1. If $\vec{a} = \vec{i} + \vec{j} + \vec{k}$ and $\vec{b} = 2\vec{i} - \vec{k}$, find the vector projection of $\vec{b}$ onto $\vec{a}$, $\text{proj}_{\vec{a}}\, \vec{b}$.
A. $\frac{1}{3}\vec{a}$ B. $\frac{1}{\sqrt{3}}\,\vec{a}$ C. $\frac{1}{\sqrt{5}}\,\vec{a}$ D. $\frac{1}{\sqrt{3}}\,\vec{b}$ E. $\frac{1}{3}\,\vec{b}$

2. Find the angle between the vectors $\vec{a} = -\vec{i} + 2\vec{j}$ and $\vec{b} = \vec{i} + 3\vec{j}$
A. $\frac{3\pi}{4}$ B. $\frac{\pi}{4}$ C. $\frac{2\pi}{3}$ D. $\frac{5\pi}{6}$ E. $\frac{11\pi}{12}$

3. Find the area of the triangle with vertices at the points $(1, 0, 2)$ $(2, 4, -3)$ and $(1, 2, 1)$.
A. $\frac{1}{2}\sqrt{41}$ B. $\sqrt{41}$ C. $\sqrt{10}$ D. $\frac{\sqrt{2}}{2}\sqrt{21}$ E. $\frac{41}{2}$

4. If $\vec{a} = \vec{i} - \vec{j}$ and $\vec{b} = 2\vec{j} - \vec{k}$, find a unit vector orthogonal to both $\vec{a}$ and $\vec{b}$.
A. $\frac{1}{\sqrt{2}}\,(\vec{i} + \vec{j})$ B. $\frac{1}{\sqrt{6}}\,(\vec{i} + \vec{j} + 2\vec{k})$ C. $\frac{1}{\sqrt{5}}\,(\vec{j} + 2\vec{k})$ D. $\vec{i} + \vec{k}$ E. $\frac{1}{\sqrt{5}}\,(\vec{i} + 2\vec{k})$

5. The radius of the sphere $x^2 + y^2 + z^2 + 2x + 4y - 6z = 3$ is
A. $3 + \sqrt{13}$ B. $\sqrt{13}$ C. $\sqrt{65}$ D. $3 + \sqrt{56}$ E. $\sqrt{17}$

6. The area of the region enclosed by the curves $y = x^2 + 1$ and $y = 2x + 9$ is given by
A. $\int_{-2}^{4} (x^2 + 1 - 2x - 9)dx$ B. $\int_{-2}^{4} (2x + 9 - x^2 - 1)dx$ C. $\int_{-2}^{2} (2x + 9 - x^2 - 1)dx$
D. $\int_{-4}^{2} (2x + 9 - x^2 - 1)dx$ E. $\int_{-4}^{2} (x^2 + 1 - 2x - 9)dx$

7. The volume of the solid obtained by rotating about the x–axis the region in the first quadrant bounded by the graphs of $y = 1 - x^2$, $y = 2x$, and $x = 0$ is given by
A. $\int_{0}^{\sqrt{2}-1} (1 - x^2 - 2x)dx$ B. $\int_{0}^{\sqrt{2}-1} \pi(1 - x^2 + 2x)dx$ C. $\int_{-\sqrt{2}-1}^{0} \pi[(1 - x^2)^2 - (2x)^2]dx$
D. $\int_{0}^{\sqrt{2}-1} \pi[(1 - x^2)^2 - (2x)^2]dx$ E. $\int_{0}^{\sqrt{2}-1} [(2x)^2 - (1 + x^2)^2]dx$

8. Let R be the region bounded by the curves $y = x^2$ and $y = 2x$. Using the method of cylindrical shells, the volume of the solid generated by rotating R about the x–axis, is given by
A. $\int_{0}^{2} \pi(2x - x^2)dx$ B. $\int_{0}^{2} 2\pi(2x - x^2)^2 dx$ C. $\int_{0}^{2} \pi x^2(x^2 - \frac{1}{2}\,x)dy$ D. $\int_{0}^{4} \pi y^2(\frac{1}{2}\,y - \sqrt{y})dy$
E. $\int_{0}^{4} 2\pi y(\sqrt{y} - \frac{1}{2}\,y)dy$

