

SOLUTIONS TO PRACTICE QUESTIONS FOR THE FINAL EXAM

1. $16x^2 - 4y^8 = 4(4x^2 - y^8) = 4((2x)^2 - (y^4)^2) = 4(2x - y^4)(2x + y^4)$

2.
$$\left(\frac{36a^{-4}b^{10}c^2}{a^2c^{-6}}\right)^{-1/2} = (36a^{-6}b^{10}c^8)^{-1/2} = (36)^{-1/2}(a^{-6})^{-1/2}(b^{10})^{-1/2}(c^8)^{-1/2} =$$

$$= \frac{1}{(36)^{1/2}} a^3 b^{-5} c^{-4} = \frac{1}{\sqrt{36}} \cdot \frac{a^3}{b^5 c^4} = \frac{a^3}{6b^5 c^4}$$

3.
$$\frac{3x}{3x+1} - \frac{x}{x-2} = \frac{3x}{(3x+1)} \cdot \frac{(x-2)}{(x-2)} - \frac{x}{(x-2)} \cdot \frac{(3x+1)}{(3x+1)} =$$

$$= \frac{3x(x-2) - x(3x+1)}{(3x+1)(x-2)} = \frac{3x^2 - 6x - 3x^2 - x}{(3x+1)(x-2)} = \frac{-7x}{(3x+1)(x-2)}$$

4.
$$(2x+1)^3(2)(3x-5)(3) + (3x-5)^2(3)(2x+1)^2(2)$$

$$= 6[(2x+1)^3(3x-5) + (3x-5)^2(2x+1)^2]$$

$$= 6(2x+1)^2[(2x+1)(3x-5) + (3x-5)^2]$$

$$= 6(2x+1)^2(3x-5)[(2x+1) + (3x-5)]$$

$$= 6(2x+1)^2(3x-5)(5x-4)$$

$$= 6(3x-5)(5x-4)(2x+1)^2$$

5.
$$\frac{xy^{-1}}{(x+y)^{-1}} = \frac{\frac{x}{y}}{\frac{1}{(x+y)}} = \frac{x}{y} \cdot \frac{(x+y)}{1} = \frac{x(x+y)}{y}$$

6. $A = P(1 + rt)$
 $A = P + Prt$
 $A - P = Prt$
 $\frac{A - P}{Pr} = \frac{Prt}{Pr}$
 $\frac{A - P}{Pr} = t$
 $t = \frac{A - P}{Pr}$

7.
$$\frac{4}{2p-3} + \frac{10}{4p^2-9} = \frac{1}{2p+3}$$

$$\frac{4}{2p-3} + \frac{10}{(2p-3)(2p+3)} = \frac{1}{2p+3}$$

$$(2p-3)(2p+3) \cdot \left[\frac{4}{2p-3} + \frac{10}{(2p-3)(2p+3)} \right] = \left[\frac{1}{2p+3} \right] \cdot (2p-3)(2p+3)$$

$$(2p+3) \cdot 4 + 10 = 1 \cdot (2p-3)$$

$$4(2p+3) + 10 = (2p-3)$$

$$8p + 12 + 10 = 2p - 3$$

$$8p - 2p = -3 - 12 - 10$$

$$6p = -25$$

$$p = -\frac{25}{6}$$

8.
$$\frac{\sqrt{x}+5}{\sqrt{x}-5} = \frac{(\sqrt{x}+5)(\sqrt{x}+5)}{(\sqrt{x}-5)(\sqrt{x}+5)} =$$

$$= \frac{x+5\sqrt{x}+5\sqrt{x}+25}{x+5\sqrt{x}-5\sqrt{x}-25} = \frac{x+10\sqrt{x}+25}{x-25}$$

9. $t = \#$ of hours the other person takes to complete the job.

