

MA 16100 FINAL EXAM PRACTICE QUESTIONS

- $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x^2 - x} =$ A. -1 B. 0 C. 1 D. 2 E. Does not exist
- If $y = (x^2 + 1) \tan x$, then $\frac{dy}{dx} =$ A. $2x \tan x + (x^2 + 1) \sec^2 x$ B. $2x \sec^2 x$
C. $2x \tan x + (x^2 + 1) \tan x$ D. $2x \tan x + 2x \sec^2 x$ E. $2x \tan x$
- If $h(x) = \begin{cases} x^2 + a, & \text{for } x < -1 \\ x^3 - 8 & \text{for } x \geq -1 \end{cases}$ determine all values of a so that h is continuous for all values of x . A. $a = -1$ B. $a = -8$ C. $a = -9$ D. $a = -10$ E. There are no values of a .
- Evaluate $\lim_{x \rightarrow 0^+} x \cos\left(\frac{1}{x}\right)$. (Hint: $-1 \leq \cos\left(\frac{1}{x}\right) \leq 1$ for all $x \neq 0$.) A. 0 B. 1
C. -1 D. $\frac{\pi}{2}$ E. Does not exist
- If $f(x) = \frac{1}{x+3}$, then $\lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1} =$ A. $\frac{1}{4}$ B. $\frac{1}{16}$ C. $-\frac{1}{16}$ D. $-\frac{1}{4}$ E. Does not exist
- The equation $x^3 - x - 5 = 0$ has one root in the interval $(-2, 2)$. This root is in the interval: A. $(-2, -1)$ B. $(-1, 0)$ C. $(0, 1)$ D. $(1, 2)$ E. $(-1, 1)$
- If $f(x) = \frac{1-x}{1+x}$, then $f'(1) =$ A. -1 B. $-\frac{1}{2}$ C. 0 D. $\frac{1}{2}$ E. 1
- If $y = \ln(1-x^2) + \sin^2 x$, then $\frac{dy}{dx} =$ A. $\frac{1}{1-x^2} + \cos^2 x$ B. $\frac{1}{1-x^2} + 2 \sin x \cos x$
C. $\frac{1}{1-x^2} + 2 \sin x$ D. $\frac{-2x}{1-x^2} + \cos^2 x$ E. $\frac{-2x}{1-x^2} + 2 \sin x \cos x$
- Find $f''(x)$ if $f(x) = \frac{1-x}{1+x}$ A. $\frac{4}{(1+x)^3}$ B. $\frac{-4}{(1+x)^3}$ C. $-\frac{4x}{(1+x)^3} + \frac{2}{(1+x)^2}$
D. $\frac{2(1+x)^2 - 2x(1+x)}{(1+x)^4}$ E. -1
- Assume that y is defined implicitly as a differentiable function of x by the equation $xy^2 - x^2 + y + 5 = 0$. Find $\frac{dy}{dx}$ at $(-2, 1)$. A. 9 B. $\frac{-5}{3}$ C. 1 D. 2 E. $\frac{5}{3}$
- Find the maximum and minimum values of the function $f(x) = 3x^2 + 6x - 10$ on the interval $-2 \leq x \leq 2$. A. max is 14, min is -10. B. max is -10, min is -13 C. max is 14, min is -13 D. no max., min is -10 E. max is 14, no min.
- For a differentiable function $f(x)$ it is known that $f(3) = 5$ and $f'(3) = -2$. Use a linear approximation to get the approximate value of $f(3.02)$. A. 6.02 B. 5.02 C. 5.04 D. 3 E. 4.96.
- Water is withdrawn from a conical reservoir, 8 feet in diameter and 10 feet deep (vertex down) at the constant rate of $5 \text{ ft}^3/\text{min}$. How fast is the water level falling when the depth of the water in the reservoir is 5 ft? ($V = \frac{1}{3}\pi r^2 h$). A. $\frac{15}{16\pi} \text{ ft/min}$
B. $\sqrt{\frac{3}{\pi}} \text{ ft/min}$ C. $\frac{2}{\pi} \text{ ft/min}$ D. $5\sqrt{3/4\pi} \text{ ft/min}$ E. $\frac{5}{4\pi} \text{ ft/min}$.

