C H A P T E R 6
Rational Expressions, Equations, and Functions

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Section 6.1 Rational Expressions and Functions

Solutions to Even-Numbered Exercises

2. $7 \neq 0$

$D = (-\infty, \infty)$

4. $x - 9 \neq 0$

$x \neq 9$

$D = (-\infty, 9) \cup (9, \infty)$

6. $6 - y \neq 0$

$y \neq 6$

$D = (-\infty, 6) \cup (6, \infty)$

8. $x + 1 \neq 0$

$x \neq -1$

$D = (-\infty, -1) \cup (-1, \infty)$

10. $x^2 + 16 \neq 0$

$D = (-\infty, \infty)$

12. $z(z - 4) \neq 0$

$z \neq 0 \quad z - 4 \neq 0$

$z \neq 4$

$D = (-\infty, 0) \cup (0, 4) \cup (4, \infty)$

14. $x^2 - 4 \neq 0$

$(x - 2)(x + 2) \neq 0$

$x - 2 \neq 0 \quad x + 2 \neq 0$

$x \neq 2 \quad x \neq -2$

$D = (-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

16. $t^2 + 5t \neq 0$

$t(t + 5) \neq 0$

$t \neq 0 \quad t \neq -5$

$D = (-\infty, -5) \cup (-5, 0) \cup (0, \infty)$

18. $t^2 - 2t - 3 \neq 0$

$(t - 3)(t + 1) \neq 0$

$t - 3 \neq 0 \quad t + 1 \neq 0$

$t \neq 3 \quad t \neq -1$

$D = (-\infty, -1) \cup (-1, 3) \cup (3, \infty)$

20. $4y^2 - 5y - 6 \neq 0$

$(4y + 3)(y - 2) \neq 0$

$4y + 3 \neq 0 \quad y - 2 \neq 0$

$4y \neq -3 \quad y \neq 2$

$y \neq -\frac{3}{4}$

$D = \left(-\infty, -\frac{3}{4}\right) \cup \left(-\frac{3}{4}, 2\right) \cup (2, \infty)$

22. (a) $f(10) = \dfrac{10 - 10}{4(10)} = \dfrac{0}{40} = 0$

(b) $f(0) = \dfrac{0 - 10}{4(0)} = \dfrac{-10}{0}$; not possible; undefined

(c) $f(-2) = \dfrac{-2 - 10}{4(-2)} = \dfrac{-12}{-8} = \dfrac{3}{2}$

(d) $f(12) = \dfrac{12 - 10}{4(12)} = \dfrac{2}{48} = \dfrac{1}{24}$

24. (a) $g(2) = \dfrac{2 - 2}{2(2) - 5} = \dfrac{0}{4 - 5} = \dfrac{0}{-1} = 0$

(b) $g\left(\dfrac{5}{2}\right) = \dfrac{\frac{5}{2} - 2}{2\left(\frac{5}{2}\right) - 5} = \dfrac{\frac{5}{2} - \frac{4}{2}}{5 - 5} = \dfrac{\frac{1}{2}}{0}$; not possible; undefined

(c) $g(-2) = \dfrac{-2 - 2}{2(-2) - 5} = \dfrac{-4}{-4 - 5} = \dfrac{-4}{-9} = \dfrac{4}{9}$

(d) $g(0) = \dfrac{0 - 2}{2(0) - 5} = \dfrac{-2}{0 - 5} = \dfrac{-2}{-5} = \dfrac{2}{5}$

26. (a) $f(-1) = \dfrac{(-1)^3 + 1}{(-1)^2 - 6(-1) + 9}$

$= \dfrac{-1 + 1}{1 + 6 + 9}$

$= \dfrac{0}{16} = 0$

(b) $f(3) = \dfrac{(3)^3 + 1}{(3)^2 - 6(3) + 9}$

$= \dfrac{27 + 1}{9 - 18 + 9}$

$= \dfrac{28}{0}$; not possible; undefined

(c) $f(-2) = \dfrac{(-2)^3 + 1}{(-2)^2 - 6(-2) + 9}$

$= \dfrac{-8 + 1}{4 + 12 + 9}$

$= -\dfrac{7}{25}$

(d) $f(2) = \dfrac{(2)^3 + 1}{(2)^2 - 6(2) + 9}$

$= \dfrac{8 + 1}{4 - 12 + 9}$

$= \dfrac{9}{1} = 9$

28. Since p is the percent of a certain illegal drug, $p \geq 0$. Since

$$\frac{258p}{100 - p}$$

must be defined, $100 - p \neq 0$. Thus, $p \neq 100$. Since p is a percent of a certain illegal drug, $p \leq 100$. Therefore, the domain is $[0, 100)$.