fraction from 1st person + fraction from 2nd person = whole job

$$\left(\frac{1}{6}\right) \frac{\text{job}}{\text{hour}} \cdot 4\text{hours} + \left(\frac{1}{t}\right) \frac{\text{job}}{\text{hour}} \cdot 4\text{hours} = \left(\frac{1}{4}\right) \frac{\text{job}}{\text{hour}} \cdot 4\text{hours}$$

$$\left(\frac{2}{3}\right) \text{job} + \left(\frac{4}{t}\right) \text{job} = 1 \text{job}$$

$$\frac{2}{3} + \frac{4}{t} = 1$$

$$3t\left(\frac{2}{3} + \frac{4}{t}\right) = 1 \cdot 3t$$

$$2t + 12 = 3t$$

$$12 = t$$

$$t = 12$$

10.
$$\begin{cases} y = x + 1 \\ y^2 - x^2 = 145 \end{cases}$$

$$(x+1)^2 - x^2 = 145$$

$$x^2 + 2x + 1 - x^2 = 145$$

$$2x + 1 = 145$$

$$2x = 144$$

$$x = 72$$

11. let $t = \#$ hours truck has been traveling

$40t = 55(t-1)$	rate	time	distance
$40t = 55t - 55$	truck	40	t
$55 = 15t$	car	55	t - 1
			$55(t-1)$

$t = \frac{55}{15} = \frac{11}{3}$ hours, so distance is $40\left(\frac{11}{3}\right) = \frac{440}{3}$ miles

12. let $x = \#$ ml of the 50% solution

let $y =$ total # of ml

$$\begin{cases} x + 40 = y \\ x(.50) + 40(.20) = y(.25) \end{cases}$$

$$x(.50) + 8 = (x + 40)(.25)$$

$$.50x + 8 = .25x + 10$$

$$.25x = 2$$

$$x = 8 \text{ ml}$$

B

18.

$$\text{slope of line} \Rightarrow m = -\frac{1}{3}$$

$$\text{slope of line perpendicular} \Rightarrow m = 3$$

D

19.

$$2x - 3y = 7$$

$$-3y = -2x + 7$$

$$y = \frac{2}{3}x - \frac{7}{3}$$

$$\text{slope } m = \frac{2}{3}$$

$$\text{slope of parallel line } m = \frac{2}{3}$$

$$\text{point is } (2, -1); m = \frac{2}{3}$$

$$y = mx + b$$

$$-1 = \left(\frac{2}{3}\right)(2) + b$$

C

$$-1 = \frac{4}{3} + b$$

$$b = -\frac{7}{3}$$

$$\text{so } y = \frac{2}{3}x - \frac{7}{3}$$

20.

$$\text{Center} \Rightarrow (0, 2)$$

$$(x - h)^2 + (y - k)^2 = r^2$$

$$\text{radius} = 2$$

$$(x - 0)^2 + (y - 2)^2 = 2^2$$

$$x^2 + (y - 2)^2 = 4$$

B

$$x^2 + y^2 - 4y + 4 = 4$$

$$x^2 + y^2 - 4y = 0$$

21.

$$f(x) = 1 - \sqrt{x}, \quad g(x) = \frac{1}{x}$$

$$(g \circ f)(x) = g[f(x)] = g(1 - \sqrt{x}) = \frac{1}{1 - \sqrt{x}}$$

D

22.

$$f(x) = \frac{x}{x^2 + 1}$$

$$\frac{1}{f(3)} = \frac{1}{\frac{1}{(3)^2 + 1}} = \frac{1}{\frac{1}{10}} = \frac{10}{1}$$

D

23.

$$y = \frac{1}{3x - 2}$$

$$x = \frac{1}{3y - 2}$$

$$x(3y - 2) = 1$$

$$3xy - 2x = 1$$

$$3xy = 1 + 2x$$

$$y = \frac{1 + 2x}{3x} = f^{-1}(x)$$

24.

$$f(x) = x^2 - 2x + 4$$

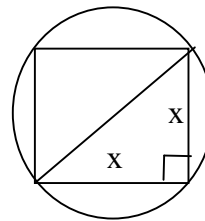
$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - 2(x+h) + 4 - (x^2 - 2x + 4)}{h}$$

A

$$= \frac{x^2 + 2xh + h^2 - 2x - 2h + 4 - x^2 + 2x - 4}{h} = \frac{2xh + h^2 - 2h}{h}$$

$$= \frac{h(2x + h - 2)}{h} = 2x + h - 2$$

25.



