MA 511 HW 38

#6.5.2

Sol. $-u'' = x$ with

$$u(0) = u(1) = 0$$

gives $u = -\frac{1}{6}x^3 + \frac{1}{6}x$

Then $A = 3\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ $b = \begin{bmatrix} \frac{1}{9} \\ \frac{2}{9} \end{bmatrix}$

Leads to

$$y = A^{-1}b = \begin{bmatrix} \frac{4}{81} \\ \frac{5}{81} \end{bmatrix}$$

$$U = \frac{4}{81}V_1 + \frac{5}{81}V_2 = \begin{cases} \frac{4}{27}x & 0 \le x \le \frac{1}{3} \\ \frac{1}{27}x + \frac{1}{27} & \frac{1}{3} \le x \le \frac{2}{3} \\ \frac{5}{27} - \frac{5}{27}x & \frac{2}{3} \le x \le 1 \end{cases}$$

This has its largest error at $x = \sqrt{7}/3\sqrt{3}$

#6.5.4

Sol. $\int (V_1')^2 dx = \frac{16}{3}$

$\int 2V_1 dx = 2$

So. $U = \frac{3}{8}V_1 = \begin{cases} \frac{3}{2}x & 0 \le x \le \frac{1}{4} \\ \frac{1}{2} - \frac{1}{2}x & \frac{1}{4} \le x \le 1 \end{cases}$