30. $x =$ units of a product

$D = \{1, 2, 3, 4, \ldots\}$

32. $x =$ units of a product

$D = \{1, 2, 3, 4, \ldots\}$

34. $\dfrac{7}{15} = \dfrac{7(x - 10)}{15(x - 10)}, \quad x \neq 10$

36. $\dfrac{5x}{12} = \dfrac{25x^2(x - 10)}{12[5x(x - 10)]}, \quad x \neq 10$

38. $\dfrac{3y - 7}{y + 2} = \dfrac{(3y - 7)(y - 2)}{y^2 - 4}, \quad y \neq 2$

40. $\dfrac{3 - z}{z^2} = \dfrac{(3 - z)(z + 2)}{z^3 + 2z^2}, \quad z \neq -2$

42. $\dfrac{32y}{24} = \dfrac{4 \cdot 8y}{3 \cdot 8} = \dfrac{4y}{3}$

44. $\dfrac{15z^3}{15z^3} = 1, \quad z \neq 0$

46. $\dfrac{16y^2z^2}{60y^5z} = \dfrac{4 \cdot 4 \cdot y^2 \cdot z \cdot z}{4 \cdot 15 \cdot y^2 \cdot y^3 \cdot z} = \dfrac{4z}{15y^3}, \quad y \neq 0, z \neq 0$

48. $\dfrac{8x^3 + 4x^2}{20x} = \dfrac{4x^2(2x + 1)}{20x} = \dfrac{x(2x + 1)}{5}, \quad x \neq 0$

50. $\dfrac{a^2b(b - 3)}{b^3(b - 3)^2} = \dfrac{a^2b(b - 3)}{b \cdot b^2(b - 3)(b - 3)}$

$= \dfrac{a^2}{b^2(b - 3)}, \quad b \neq 0, 3$

52. $\dfrac{y^2 - 81}{2y - 18} = \dfrac{(y + 9)(y - 9)}{2(y - 9)} = \dfrac{y + 9}{2}, \quad y \neq 9$

54. $\dfrac{x^2 - 36}{6 - x} = \dfrac{(x + 6)(x - 6)}{-(x - 6)}$

$= \dfrac{(x + 6)}{-1}$

$= -(x + 6)$ or $-x - 6, \quad x \neq 6$

56. $\dfrac{u^2 - 12u + 36}{u - 6} = \dfrac{(u - 6)(u - 6)}{u - 6}$

$= u - 6, \quad u \neq 6$

58. $\dfrac{z^2 + 22z + 121}{3z + 33} = \dfrac{(z + 11)(z + 11)}{3(z + 11)}$

$= \dfrac{z + 11}{3}, \quad z \neq -11$

60. $\dfrac{x^2 - 7x}{x^2 - 4x - 21} = \dfrac{x(x - 7)}{(x + 3)(x - 7)}$

$= \dfrac{x}{x + 3}, \quad x \neq 7$

62. $$\frac{x^4 - 25x^2}{x^2 + 2x - 15} = \frac{x^2(x^2 - 25)}{(x + 5)(x - 3)} = \frac{x^2(x - 5)(x + 5)}{(x + 5)(x - 3)} = \frac{x^2(x - 5)}{x - 3}, \quad x \neq -5$$

64. $$\frac{2x^2 + 3x - 5}{7 - 6x - x^2} = \frac{(2x + 5)(x - 1)}{-1(x^2 + 6x - 7)} = \frac{(2x + 5)(x - 1)}{-1(x + 7)(x - 1)} = -\frac{2x + 5}{x + 7}, \quad x \neq 1$$

66. $$\frac{2y^2 + 13y + 20}{2y^2 + 17y + 30} = \frac{(2y + 5)(y + 4)}{(2y + 5)(y + 6)} = \frac{y + 4}{y + 6}, \quad y \neq -\frac{5}{2}$$

68. $$\frac{56z^2 - 3z - 20}{49z^2 - 16} = \frac{(7z + 4)(8z - 5)}{(7z + 4)(7z - 4)} = \frac{8z - 5}{7z - 4}, \quad z \neq -\frac{4}{7}$$

70. $$\frac{x + 3x^2y}{3xy + 1} = \frac{x(1 + 3xy)}{1 + 3xy} = x, \quad 3xy \neq -1$$