Let A = area of circle

$$\text{Area of circle} \Rightarrow A(r) = \pi r^2$$

$$\text{Diameter } (d) \text{ of circle} \Rightarrow x^2 + x^2 = d^2$$

$$2x^2 = d^2$$

$$d = \pm\sqrt{2x^2}$$

$$d = x\sqrt{2}$$

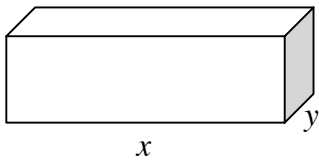
$$\text{Radius } (r) \text{ of circle} \Rightarrow r = \frac{x\sqrt{2}}{2}$$

$$\text{So, } A(x) = \pi \left(\frac{x\sqrt{2}}{2}\right)^2 = \pi \left(\frac{x^2(2)}{4}\right)$$

$$= \frac{\pi x^2}{2} \text{ or } \frac{\pi}{2} x^2$$

A

26.



$$\text{Volume} = 6 \text{ ft.}^3$$

$$xy(1.5) = 6$$

$$y = \frac{6}{1.5x} \quad \mathbf{B}$$

$$y = \frac{4}{x}$$

27.

$$T = k \frac{a^3}{\sqrt{d}}$$

$$4 = k \frac{2^3}{\sqrt{9}}$$

$$4 = k \frac{8}{3}$$

$$k = \frac{4}{1} \cdot \frac{3}{8}$$

$$k = \frac{3}{2}$$

$$T = \frac{3}{2} \cdot \frac{(-1)^3}{\sqrt{4}} \quad \mathbf{A}$$

$$T = \frac{3}{2} \cdot \left(-\frac{1}{2}\right)$$

$$T = -\frac{3}{4}$$

28.

$$x^2 - 4x - 2y - 4 = 0$$

$$2y = x^2 - 4x - 4$$

$$2y = (x^2 - 4x + 4) - 4 - 4$$

$$2y = (x - 2)^2 - 8$$

$$y = \frac{1}{2}(x - 2)^2 - 4$$

$$y = a(x - h)^2 + k$$

$$\text{Vertex}(h, k) = (2, -4)$$

29.

$$\text{Vertex} \Rightarrow V(0, 2)$$

$$y = a(x - h)^2 + k$$

$$\text{point on parabola} \Rightarrow (1, 0)$$

$$y = a(x - 0)^2 + 2$$

$$y = ax^2 + 2 \quad \mathbf{B}$$

$$0 = a(1)^2 + 2$$

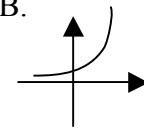
$$a = -2$$

$$y = -2x^2 + 2$$

30.

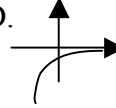
A.

B.

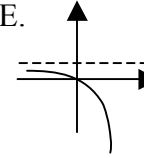


C.

D.



E.



31.

$$\log_b y^3 + \log_b y^2 - \log_b y^4 = \log_b (y^3 y^2) - \log_b y^4$$

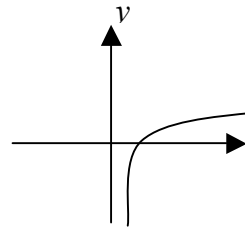
$$= \log_b y^5 - \log_b y^4 = \log_b \left(\frac{y^5}{y^4} \right) = \log_b y \quad \mathbf{B}$$

32.

$$f(x) = \log_a x \text{ if } a > 1$$

$$\text{example: if } a = 2, \text{ then } f(x) = \log_2 x, \quad \mathbf{D}$$

$$\text{Graph of } y = \log_2 x \Rightarrow 2^y = x$$



f is increasing, f does not have a y -intercept, the domain of f is $(0, \infty)$.

