MA 511 HW8 solution.

2.3.2

Find the largest number of independent vectors among

$$v_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \quad v_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} \quad v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \quad v_4 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix} \quad v_5 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}$$

$$v_6 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$$

v_1, v_2, v_3 are independent. All six vectors are on the plane $(1,1,1,1) \cdot v = 0$ so no four of these six vectors can be independent.

The number $\underline{\underline{3}}$ is the $\underline{\underline{\text{dimension}}}$.

2.3.8

If w_1, w_2, w_3 are independent vectors. Show that the sum $v_1 = w_2 + w_3$, $v_2 = w_1 + w_3$ and $v_3 = w_1 + w_2$ are independent.

proof: If $c_1(w_2+w_3) + c_2(w_1+w_3) + c_3(w_1+w_2) = 0$

then $(c_2+c_3)w_1 + (c_1+c_3)w_2 + (c_1+c_2)w_3 = 0$

Since the w's independent, this requires

$$\begin{cases} c_2 + c_3 = 0 \\ c_1 + c_3 = 0 \\ c_1 + c_2 = 0 \end{cases} \Rightarrow \quad c_1 = c_2 = c_3 = 0$$

$\therefore$ w_1, w_2, w_3 are independent.