72. $$\frac{x^2 - 25z^2}{x + 5z} = \frac{(x + 5z)(x - 5z)}{x + 5z} = x - 5z, \quad x \neq -5z$$

74. $$\frac{4u^2v - 12uv^2}{18uv} = \frac{4uv(u - 3v)}{18uv} = \frac{2 \cdot 2 \cdot u \cdot v \cdot (u - 3v)}{2 \cdot 9 \cdot u \cdot v} = \frac{2(u - 3v)}{9}, \quad u \neq 0, v \neq 0$$

76. $$\frac{x^2 + 4xy}{x^2 - 16y^2} = \frac{x(x + 4y)}{(x + 4y)(x - 4y)} = \frac{x}{x - 4y}, \quad x \neq -4y$$

78. $$\frac{x^2 + xy - 2y^2}{x^2 + 3xy + 2y^2} = \frac{(x + 2y)(x - y)}{(x + 2y)(x + y)} = \frac{x - y}{x + y}, \quad x \neq -2y$$

80.

x	-2	-1	0	1	2	3	4
$\frac{x^2 + 5x}{x}$	3	4	undefined	6	7	8	9
$x + 5$	3	4	5	6	7	8	9

$\frac{x^2 + 5x}{x} = \frac{x(x + 5)}{x} = x + 5, \quad x \neq 0$

Domain of $\frac{x^2 + 5x}{x}$ is $(-\infty, 0) \cup (0, \infty)$.

Domain of $x + 5$ is $(-\infty, \infty)$.

The two expressions are equal for all replacements of the variable except $x = 0$.

82. $$\frac{\text{Area of shaded portion}}{\text{Area of total figure}} = \frac{(x + 2)(2x)}{(2x)(2x + 5)} = \frac{x + 2}{2x + 5}, \quad x > 0$$

84. $$\frac{\text{Area of shaded portion}}{\text{Area of total figure}} = \frac{\frac{1}{2}x(0.6x)}{\frac{1}{2}(3x)(1.8x)} = \frac{0.6x^2}{5.4x^2} = \frac{0.6}{5.4} = \frac{6}{54} = \frac{1}{9}, \quad x > 0$$

86. (a) *Verbal Model:* [Total cost] = [Number of units] · [Cost per unit] + [Initial cost]

Labels: Total cost = C

Number of units = x

Equation: $C = 6.50x + 60{,}000$

(b) *Verbal Model:* [Average cost] = [Total cost] ÷ [Number of units]

Label: Average cost = $\overline{C}$

Equation: $\overline{C} = \frac{C}{x}$; $\overline{C} = \frac{6.50x + 60{,}000}{x}$

(c) $D = \{1, 2, 3, 4, \ldots\}$

(d) $\overline{C}(11{,}000) = \frac{6.50(11{,}000) + 60{,}000}{11{,}000} \approx \$11.95$

88. (a) *Verbal Model:* $\boxed{\text{Distance}} = \boxed{\text{Rate}} \cdot \boxed{\text{Time}}$

Labels: Car 1: $55(t + 2)$

Car 2: $65t$

(b) Distance between car 1 and car 2:

$$\begin{aligned} d &= |55(t + 2) - 65t| \\ &= |55t + 110 - 65t| \\ &= |110 - 10t| \\ &= |10(11 - t)| \end{aligned}$$

(c) $\dfrac{\text{Distance of car 2}}{\text{Distance of car 1}} = \dfrac{65t}{55(t + 2)} = \dfrac{13t}{11(t + 2)}$

90. $\dfrac{\text{Rectangular pool volume}}{\text{Circular pool volume}} = \dfrac{d(6d - 2)(3d + 4)}{\pi(5d)^2 d} = \dfrac{2(3d - 1)(3d + 4)}{25\pi d^2}$

92.

Year	1995	1996	1997	1998	1999	2000
Average revenue	277.77	296.33	314.12	331.19	347.57	363.31

(a) Average Revenue $= \dfrac{1531.1(5) + 9358}{1.33(5) + 54.6} = 277.77$

(b) Average Revenue $= \dfrac{1531.1(6) + 9358}{1.33(6) + 54.6} = 296.33$

(c) Average Revenue $= \dfrac{1531.1(7) + 9358}{1.33(7) + 54.6} = 314.12$

(d) Average Revenue $= \dfrac{1531.1(8) + 9358}{1.33(8) + 54.6} = 331.19$

(e) Average Revenue $= \dfrac{1531.1(9) + 9358}{1.33(9) + 54.6} = 347.57$

(f) Average Revenue $= \dfrac{1531.1(10) + 9358}{1.33(10) + 54.6} = 363.31$

94. $\dfrac{1}{x^2 + 1}$

There are many correct answers.