33.

$$\log \left(\frac{432}{(\sqrt{.095})(\sqrt[3]{72.1})} \right) = \log \left(\frac{432}{(.095)^{\frac{1}{2}}(72.1)^{\frac{1}{3}}} \right)$$

$$= \log 432 - \left(\log \left[(.095)^{\frac{1}{2}}(72.1)^{\frac{1}{3}} \right] \right) \quad \mathbf{B}$$

$$= \log 432 - \left(\log(.095)^{\frac{1}{2}} + \log(72.1)^{\frac{1}{3}} \right)$$

$$= \log 432 - \frac{1}{2} \log .095 - \frac{1}{3} \log 72.1$$

34. $\log_x 2 = 5$

$$x^5 = 2$$

$$(x^5)^{\frac{1}{5}} = (2)^{\frac{1}{5}}$$

$$x = \sqrt[5]{2}$$

$$x \approx 1.1487$$

D

35.

$$\frac{\log_5\left(\frac{1}{8}\right)}{\log_5(2)} = \log_2\left(\frac{1}{8}\right) = \log_2(2^{-3}) = -3$$

36. $3^{x-5} = 4$

$$\log 3^{x-5} = \log 4$$

$$(x-5)\log 3 = \log 4$$

$$x-5 = \frac{\log 4}{\log 3}$$

$$x = \frac{\log 4}{\log 3} + 5$$

C

37. $\log_3 \sqrt{2x+3} = 2$

$$3^2 = \sqrt{2x+3}$$

$$\sqrt{2x+3} = 9$$

$$(\sqrt{2x+3})^2 = (9)^2$$

$$2x+3 = 81$$

$$2x = 78$$

$$x = 39$$

Check: $\sqrt{2(39)+3} = 9$

$$9 = 9$$

Check: $\log_3 \sqrt{2(39)+3} = 2$

$$3^2 = \sqrt{81}$$

C

38.

$$\log_3 m = 8 \quad \log_3 \left(\frac{\sqrt{mn}}{p^3} \right) = \log_3 (mn)^{\frac{1}{2}} - \log_3 p^3$$

$$\log_3 n = 10 \Rightarrow = \log_3 \left(m^{\frac{1}{2}} n^{\frac{1}{2}} \right) - \log_3 p^3$$

A

$$\log_3 p = 6 \quad = \log_3 m^{\frac{1}{2}} + \log_3 n^{\frac{1}{2}} - \log_3 p^3$$

$$= \frac{1}{2} \log_3 m + \frac{1}{2} \log_3 n - 3 \log_3 p$$

$$= \frac{1}{2}(8) + \frac{1}{2}(10) - 3(6)$$

$$= 4 + 5 - 18 = -9$$

39. Half-life means when half of the initial amount still remains, $\frac{1}{2}q_0$.

$$\frac{1}{2}q_0 = q_0 e^{-0.0063t}$$

$$\frac{1}{2} = e^{-0.0063t}$$

$$\ln\left(\frac{1}{2}\right) = \ln\left(e^{-0.0063t}\right)$$

$$\ln\left(\frac{1}{2}\right) = -0.0063t$$

$$\frac{\ln(0.5)}{-0.0063} = t \approx 110.0 \text{ days}$$

40.

$$y = 2 + 2^x$$

When $x = 0$, $y = 2 + 2^0$

$$y = 2 + 1 = 3$$

D

41.

$$\begin{cases} x + 4y = 3 \Rightarrow x = 3 - 4y \\ 2x - 6y = 8 \end{cases}$$

$$2(3 - 4y) - 6y = 8$$

$$6 - 8y - 6y = 8$$

$$y = \frac{2}{-14} = -\frac{1}{7} \Rightarrow x = 3 - 4\left(-\frac{1}{7}\right) = \frac{25}{7}$$

$$\left(\frac{25}{7}, -\frac{1}{7}\right)$$

42.

