = \frac{x\cdot y}{\|x\|\|y\|} \quad (\theta \text{ angle between } x \text{ and } y)$$

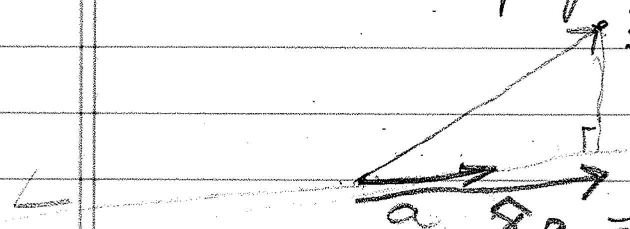
Projection onto a subspace (or line as special case)



<u>Question</u>: what vector in S is closest to x?
Solution: choose $x' \in S$ so that $(x-x') \perp S$,
(so that $(x-x')\cdot v = 0$ for all v in S).

① (Section 3.2) projection on a line, say $L = \{ta\}$

Problem: to find $\hat{x}$.



Use equation (geometry!)

$$(b-\hat{x}a) \perp a : \quad b\cdot a = \hat{x}\, a\cdot a \quad \text{or} \quad \hat{x} = \frac{b\cdot a}{a\cdot a}$$

(Book writes as $a^T(b-\hat{x}a)=0$, $\hat{x} = \frac{a^Tb}{a^Ta}$, I accept either.)

Formula $p = \frac{b\cdot a}{a\cdot a}\, a$ ← notice — to get a vector.

<u>Matrix form</u> Following the book, we write

$$p = \frac{b\cdot a}{a\cdot a}\, a = \frac{a^Tb}{a\cdot a}\, a \quad (b \text{ not changed})$$

as

matrix → $\frac{aa^T}{a^Ta}\, b$

so we have the projection matrix

$$P = \frac{aa^T}{a^Ta} \leftarrow \text{scalar}$$

-1-

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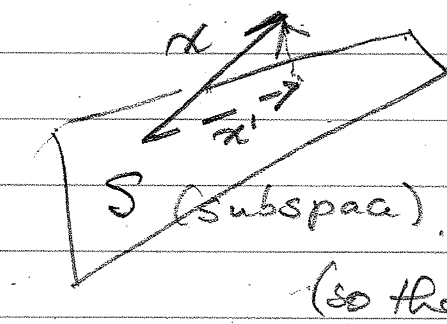
Projection onto a subspace (or line as special case)



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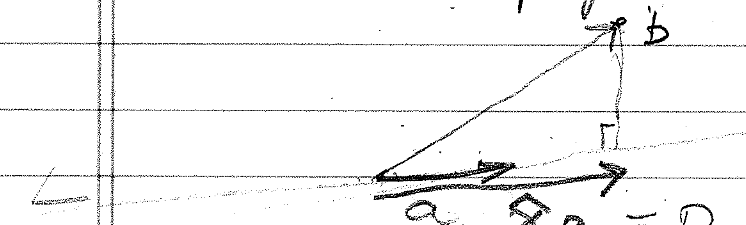

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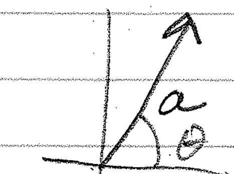
$$\text{matrix} \longrightarrow \frac{aa^T}{a^Ta}b$$

So we have the projection matrix

$$P = \frac{aa^T}{a^Ta} \longleftarrow \text{scalar}$$

Note that $P^2 = P$ and $P = P^T$ (check!)
Question: If we replace a by $2a$, what happens to P?

Ex  $a = (\cos\theta, \sin\theta)$ works.

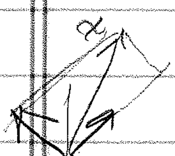
Ex P 158 # 11

(a) $a = \begin{pmatrix}1\\3\end{pmatrix}$ (no reason to care about length, keep nice numbers)

$$P_1 = \frac{a\,a^T}{a^T a} = \frac{\begin{pmatrix}1\\3\end{pmatrix}(1\ 3)}{(1\ 3)\begin{pmatrix}1\\3\end{pmatrix}} = \frac{\begin{pmatrix}1 & 3\\ 3 & 9\end{pmatrix}}{10} = \begin{pmatrix}1/10 & 3/10\\ 3/10 & 9/10\end{pmatrix}$$

Now to vector $\perp$ to a, call it (α,β) then $(1,3)\cdot(\alpha,\beta) = 0$ Try $(-3,1)$ Then if $a = \begin{pmatrix}-3\\1\end{pmatrix}$

$$P_2 = \frac{a\,a^T}{a^T a}$$ for this a, $P_2 = \begin{pmatrix}9/10 & -3/10\\ -3/10 & 1/10\end{pmatrix}$



(b) $P_1 + P_2 = Id$ (Geometry)

$P_1 \cdot P_2 = 0$ (Think: P_2 sends to line, which P_1 sends to 0)

P 158 17(a) only. $P = \frac{\begin{pmatrix}1\\1\\1\end{pmatrix}(1\,1\,1)}{3} = \begin{pmatrix}1/3 & 1/3 & 1/3\\ 1/3 & 1/3 & 1/3\\ 1/3 & 1/3 & 1/3\end{pmatrix}$; $b = \begin{pmatrix}1\\2\\2\end{pmatrix}$

$Pb = \begin{pmatrix}5/3\\5/3\\5/3\end{pmatrix}$ (so $\parallel$ to a!)

error: $e = b - Pb = \begin{pmatrix}1\\2\\2\end{pmatrix} - \begin{pmatrix}5/3\\5/3\\5/3\end{pmatrix}$

$= \begin{pmatrix}-2/3\\1/3\\1/3\end{pmatrix}$ (this is $\perp$ to b, as expected)