9. A right circular conical tank of height 20 ft. and base radius 5 ft. has its vertex at the bottom, and its axis vertical. If the tank is full of water at 62.5 lb./cu. ft., the work required to pump all the water over the top is: (Take the y-axis upwards along the axis of the tank and the origin at its vertex).
A. $62.5\pi \int_{0}^{20} (20 - y)(\frac{y}{4})^2 dy$ B. $62.5\pi \int_{0}^{2\pi} y(\frac{y}{4})^2 dy$ C. $62.5\pi \int_{0}^{20} (20 - y)^2(\frac{y}{4})dy$
D. $62.5\pi \int_{0}^{20} (20 - y)(\frac{y}{2})^2 dy$ E. $62.5\pi \int_{0}^{20} (20 - y)(2y)^2 dy$

10. A force of 9 lb. is required to stretch a spring from its natural length of 6 in. to a length of 8 in. Find the work required to stretch it from its natural length to 10 in.
A. 1 ft.–lb. B. 1.5 ft.–lb. C. 2 ft.–lb. D. 3 ft.–lb. E. 4 ft.–lb.

11. $\int_{0}^{1} xe^{3x}dx =$
A. $\frac{2}{9}\,e^3$ B. $\frac{1}{9} + \frac{2}{9}\,e^3$ C. 1 D. $\frac{1}{9}$ E. $\frac{1}{9}\,e^3 - 1$

12. $\int_{0}^{\pi/2} \cos^3 x dx =$

A. $\frac{\pi}{2} - \frac{1}{3}$ B. $\frac{\pi}{2} + \frac{1}{3}$ C. 0 D. $-\frac{2}{3}$ E. $\frac{2}{3}$

13. For the integral $\int (1-x^2)^{3/2} dx$, (i) choose a trigonometric substitution to simplify the integral and (ii) give the resulting integral

A. (i) $x = \sec\theta$, (ii) $\int \tan^3\theta d\theta$ B. (i) $x = \sec\theta$, (ii) $\int \tan^4\theta \sec\theta d\theta$

C. (i) $x = \sec\theta$, (ii) $\int \tan^3\theta \sec^2\theta d\theta$ D. (i) $x = \sin\theta$, (ii) $\int \cos^3\theta d\theta$ E. (i) $x = \sin\theta$, (ii) $\int \cos^4\theta d\theta$

14. Give the form of the partial fraction decomposition of $\frac{3x+2}{(x^2+1)(x-1)^2}$.

A. $\frac{A}{x^2+1} + \frac{B}{(x-1)^2}$ B. $\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-1} + \frac{D}{(x-1)^2}$ C. $\frac{Ax+B}{x^2+1} + \frac{C}{x-1} + \frac{D}{(x-1)^2}$

D. $\frac{A}{x^2+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$ E. $\frac{Ax+B}{x^2+1} + \frac{C}{(x-1)^2}$

15. $\int_{1}^{2} \frac{1}{x^3+x} dx =$

A. $\ln 2 + \ln\frac{2}{5}$ B. $\ln 2 + \frac{1}{2}\ln\frac{2}{5}$ C. $\ln 2 + \tan^{-1} 5 - \tan^{-1} 2$ D. $2 + \frac{1}{2}\tan^{-1} 5 - \frac{1}{2}\tan^{-1} 2$ E. $\ln\frac{2}{5}$

16. Indicate convergence or divergence for each of the following improper integrals:

$$\text{(I)} \int_2^\infty \frac{1}{(x-1)^2} dx \qquad \text{(II)} \int_0^2 \frac{1}{(x-1)^2} dx$$

A. (I) converges; (II) diverges B. (I) converges; (II) converges

C. (I) diverges; (II) converges D. (I) diverges; (II) diverges

17. The length of the graph of $y = x^{\frac{3}{2}}$ for $0 \le x \le 1$ is

A. $\frac{2}{3}\sqrt{\frac{5}{2}}$ B. $\frac{4}{27}\sqrt{13}$ C. $\frac{4}{9}\sqrt{\frac{5}{2}}$ D. $\frac{8}{27}((\frac{13}{4})^{\frac{3}{2}} - 1)$ E. $\frac{4}{9}(\sqrt{\frac{5}{2}} - 1)$

18. The semicircular lamina bounded by the x–axis and $y = \sqrt{4-x^2}$, $-2 < x < 2$, has density $\rho = 1$. Its center of mass $(\overline{x}, \overline{y})$ is