$$\begin{cases} x^2 + y^2 = 16 \\ 2y - x = 4 \Rightarrow x = 2y - 4 \end{cases}$$

$$(2y - 4)^2 + y^2 = 16$$

$$4y^2 - 16y + 16 + y^2 = 16$$

$$5y^2 - 16y = 0$$

$$y(5y - 16) = 0$$

$$y = 0 \Rightarrow x = 2(0) - 4 = -4$$

$$y = \frac{16}{5} \Rightarrow x = 2\left(\frac{16}{5}\right) - 4 = \frac{12}{5}$$

$$(-4, 0) \& \left(\frac{12}{5}, \frac{16}{5}\right)$$

43.

$$\begin{cases} x + y - z = -1 \Rightarrow x = -y + z - 1 \\ 4x - 3y + 2z = 16 \\ 2x - 2y - 3z = 5 \end{cases}$$

$$\begin{cases} 4(-y + z - 1) - 3y + 2z = 16 \\ 2(-y + z - 1) - 2y - 3z = 5 \end{cases}$$

$$\begin{cases} -7y + 6z - 4 = 16 \\ -4y - z - 2 = 5 \end{cases}$$

$$\begin{cases} -7y + 6z = 20 \\ -4y - z = 7 \Rightarrow z = -4y - 7 \end{cases}$$

$$-7y + 6(-4y - 7) = 20$$

$$-31y = 62$$

$$y = -2$$

$$y = -2 \Rightarrow z = -4(-2) - 7$$

$$z = 1$$

44.

$$\begin{array}{r} x^2 + 6x + 34 \\ x^2 - 6x + 0 \overline{) x^4 + 0x^3 - 2x^2 + 0x - 3} \\ \underline{+(-x^4 + 6x^3 + 0x^2)} \end{array}$$

$$\begin{array}{r} 6x^3 - 2x^2 + 0x \\ +(-6x^3 + 36x^2 + 0x) \end{array} \quad \mathbf{C}$$

$$\begin{array}{r} 34x^2 + 0x - 3 \\ +(-34x^2 + 204x + 0) \end{array}$$

$$q(x) = x^2 + 6x + 34$$

$$r(x) = 204x - 3$$

$$204x - 3$$

45. If the denominator of the function is equal to zero the function will be undefined.

$$f(x) = \frac{(x+3)(x-3)}{x(x+2)}$$

when $x = 0$ or $x = -2$

46.

$$y = x^2(x-1)(x+1)^2$$

$$x \text{ - intercepts: } x^2(x-1)(x+1)^2 = 0$$

$$x = 0, x = 1, x = -1$$

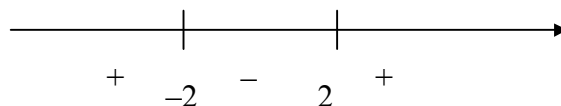
A

Interval	$(-\infty, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \infty)$
x^2	+	+	+	+
$x - 1$	-	-	-	+
$(x + 1)^2$	+	+	+	+
Result	-	-	-	+
	below	below	below	above
	x - axis	x - axis	x - axis	x - axis

47.

$$x = 2 \Rightarrow x \text{ - intercept}$$

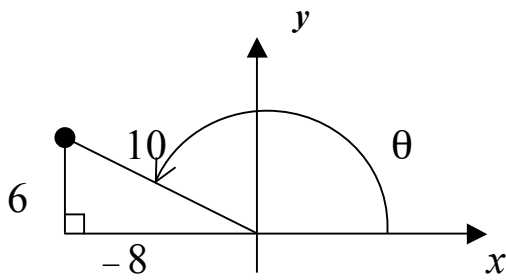
$$x = -2 \Rightarrow \text{vertical asymptote}$$