96. No. You can divide out only common factors.

98. True.

$$\frac{6x - 5}{5 - 6x} = \frac{-(5 - 6x)}{5 - 6x} = -1$$

Section 6.2 Multiplying and Dividing Rational Expressions

2. $\dfrac{2x}{x - 3} = \dfrac{14x(x - 3)^2}{(x - 3)(7(x - 3)^2)}, \quad x \neq 0$

4. $\dfrac{x + 1}{x} = \dfrac{(x + 1)^3}{x(x + 1)^2}, \quad x \neq -1$

6. $\dfrac{3t + 5}{t} = \dfrac{(3t + 5)[5t(3t - 5)]}{5t^2(3t - 5)}, \quad t \neq \dfrac{5}{3}$

8. $\dfrac{x^2}{10 - x} = \dfrac{x^2(-x)}{x^2 - 10x}, \quad x \neq 0$

10. $\dfrac{6}{5a} \cdot (25a) = \dfrac{6 \cdot 5 \cdot 5 \cdot a}{5a} = 30, \quad a \neq 0$

12. $\dfrac{3x^4}{7x} \cdot \dfrac{8x^2}{9} = \dfrac{3 \cdot x \cdot x^3 \cdot 8 \cdot x^2}{7 \cdot x \cdot 3 \cdot 3} = \dfrac{8x^3x^2}{7 \cdot 3} = \dfrac{8x^5}{21}, \quad x \neq 0$

14. $25x^3 \cdot \dfrac{8}{35x} = \dfrac{5 \cdot 5 \cdot x \cdot x \cdot x \cdot 8}{7 \cdot 5 \cdot x} = \dfrac{40x^2}{7}, \quad x \neq 0$

16. $(6 - 4x) \cdot \dfrac{10}{3 - 2x} = 2(3 - 2x) \cdot \dfrac{10}{3 - 2x} = 20, \quad x \neq \dfrac{3}{2}$

18. $\dfrac{1 - 3xy}{4x^2y} \cdot \dfrac{46x^4y^2}{15 - 45xy} = \dfrac{1 - 3xy}{4x^2y} \cdot \dfrac{2(23)x^4y^2}{15(1 - 3xy)} = \dfrac{23x^2y}{30}$

$x \neq 0, y \neq 0, 1 - 3xy \neq 0$

20. $\dfrac{8 - z}{8 + z} \cdot \dfrac{z + 8}{z - 8} = \dfrac{-(z - 8)(z + 8)}{(z + 8)(z - 8)} = -1, \quad z \neq -8, 8$

22. $\dfrac{x+14}{x^3(10-x)} \cdot \dfrac{x(x-10)}{5} = \dfrac{(x+14)(x)(x-10)}{-x \cdot x^2(x-10)(5)}$

$= \dfrac{x+14}{-5x^2}$

$= -\dfrac{x+14}{5x^2}, \quad x \neq 10$

24. $\dfrac{5y-20}{5y+15} \cdot \dfrac{2y+6}{y-4} = \dfrac{5(y-4) \cdot 2(y+3)}{5(y+3)(y-4)} = 2, \quad y \neq -3, 4$

26. $\dfrac{y^2-16}{y^2+8y+16} \cdot \dfrac{3y^2-5y-2}{y^2-6y+8} = \dfrac{(y-4)(y+4)}{(y+4)(y+4)} \cdot \dfrac{(3y+1)(y-2)}{(y-4)(y-2)} = \dfrac{3y+1}{y+4}, \quad y \neq 2, 4$

28. $(u-2v)^2 \cdot \dfrac{u+2v}{u-2v} = \dfrac{(u-2v)(u-2v)(u+2v)}{u-2v} = (u-2v)(u+2v), \quad u \neq 2v$

30. $\dfrac{(x-2y)^2}{x+2y} \cdot \dfrac{x^2+7xy+10y^2}{x^2-4y^2} = \dfrac{(x-2y)^2}{x+2y} \cdot \dfrac{(x+5y)(x+2y)}{(x-2y)(x+2y)} = \dfrac{(x-2y)(x+5y)}{x+2y}, \quad x \neq 2y$

