Name: Answes **Math 511** **June 25, 2008**

(*Work will be graded on the basis of clarity as well as accuracy.*)

(15) 1. True-False. Write T or F and give a reason in each case If the answer is F, it would be convincing to give an example.

(*a*) If $Ax = Ay$ then $x = y$.

F dim $N(A)$ can be any # from 0 to n (A is $n \times n$), Or example: $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

(*b*) The zero vector belongs to every subspace of any vector space.

yes

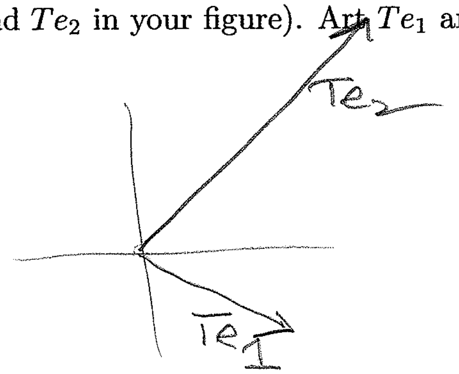
(*c*) If A is a 3×3 matrix whose columns are linearly independent, then so are its rows.

yes 3 pivot rows, so 3 pivot columns

(15) 2. Let T be the linear transformtion with matrix

$$\begin{bmatrix} 1 & 5 \\ -1 & 4 \end{bmatrix}.$$

Draw the $x-y$-plane and show the image of the basis vectors e_1 and e_2 under T (label them as Te_1 and Te_2 in your figure). Art Te_1 and Te_2 linearly independent?



(25) 3. Find the row space and the nullspace of the matrix

$$A = \begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 2 \\ 1 & 1 & 1 & 0 \end{bmatrix}.$$

What are their dimensions, and how do you know this is true (use results from class)? Give a basis for each of these subspaces.

$R(A)$ has dimension 2, basis $(0,0,1,2)$, $(1,1,1,0)$.
$N(A)$ has dim 2 then ~~too~~ too. But there are many possibilites for (x_1, x_2, x_3, x_4) so long as $x_3 + 2x_4 = 0$ and $x_1 + x_2 + x_3 = 0$. Here's one possibility:

$(0, 0, -2, 1)$, $(-1, 1, 0, 0)$

(20) 4 (*a*). Complete the definition: the vectors v_1, v_2, v_3 are *linearly independent* if:

$c_1 v_1 + c_2 v_2 + c_3 v_3 = 0$ is possible only if $c_1 = c_2 = c_3 = 0$

Use the answer in (*a*) to show that the vectors

$$\begin{array}{ccc} v_1 & v_2 & v_3 \\ [1, 2, 1], & [2, 4, 2], & [1, 1, 1] \end{array}$$

are not linearly independent (and thus are linearly dependent).

$-2v_1 + v_2 = 0$ So there is a "nontrivial" linear relation,

(20) 5(a). Let P_3 be the (4-dimensional) vector space of polynomials of degree at most 3, and D be the differentiation transformation. How do we know that D is a linear transformation on P_3 (you should refer to things you learned in an earlier class or two!)?

If p_1 and p_2 are polynomials we have learned that

$$(cp_1+dp_2)' = cp_1' + dp_2',$$

which means that $p \longrightarrow p'$ is a linear transformation

(b) Is the operation that takes a polynomial $P(x)$ to

$$\int_0^x P(t)\,dt$$

a linear transformation on P_3? Explain.

It is linear, but if $P \in P_3$,

$$\int^x P$$

can be a degree 4 polynomial, and so not in P_3