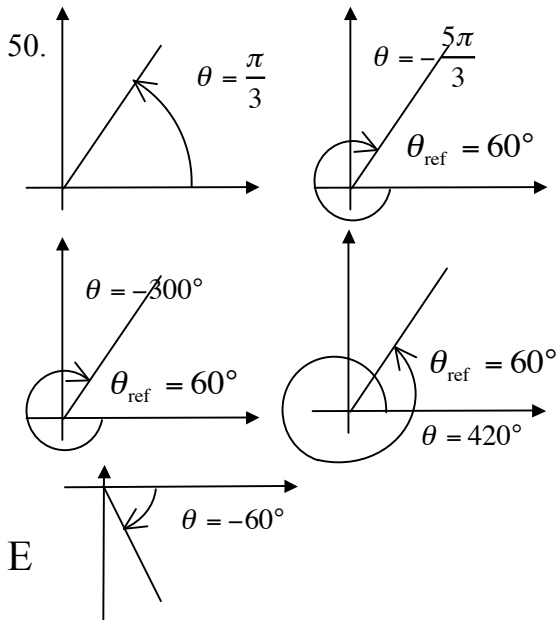
E is the closest answer. The scale is a bit off on the x -axis.

48. Shifted left 1 unit, then reflected about x -axis, then shifted down 2 units -- Answer: C

49.



$$\begin{aligned} \sin \theta &= \frac{6}{10} = \frac{y}{r} \\ x^2 + y^2 &= r^2 \\ x^2 + 6^2 &= 10^2 \\ x^2 &= 64 \\ x &= \pm 8 \\ x &= -8 \\ \cos \theta &= \frac{x}{r} = \frac{-8}{10} = -0.8 \end{aligned}$$



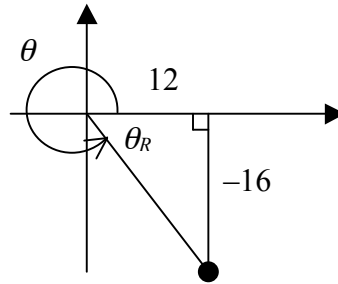
51.

$$135^\circ \cdot \frac{\pi \text{ radians}}{180^\circ} = \frac{3\pi}{4}$$

52.

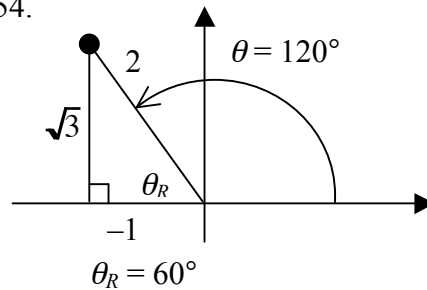
$$\sec 126^\circ = \frac{1}{\cos 126^\circ} \approx \frac{1}{-0.587785} \approx -1.7013$$

53.



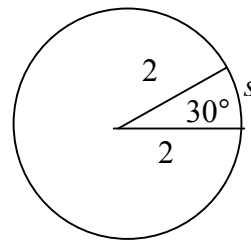
$$\tan \theta = \frac{y}{x} = \frac{-16}{12} = -\frac{4}{3}$$

54.



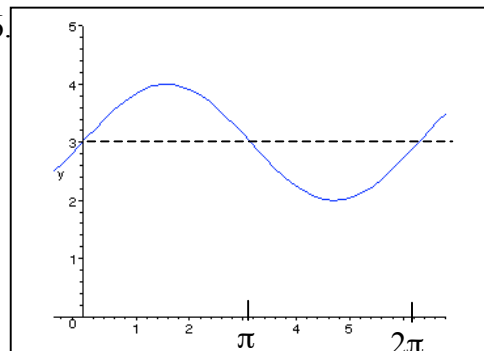
$$\tan \theta = \frac{y}{x} = \frac{\sqrt{3}}{-1} = -\sqrt{3}$$

55.