32. $\dfrac{t^2+4t+3}{2t^2-t-10} \cdot \dfrac{t}{t^2+3t+2} \cdot \dfrac{2t^2+4t^3}{t^2+3t} = \dfrac{(t+3)(t+1)}{(2t-5)(t+2)} \cdot \dfrac{t}{(t+2)(t+1)} \cdot \dfrac{2t^2(1+2t)}{t(t+3)}$

$= \dfrac{2t^2(1+2t)}{(2t-5)(t+2)^2}, \quad t \neq -1, -3, 0$

34. $\dfrac{16x^2-1}{4x^2+9x+5} \cdot \dfrac{5x^2-9x-18}{x^2-12x+36} \cdot \dfrac{12+4x-x^2}{4x^2-13x+3} = \dfrac{\cancel{(4x-1)}(4x+1)}{4x^2+9x+5} \cdot \dfrac{(5x+6)\cancel{(x-3)}}{(x-6)(x-6)} \cdot \dfrac{(6-x)(2+x)}{\cancel{(4x-1)}\cancel{(x-3)}}$

$= \dfrac{(4x+1)(5x+6)-1(x-6)(2+x)}{(4x^2+9x+5)(x-6)(x-6)}$

$= \dfrac{(4x+1)(5x+6)(2+x)(-1)}{(4x^2+9x+5)(x-6)}$

$= \dfrac{(4x+1)(5x+6)(2+x)}{(4x^2+9x+5)(6-x)}, \quad x \neq \dfrac{1}{4}, 3$

36. $\dfrac{xu-yu+xv-yv}{xu+yu-xv-yv} \cdot \dfrac{xu+yu+xv+yv}{xu-yu-xv+yv} = \dfrac{u(x-y)+v(x-y)}{u(x+y)-v(x+y)} \cdot \dfrac{u(x+y)+v(x+y)}{u(x-y)-v(x-y)}$

$= \dfrac{(x-y)(u+v)}{(x+y)(u-v)} \cdot \dfrac{(x+y)(u+v)}{(x-y)(u-v)}$

$= \dfrac{(x-y)(u+v)(x+y)(u+v)}{(x+y)(u-v)(x-y)(u-v)}$

$= \dfrac{(u+v)(u+v)}{(u-v)(u-v)}$ or $\dfrac{(u+v)^2}{(u-v)^2}, \quad x \neq y, x \neq -y$

38. $\dfrac{u}{10} \div u^2 = \dfrac{u}{10} \cdot \dfrac{1}{u^2} = \dfrac{1}{10u}$

40. $\dfrac{3y^2}{20} \div \dfrac{y}{15} = \dfrac{3y^2}{20} \cdot \dfrac{15}{y}$

$= \dfrac{3(5)(3)y^2}{4(5)y}$

$= \dfrac{9y}{4}, \quad y \neq 0$

42. $\dfrac{25x^2y}{60x^3y^2} \div \dfrac{5x^4y^3}{16x^2y} = \dfrac{25x^2y}{60x^3y^2} \cdot \dfrac{16x^2y}{5x^4y^3}$

$$= \frac{5 \cdot 5 \cdot x^2 \cdot y \cdot 4 \cdot 4 \cdot x^2 \cdot y}{3 \cdot 4 \cdot 5 \cdot x^2 \cdot x \cdot y \cdot y \cdot 5 \cdot x^2 \cdot x^2 \cdot y \cdot y^2}$$

$$= \frac{4}{3 \cdot x \cdot yx^2y^2} = \frac{4}{3x^3y^3}$$

44. $\dfrac{x^2+9}{5(x+2)} \div \dfrac{x+3}{5(x^2-4)} = \dfrac{x^2+9}{5(x+2)} \cdot \dfrac{5(x-2)(x+2)}{x+3}$

$$= \frac{(x-2)(x^2+9)}{(x+3)}, \quad x \neq \pm 2$$

46. $\dfrac{x^2-y^2}{2x^2-8x} \div \dfrac{(x-y)^2}{2xy} = \dfrac{x^2-y^2}{2x^2-8x} \cdot \dfrac{2xy}{(x-y)^2}$

$$= \frac{(x-y)(x+y) \cdot 2 \cdot x \cdot y}{2 \cdot x(x-4)(x-y)(x-y)}$$

$$= \frac{y(x+y)}{(x-4)(x-y)}, \quad x \neq 0, y \neq 0$$

48. $\dfrac{x+3}{x^2+7x+10} \div \dfrac{x^2+6x+9}{x^2+5x+6} = \dfrac{x+3}{(x+5)(x+2)} \cdot \dfrac{(x+3)(x+2)}{(x+3)(x+3)}$

