MA511 HW15 Sol.

3.1.2 Solve:

$(1,1)$ and $(1,0)$

$\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} \neq 0 \Rightarrow$ linearly independent

$(1,1)\cdot(1,0) = 1\times 1 + 1\times 0 \neq 0 \Rightarrow$ not orthogonal

3.1.6 Solve:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & -2 \end{bmatrix} \cancel{\neq 0 \Rightarrow \text{they're}}$$

Suppose $u = \langle x, y, z\rangle$ is the desire vector

$$\left.\begin{cases} \langle x,y,z\rangle \cdot \langle 1,1,1\rangle = 0 \\ \langle x,y,z\rangle \cdot \langle 1,-1,0\rangle = 0 \end{cases}\right\} \Rightarrow u = k\langle 1,1,-2\rangle \quad k\in\mathbb{R}$$

$\therefore \{u\in\mathbb{R}^3 : u = k\langle 1,1,-2\rangle, k\in\mathbb{R}\}$ is all vectors we're looking for.

Unify $\langle 1,1,1\rangle, \langle 1,-1,0\rangle, \langle 1,1,-2\rangle$, we have a othogonal basis

$$\left\{\left\langle \tfrac{1}{\sqrt3}, \tfrac{1}{\sqrt3}, \tfrac{1}{\sqrt3}\right\rangle, \left\langle \tfrac{1}{\sqrt2}, -\tfrac{1}{\sqrt2}, 0\right\rangle, \left\langle \tfrac{1}{\sqrt6}, \tfrac{1}{\sqrt6}, \tfrac{-2}{\sqrt6}\right\rangle\right\}$$