HW 9 Solution

2.3.14 Choose $x = (x_1, x_2, x_3, x_4)$ in $\mathbb{R}^4$. It has 24 rearrangement like (x_2, x_1, x_3, x_4) and (x_4, x_3, x_1, x_2). Those 24 vectors, including x itself, span a space S. Find specific vector x so that the dimension of S is: (a) 0, (b) 1, (c) 3, (d) 4.

Sol: (a) $x = 0$ (b) $x = (1, 1, 1, 1)$ (c) $x = (1, 1, -1, -1)$

(d) when the x's are not equal and don't add to zero,

2.3.16 Decide whether or not the following vectors are linearly independent, by solving $c_1 v_1 + c_2 v_2 + c_3 v_3 + c_4 v_4 = 0$

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad v_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \quad v_4 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

Decide also if they span $\mathbb{R}^4$, by trying to solve $c_1 v_1 + c_2 v_2 + c_3 v_3 + c_4 v_4 = (0, 0, 0, 1)$.

Sol.: Dependent with $v_1 - v_2 + v_3 - v_4 = 0$

Can't span $\mathbb{R}^4$ or produce $(0, 0, 0, 1)$