$$= \frac{1}{x+5}, \quad x \neq -3, -2$$

50. $\dfrac{y^2+5y-14}{y^2+10y+21} \div \dfrac{y^2+5y+6}{y^2+7y+12} = \dfrac{(y+7)(y-2)}{(y+7)(y+3)} \cdot \dfrac{(y+4)(y+3)}{(y+3)(y+2)}$

$$= \frac{(y-2)(y+4)}{(y+3)(y+2)}, \quad y \neq -4, -7$$

52. $\left(\dfrac{x^2+6x+9}{x^2} \cdot \dfrac{2x+1}{x^2-9}\right) \div \dfrac{4x^2+4x+1}{x^2-3x} = \left(\dfrac{(x+3)^2}{x^2} \cdot \dfrac{2x+1}{(x-3)(x+3)}\right) \div \dfrac{(2x+1)^2}{x(x-3)}$

$$= \frac{(x+3)(2x+1)}{x^2(x-3)} \cdot \frac{x(x-3)}{(2x+1)^2}$$

$$= \frac{x+3}{x(2x+1)}, \quad x \neq \pm 3$$

54. $\dfrac{3u^2-u-4}{u^2} \div \dfrac{3u^2+12u+4}{u^4-3u^3} = \dfrac{(3u-4)(u+1)}{u^2} \div \dfrac{3u^2+12u+4}{u^3(u-3)}$

$$= \frac{(3u-4)(u+1)}{u^2} \cdot \frac{u^3(u-3)}{3u^2+12u+4}$$

$$= \frac{u(u-3)(3u-4)(u+1)}{3u^2+12u+4}, \quad u \neq 3, 0$$

56. $\dfrac{t^2-100}{4t^2} \cdot \dfrac{t^3-5t^2-50t}{t^4+10t^3} \div \dfrac{(t-10)^2}{5t} = \dfrac{(t-10)(t+10)}{4t^2} \cdot \dfrac{t(t^2-5t-50)}{t^3(t+10)} \cdot \dfrac{5t}{(t-10)^2}$

$$= \frac{(t-10)(t+10)t(t-10)(t+5)5t}{4t^2 \cdot t^3(t+10)(t-10)^2}$$

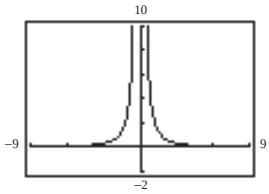
$$= \frac{5(t+5)}{4t^3}, \quad t \neq -10, 10$$

58. $\dfrac{x^{n+1}-8x}{x^{2n}+2x^n+1} \cdot \dfrac{x^{2n}-4x^n-5}{x} \div x^n = \dfrac{x(x^n-8)}{(x^n+1)(x^n+1)} \cdot \dfrac{(x^n-5)(x^n+1)}{x} \cdot \dfrac{1}{x^n} = \dfrac{(x^n-8)(x^n-5)}{x^n(x^n+1)}$

60. *Keystrokes:*

y_1 [Y=] [(] [(] 3 [X,T,θ] [+] 15 [)] [÷] [X,T,θ] [^] 4 [)] [÷]
[(] [(] [X,T,θ] [+] 5 [)] [÷] [X,T,θ] [x^2] [)] [ENTER]

y_2 3 [÷] [X,T,θ] [x^2] [GRAPH]



62. $\text{Area} = \left(\frac{2w-1}{2}\right)\left(\frac{w}{3}\right)$

$= \frac{(2w-1)w}{6}$

$= \frac{2w^2 - w}{6}$

64. $\frac{\text{Unshaded Area}}{\text{Total Area}} = \frac{\frac{1}{2}(2x)(x)}{(2x)(4x+2)}$

$= \frac{x}{(2)(2)(2x+1)}$

$= \frac{x}{4(2x+1)}$

66. $\frac{\text{Unshaded Area}}{\text{Total Area}} = \frac{(2x)(x)}{(2x)(4x+2)}$

$= \frac{x}{2(2x+1)}$

68. (a) $\frac{15 \text{ gallons}}{1 \text{ minute}} = \frac{15 \text{ gallons}}{60 \text{ seconds}} = \frac{1 \text{ gallon}}{4 \text{ seconds}}$, $t = 4$ seconds or $\frac{1}{15}$ minute

(b) $\frac{4 \text{ seconds}}{1 \text{ gallon}} \cdot x \text{ gallons} = 4x$ seconds or $\frac{x}{15}$ minutes