A. $(0, \frac{8}{3\pi})$ B. $(\frac{8}{3\pi}, 0)$ C. $(0, 1)$ D. $(1, 0)$ E. $(0, 0)$

19. Evaluate the limit $\lim_{n\longrightarrow\infty} \left[1 + \frac{(-1)^n}{n}\right]$.

A. 0 B. 1 C. -1 D. 2 E. limit does not exist

20. Evaluate the limit $\lim_{n\longrightarrow\infty} (\sqrt[n]{n} + \frac{1}{n!})$.

A. 0 B. 1 C. e D. $1/e$ E. limit does not exist

21. If $s = \sum_{n=0}^{\infty} \frac{2^{n+1}}{3^n}$, then $s =$

A. 3 B. 6 C. 9 D. 2 E. 4/3

22. If $L = \sum_{n=1}^{\infty} \frac{1}{2^n} + \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n}$, then $L =$

A. 1/3 B. 2/3 C. 1 D. 4/3 E. 5/3

23. Find all values of p for which $\sum_{n=1}^{\infty} \frac{1}{(n^2+1)^p}$ converges.

A. $p > 1$ B. $p \le 1$ C. $p \ge 1$ D. $p > \frac{1}{2}$ E. $p \le \frac{1}{2}$

24. $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^p$ converges for

A. $p \le 1$ B. $p > 1$ C. $p < 0$ D. $p > 0$ E. no values of p

25. Which of the following series converge conditionally?

$$\text{(i)} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \qquad \text{(ii)} \sum_{n=2}^{\infty} \frac{(-1)^n n}{\ln n} \qquad \text{(iii)} \sum_{n=1}^{\infty} \frac{(-1)^n n}{e^n}$$

A. only (ii) B. only (i) and (iii) C. only (i) and (ii) D. all three E. none of them

26. Which of the following series converge?

$$\text{(i)} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^{\frac{1}{4}}} \qquad \text{(ii)} \sum_{n=1}^{\infty} \frac{n!}{1 \cdot 3 \cdot 5 \cdots (2n-1)} \qquad \text{(iii)} \sum_{n=1}^{\infty} \frac{4}{3}\left(\frac{1}{2}\right)^n$$

A. only (ii) B. only (i) and (iii) C. only (i) and (ii) D. all three E. none of them

27. The interval of convergence for the power series $\sum_{n=2}^{\infty} \frac{3^n x^n}{n \ln n}$ is

A. $-\frac{1}{3} \le x < \frac{1}{3}$ B. $-\frac{1}{3} < x \le \frac{1}{3}$ C. $0 \le x \le \frac{1}{3}$ D. $-1 \le x \le 2$ E. $-1 < x < 1$

28. Find the interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{n x^n}{2^n}$$

A. $-\frac{1}{2} < x < \frac{1}{2}$ B. $-2 < x < 2$ C. $-2 \le x \le 2$ D. $-2 < x \le 2$ E. $-\infty < x < \infty$

29. The fourth term of the Maclaurin series for $\frac{x^2+3}{x-1}$ is

A. $-x^3$ B. $3x^3$ C. $-3x^3$ D. $-4x^3$ E. $4x^3$

30. The first three nonzero terms of the Maclaurin series for $f(x) = (1-x^2)\sin x$ are

A. $x - \frac{5}{6}x^3 + \frac{31}{150}x^5$ B. $1 - \frac{3}{2}x^2 + \frac{13}{24}x^4$ C. $x - \frac{7}{6}x^3 + \frac{31}{150}x^5$ D. $x^2 - \frac{7}{6}x^3 + \frac{1}{25}x^5$ E. $x - \frac{7}{6}x^3 + \frac{21}{120}x^5$

31. The fourth term of the Taylor series for $f(x) = \ln x$ centered at $a = 2$ is

A. $\frac{1}{6}(x-2)^3$ B. $\frac{1}{12}(x-2)^3$ C. $\frac{1}{24}(x-2)^3$ D. $-\frac{1}{3}(x-2)^3$ E. $-(x-2)^3$

32. Using Maclaurin series and the Estimation Theorem for alternating series, we can obtain the approximation

$$\int_0^{0.1} e^{-x^2} dx \approx 0.1 - \frac{1}{3000}, \text{ with error } \le c$$