14. A rectangle is inscribed in the upper half of the circle $x^2 + y^2 = a^2$ with its base on the x-axis. Calculate the area of the largest such rectangle.
 A. $\frac{a^2}{2}$ B. $3a\sqrt{2}$ C. $2a^2$ D. $4a^2$ E. a^2 .
15. Given that $f(x)$ is differentiable for all x , $f(2) = 4$, and $f(7) = 10$, then the Mean Value Theorem states that there is a number c such that
 A. $2 < c < 7$ and $f'(c) = \frac{6}{5}$ B. $2 < c < 7$ and $f'(c) = \frac{5}{6}$ C. $4 < c < 10$ and $f'(c) = \frac{6}{5}$ D. $2 < c < 7$ and $f'(c) = 0$ E. $4 < c < 10$ and $f'(c) = 0$.
16. Suppose that the mass of a radioactive substance decays from 18 gms to 2 gms in 2 days. How long will it take for 12 gms of this substance to decay to 4 gms?
 A. $\frac{\ln 3}{\ln 2}$ days B. 1 day C. $\frac{\ln 2}{\ln 3}$ days D. 2 days E. $(\ln 3)^2$ days
17. Which of the following is/are true about the function $g(x) = 4x^3 - 3x^4$? (1) g is decreasing for $x > 1$. (2) g has a relative extreme value at $(0, 0)$. (3) the graph of g is concave up for all $x < 0$.
 A. (1), (2) and (3) B. only (2) C. only (1) D. (1) and (2) E. (1) and (3).
18. Find where the function $f(x) = 2/\sqrt{1+x^2}$ is increasing
 A. all x B. no x
 C. $x < 0$ D. $x > 0$ E. $x = 0$.
19. Let f be a function whose derivative, f' , is given by $f'(x) = (x-1)^2(x+2)(x-5)$. The function has
 A. a relative maximum at $x = -2$ and a relative minimum at $x = 5$. B. a relative maximum at $x = 5$ and a relative minimum at $x = -2$.
 C. relative maxima at $x = 1$, $x = -2$ and a relative minimum at $x = 5$. D. a relative maximum at $x = 5$ and relative minima at $x = 1$, $x = -2$ E. a relative maximum at $x = 1$ and relative minima at $x = -2$, $x = 5$.
20. Find $\frac{d}{dx} \int_1^{2x} \sqrt{t^2+1} dt$ at $x = \sqrt{2}$.
 A. 6 B. 3 C. $\sqrt{2}$ D. $\sqrt{4x^2+1}$ E. $\frac{1}{2\sqrt{3}}$.
21. $\int_3^4 x\sqrt{25-x^2} dx =$
 A. 0 B. -37 C. $\frac{37}{3}$ D. $-\frac{74}{3}$ E. $\frac{7}{12}$
22. $\lim_{x \rightarrow \infty} \frac{x^2+2x}{3x^2+4} =$
 A. 1 B. $\frac{3}{7}$ C. $\frac{1}{4}$ D. 0 E. $\frac{1}{3}$.
23. $\lim_{x \rightarrow 0} \frac{2x - \sin^{-1} x}{2x + \tan^{-1} x} =$
 A. $\frac{1}{2}$ B. 2 C. $\frac{1}{3}$ D. 1 E. 0

24. For a certain function $f(x)$, $f''(x) = 6x$, $f'(1) = 2$, and $f(2) = 3$. On which intervals is $f(x)$ increasing? A. $-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$ B. $x < -\frac{1}{\sqrt{3}}$ and $x > \frac{1}{\sqrt{3}}$ C. all x except $x = 0$ D. all x E. no x
25. Let R be the region between the graph of $y = \frac{1}{x}$ and the x -axis, from $x = a$ to $x = b$ ($0 < a < b$). If the vertical line $x = c$ cuts R into two parts of equal area, then $c =$
 A. \sqrt{ab} B. $\frac{a+b}{2}$ C. $\frac{\ln a + \ln b}{2}$ D. $\ln\left(\frac{a+b}{2}\right)$ E. $\ln\left(\frac{b-a}{2}\right)$
26. The area of the region between the graph of $y = \frac{1}{1+x^2}$ and the x -axis, from $x = -\sqrt{3}$ to $x = 1$ is
 A. $\frac{\pi}{2}$ B. $\frac{3\pi}{4}$ C. $\frac{5\pi}{12}$ D. $\frac{\pi}{3}$ E. $\frac{7\pi}{12}$
27. $\frac{d}{dx}(e^{2x} \ln \sqrt{1+x}) =$
 A. $e^{2x} \ln(1+x) + \frac{e^{2x}}{2(1+x)}$ B. $\frac{e^{2x}}{\sqrt{1+x}} + 2e^{2x} \ln \sqrt{1+x}$
 C. $\frac{1}{2}e^{2x} \ln(1+x) + \frac{e^{2x}}{2(1+x)}$ D. $\frac{2e^{2x}}{\sqrt{1+x}}$ E. $\frac{e^{2x}}{1+x}$
28. $\frac{d}{dx} x^{\sin x} =$
 A. $(\cos x)x^{\sin x}$ B. $(\sin x)x^{\sin x-1}$ C. $x^{\cos x}$ D. $x^{\sin x} \left[\frac{\sin x}{x} + (\cos x) \ln x \right]$
 E. $(\ln x)x^{\sin x}$
29. $\frac{d}{dx} \tan^{-1} e^{3x} =$
 A. $\frac{1}{1+e^{3x}}$ B. $\frac{e^{3x}}{1+e^{3x}}$ C. $\frac{3e^{3x}}{1+e^{6x}}$ D. $\frac{3e^{3x}}{1+e^{9x^2}}$ E. $\frac{3e^{3x}}{\sqrt{1-e^{6x}}}$
30. $\int_0^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}} dx =$
 A. $\frac{\pi}{2}$ B. $\frac{\pi}{6}$ C. $\sin^{-1} \sqrt{3}$ D. $\frac{\pi}{3}$ E. 1
31. $\int_0^4 \frac{x}{\sqrt{1+2x}} dx =$
 A. $\frac{7}{2}$ B. $\frac{10}{3}$ C. $\frac{11}{4} \tan^{-1} 3$ D. 3 E. 4
32. $\int_0^1 \frac{e^x}{1+e^x} dx =$
 A. $\ln \frac{1+e}{2}$ B. $\ln(1+e)$ C. $\frac{1}{2}$ D. $1 - \ln 2$ E. e

33. If $f(x) = \frac{4x-1}{2x+3}$, then $f^{-1}(x)$ is

- A. $\frac{4+2x}{1-3x}$ B. $\frac{1+3x}{4-2x}$ C. $\frac{6+2x}{1+3x}$ D. $\frac{4+2x}{1+3x}$ E. $\frac{4-2x}{1-3x}$

Answers: 1.D, 2.A, 3.D, 4.A, 5.C, 6.D, 7.B, 8.E, 9.A, 10.E, 11.C, 12.E, 13.E, 14.E, 15.A, 16.B, 17.C, 18.C, 19.A, 20.A, 21.C, 22.E, 23.C, 24.B, 25.A, 26.E, 27.A, 28.D, 29.C, 30.D, 31.B, 32.A, 33.B.