(c) $\frac{4 \text{ seconds}}{1 \text{ gallon}} \cdot 130 \text{ gallons} = 520$ seconds or $\frac{520}{60} = \frac{26}{3}$ minutes

70. Divide a rational expression by a polynomial by inverting the divisor and multiplying.

72.

x	60	100	1000	10,000	100,000	1,000,000
$\frac{x-10}{x+10}$	0.71429	0.81818	0.98020	0.99800	0.99980	0.99998
$\frac{x+50}{x-50}$	11	3	1.10526	1.01005	1.00100	1.00010
$\frac{x-10}{x+10} \cdot \frac{x+50}{x-50}$	7.85714	2.45455	1.08338	1.00803	1.00080	1.00008

The value of the first row gets larger and closer to 1 as the value of x increases (because as x becomes larger, the value of 10 becomes much smaller in comparison). The value of the second row gets smaller and closer to 1 as the value of x increases (because as x becomes larger, the value of 50 becomes much smaller in comparison). The value of the third row is in between the values of the other two rows and gets smaller and closer to 1 as the value of x increases.

Section 6.3 Adding and Subtracting Rational Expressions

2. $\frac{7y}{12} + \frac{9y}{12} = \frac{16y}{12} = \frac{4y}{3}$

4. $\frac{6}{19x} - \frac{7}{19x} = -\frac{1}{19x}$

6. $\frac{4-y}{4} + \frac{3y}{4} = \frac{4-y+3y}{4}$

$= \frac{4+2y}{4} = \frac{2(2+y)}{4} = \frac{2+y}{2}$

8. $\frac{10x^2+1}{3} - \frac{10x^2}{3} = \frac{10x^2+1-10x^2}{3} = \frac{1}{3}$

10. $\dfrac{16+z}{5z}-\dfrac{11-z}{5z}=\dfrac{16+z-(11-z)}{5z}$

$=\dfrac{16+z-11+z}{5z}=\dfrac{2z+5}{5z}$

12. $\dfrac{-16u}{9}-\dfrac{27-16u}{9}+\dfrac{2}{9}=\dfrac{-16u-(27-16u)+2}{9}$

$=\dfrac{-16u-27+16u+2}{9}$

$=-\dfrac{25}{9}$

14. $\dfrac{5x-1}{x+4}+\dfrac{5-4x}{x+4}=\dfrac{5x-1+5-4x}{x+4}$

$=\dfrac{x+4}{x+4}=1, \quad x\neq -4$

16. $\dfrac{7s-5}{2s+5}+\dfrac{3(s+10)}{2s+5}=\dfrac{7s-5+3(s+10)}{2s+5}$

$=\dfrac{7s-5+3s+30}{2s+5}$

$=\dfrac{10s+25}{2s+5}$

$=\dfrac{5(2s+5)}{2s+5}$

$=5, \quad s\neq -\dfrac{5}{2}$

18. $14t^2=2\cdot 7\cdot t\cdot t$

$42t^5=2\cdot 3\cdot 7\cdot t\cdot t\cdot t\cdot t\cdot t$

LCM $=42t^5$

20. $44m^2=2\cdot 2\cdot 11\cdot m\cdot m$

$10m=2\cdot 5\cdot m$

LCM $=220m^2$

22. $6x^2=2\cdot 3\cdot x\cdot x$

$15x(x-1)=5\cdot 3\cdot x\cdot (x-1)$

LCM $=2\cdot 3\cdot 5\cdot x\cdot x\cdot (x-1)=30x^2(x-1)$

24. $18y^3=2\cdot 3\cdot 3\cdot y\cdot y\cdot y$

$27y(y-3)^2=3\cdot 3\cdot 3\cdot y\cdot (y-3)(y-3)$

LCM $=54y^3(y-3)^2$

26. $2y^2+y-1=(2y-1)(y+1)$

$4y^2-2y=2y(2y-1)$

LCM $=2y(2y-1)(y+1)$

28. $t^3+3t^2+9t=t(t^2+3t+9)$

$2t^2(t^2-9)=2\cdot t\cdot t(t-3)(t+3)$

LCM $=2t^2(t-3)(t+3)(t^2+3t+9)$

30. $\dfrac{3y(x-3)^2}{(x-3)\left(\frac{1}{7}(x-3)^2\right)}, \quad x\neq 3$

32. $\dfrac{(3t+5)(5t(3t-5))}{10t^2(3t-5)}=\dfrac{3t+5}{2t}, \quad t\neq \dfrac{5}{3}$