The value of c is

A. 10^{-5} B. 10^{-6} C. $\frac{1}{2}\,10^{-6}$ D. $\frac{10^{-7}}{7}$ E. $\frac{10^{-5}}{2}$

33. The parametric equations of a curve C are

$$x = 2\cos t,\ y = 3\sin t \quad 0 \le t \le \frac{\pi}{2}$$

The curve C is

A. a quarter of a circle B. an ellipse C. a half of an ellipse D. a half of a circle E. a quarter of an ellipse

34. The length of the parametric curve

$$x = \frac{1}{2}t^2, \qquad y = 2 + \frac{1}{3}t^3, \qquad 0 \le t \le \sqrt{3}$$

is

A. $\frac{21}{4}$ B. $\frac{7}{2}$ C. $\frac{7}{3}$ D. $\frac{14}{3}$ E. $\frac{8}{3}$

35. A point P has polar coordinates $(2, \frac{2\pi}{3})$. The Cartesian coordinates of P are

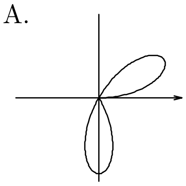
A. $(-1, \sqrt{3})$ B. $(-2, \sqrt{3})$ C. $(1, \frac{\sqrt{3}}{2})$ D. $(-1, \frac{1}{2})$ E. $(1, -\frac{1}{2})$

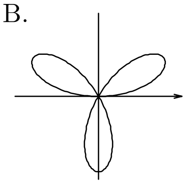
36. A point P has polar coordinates $(3, \frac{\pi}{4})$. Which of the following are also polar coordinates of P?

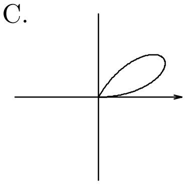
$$\text{I. } \left(-3, -\frac{\pi}{4}\right) \quad \text{II. } \left(-3, \frac{5\pi}{4}\right), \quad \text{III. } \left(3, -\frac{7\pi}{4}\right) \quad \text{IV. } \left(3, -\frac{5\pi}{4}\right).$$

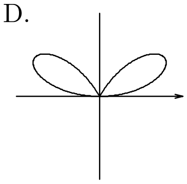
A. I and II only B. I and III only C. I and IV only D. II and III only E. II and IV only

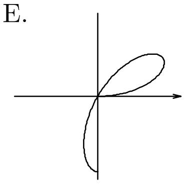
37. Which of the following looks most like the graph of the polar curve $r = \sin(3\theta)$, for $0 \leq \theta \leq \frac{\pi}{2}$?

A.


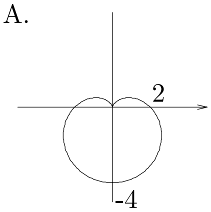
B.


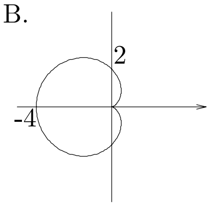
C.


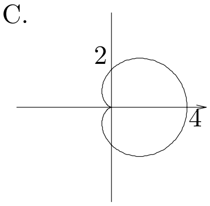
D.


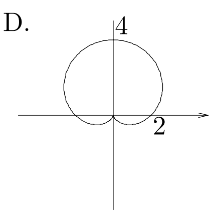
E.


38. Which of the following looks most like the graph of the polar curve $r = 2\cos\theta - 2$?

A.


B.


C.


D.


39. The rectangular coordinates of the center C and radius a of the circle $r = -2\sin\theta$ are
A. $C(0, -\frac{1}{2})$, $a = 1$ B. $C(0, 1)$, $a = 2$ C. $C(-1, 0)$, $a = 2$ D. $C(0, -1)$, $a = 1$ E. $C(-\frac{1}{2}, 0)$, $a = 1$

40. Write the complex number $1 - \sqrt{3}i$ in polar form with argument between 0 and 2π.
A. $4(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$ B. $2(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3})$ C. $4(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$
D. $2(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3})$ E. $2(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$

Answers: 1–A, 2–B, 3–A, 4–B, 5–E, 6–B, 7–D, 8–E, 9–A, 10–D, 11–B, 12–E, 13–E, 14–C 15–B, 16–A, 17–D, 18–A, 19–B, 20–B, 21–B, 22–E, 23–D, 24–E, 25–E, 26–D, 27–A, 28–B, 29–D, 30–E, 31–C, 32–B, 33–E, 34–C, 35–A, 36–D, 37–E, 38–C, 39–D, 40–D