MA511 HW11 Sol.

2.5.2

In each columns, the sum of the first two entries equals to the third. Therefore, any combination will have $b_1+b_2-b_3=0$; $Ax=b \Rightarrow x_1-x_2=b_1,\ x_2-x_3=b_2,\ x_1-x_3=b_3 \Rightarrow b_1+b_2-b_3=0$. It means the sum of potential differences around a loop is zero.

2.5.4

$$A^TA=\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

(c, c, c) in its nullspace

$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ has pivots 2 and $3/2$