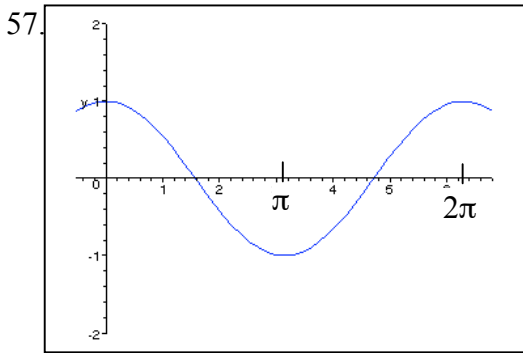
$$s = r\theta = 2\left(\frac{\pi}{6}\right) = \frac{\pi}{3} \approx 1.047$$

56.



The graph is the $y = \sin x$ shifted up three units.

I(yes), II(no), III(yes), IV(yes), **B**



$D = \text{all real number} = (-\infty, \infty)$
 $R = \text{all possible outputs/y-values} = [-1, 1]$

58.

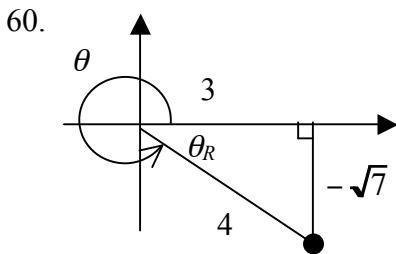
$$\frac{\tan^2 x}{1 + \sec x} = \frac{\sec^2 x - 1}{\sec x + 1} = \frac{(\sec x + 1)(\sec x - 1)}{(\sec x + 1)}$$

$$= \sec x - 1, \text{ remember } \tan^2 x = (\tan x)^2$$

59.

$$\frac{\tan x \cdot \cos x \cdot \csc x}{\cot x \cdot \sec x \cdot \sin x} = \frac{\tan x \cdot \tan x \cdot \cos x \cdot \cos x}{\sin x \cdot \sin x}$$

$$= \frac{\tan^2 x \cdot \cos^2 x}{\sin^2 x} = \frac{\sin^2 x}{\cos^2 x} \cdot \frac{\cos^2 x}{\sin^2 x} = 1$$



$$x^2 + y^2 = r^2$$

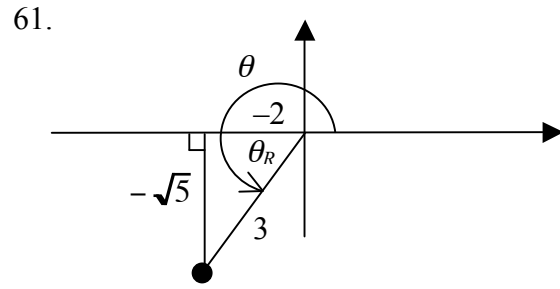
$$y^2 = 4^2 - 3^2$$

$$y = \pm \sqrt{7}$$

$$y = -\sqrt{7}$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

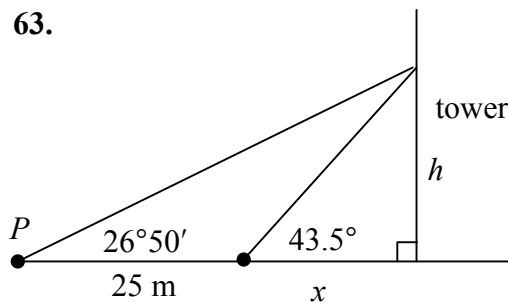
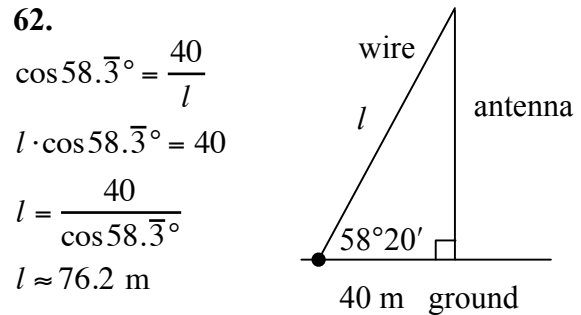
$$= 2 \left(\frac{-\sqrt{7}}{4} \right) \left(\frac{3}{4} \right) = -\frac{3\sqrt{7}}{8}$$



Given: $\tan \theta = \frac{\sqrt{5}}{2} = \frac{-\sqrt{5}}{-2} = \frac{y}{x} \uparrow$

$\frac{\theta}{2}$ is in QII, $\cos\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 + \cos \theta}{2}}$

$\cos\left(\frac{\theta}{2}\right) = -\sqrt{\frac{1 + \left(\frac{-2}{3}\right)}{2}} = -\sqrt{\frac{1 - \frac{2}{3}}{2}}$



$$\tan 43.5^\circ = \frac{h}{x}, \quad \tan 26.83^\circ = \frac{h}{x + 25}$$

$$h = x \cdot \tan 43.5^\circ$$

$$\tan 26.83^\circ = \frac{x \cdot \tan 43.5^\circ}{x + 25}$$

$$\tan 26.83^\circ (x + 25) = x \cdot \tan 43.5^\circ$$

$$x \tan 26.83^\circ + 25 \tan 26.83^\circ = x \cdot \tan 43.5^\circ$$

$$25 \tan 26.83^\circ = x \cdot \tan 43.5^\circ - x \tan 26.83^\circ$$

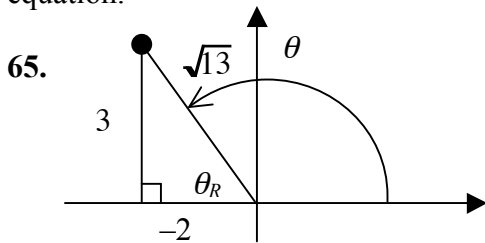
$$x = \frac{25 \tan 26.83^\circ}{\tan 43.5^\circ - \tan 26.83^\circ} \approx 28.541487$$

$$h = x \cdot \tan 43.5^\circ \approx 27.1 \text{ meters}$$

64. Examine r when $\theta = 0$ and as $\theta \rightarrow 90^\circ$

- A. $r = 1$ when $\theta = 0$ and as $\theta \rightarrow 90^\circ$ $r \rightarrow 2$, looks right as the angle changes from 0° to 90° .
- B. $r = 2$ when $\theta = 0$ and as $\theta \rightarrow 90^\circ$ $r \rightarrow 1$, looks incorrect as the angle changes from 0° to 90° .
The radial distance should be getting bigger.
- C. $r = 1$ when $\theta = 0$ and as $\theta \rightarrow 90^\circ$ $r \rightarrow 0$, looks incorrect as the angle changes from 0° to 90° .
The radial distance should be getting bigger.
- D. $r = 2$ when $\theta = 0$ and as $\theta \rightarrow 90^\circ$ $r \rightarrow 0$, looks incorrect as the angle changes from 0° to 90° .
The radial distance should be getting bigger.
- E. $r = 0$ when $\theta = 0$ and as $\theta \rightarrow 90^\circ$ $r \rightarrow 2$, looks incorrect as the angle changes from 0° to 90° .
When the angle is zero the radial distance should greater than zero.

Plugging in further angles would yield more points that will confirm that **A** is the correct polar equation.



$$r^2 = (-2)^2 + 3^2$$

$$r = \sqrt{13}$$

$$\tan \theta = \frac{y}{x} = \frac{3}{-2} \Rightarrow \tan^{-1}\left(-\frac{3}{2}\right) \approx -56.301^\circ$$

$$\theta_R = +56.301^\circ$$

$$\theta = 180^\circ - \theta_R \approx 123.7^\circ$$

$$(\sqrt{13}, 123.7^\circ)$$

66.

$$x^2 - 2x + y^2 = 0$$

$$x^2 + y^2 - 2x = 0$$

$$r^2 - 2r \cos \theta = 0$$

$$r(r - 2 \cos \theta) = 0$$

$r = 0$, which is not an equation of the given circle

or $r = 2 \cos \theta$