34. $\dfrac{4x^2(-x)}{x^2-10x}=\dfrac{4x^2}{10-x}, \quad x\neq 0$

36. $\dfrac{8s}{(s+2)^2}=\dfrac{8s\cdot s(s-1)}{s(s+2)^2(s-1)}=\dfrac{8s^2(s-1)}{s(s+2)^2(s-1)}$

$\dfrac{3}{s^3+s^2-2s}=\dfrac{3}{s(s^2+s-2)}$

$=\dfrac{3}{s(s+2)(s-1)}=\dfrac{3(s+2)}{s(s+2)^2(s-1)}$

LCD $=s(s+2)^2(s-1)$

38. $\dfrac{5t}{2t(t-3)^2}=\dfrac{5t}{2t(t-3)^2}$

$\dfrac{4}{t(t-3)}=\dfrac{4(2)(t-3)}{t(t-3)\cdot 2(t-3)}=\dfrac{8(t-3)}{2t(t-3)^2}$

LCD $=2t(t-3)^2$

40. $\dfrac{4x}{(x+5)^2} = \dfrac{4x(x-5)}{(x+5)^2(x-5)} = \dfrac{4x^2-20x}{(x+5)^2(x-5)}$

$\dfrac{x-2}{x^2-25} = \dfrac{x-2}{(x-5)(x+5)}$

$= \dfrac{(x-2)(x+5)}{(x-5)(x+5)(x+5)} = \dfrac{x^2+3x-10}{(x+5)^2(x-5)}$

LCD $= (x+5)^2(x-5)$

42. $\dfrac{3y}{y^2-y-12} = \dfrac{3y}{(y-4)(y+3)}$

$= \dfrac{3y \cdot y}{(y-4)(y+3) \cdot y} = \dfrac{3y^2}{y(y+3)(y-4)}$

$\dfrac{y-4}{y^2+3y} = \dfrac{y-4}{y(y+3)} = \dfrac{(y-4)^2}{y(y+3)(y-4)}$

LCD $= y(y+3)(y-4)$

44. $\dfrac{10}{b} + \dfrac{1}{10b} = \dfrac{10(10)}{b(10)} + \dfrac{1}{10b}$

$= \dfrac{100}{10b} + \dfrac{1}{10b}$

$= \dfrac{100+1}{10b}$

$= \dfrac{101}{10b}$

46. $\dfrac{1}{6u^2} - \dfrac{2}{9u} = \dfrac{1(3)}{6u^2(3)} - \dfrac{2(2u)}{9u(2u)}$

$= \dfrac{3}{18u^2} - \dfrac{4u}{18u^2}$

$= \dfrac{3-4u}{18u^2}$

48. $\dfrac{15}{2-t} - \dfrac{7}{t-2} = \dfrac{15(-1)}{(2-t)(-1)} - \dfrac{7}{t-2}$

$= \dfrac{-15}{t-2} - \dfrac{7}{t-2}$

$= \dfrac{-15-7}{t-2}$

$= \dfrac{-22}{t-2}$

$= \dfrac{22}{2-t}$

50. $\dfrac{1}{y-6} + \dfrac{y}{6-y} = \dfrac{1}{y-6} + \dfrac{y(-1)}{(6-y)(-1)}$

$= \dfrac{1}{y-6} - \dfrac{y}{y-6}$

$= \dfrac{1-y}{y-6}$

52. $\dfrac{100}{x-10} - 8 = \dfrac{100}{x-10} - \dfrac{8(x-10)}{x-10}$

$= \dfrac{100-8(x-10)}{x-10}$

$= \dfrac{100-8x+80}{x-10}$

$= \dfrac{-8x+180}{x-10}$

$= \dfrac{-4(2x-45)}{-1(10-x)} = \dfrac{4(2x-45)}{10-x} = \dfrac{4(45-2x)}{x-10}$

54. $\dfrac{y}{5y-3} - \dfrac{3}{3-5y} = \dfrac{y}{5y-3} - \dfrac{3(-1)}{(3-5y)(-1)}$

$= \dfrac{y}{5y-3} + \dfrac{3}{5y-3}$

$= \dfrac{y+3}{5y-3}$

56. $\dfrac{3}{y-1} + \dfrac{5}{4y} = \dfrac{3(4y)}{4y(y-1)} + \dfrac{5(y-1)}{4y(y-1)}$

$= \dfrac{12y+5y-5}{4y(y-1)}$

$= \dfrac{17y-5}{4y(y-1)}$

58. $\dfrac{1}{x+4} - \dfrac{1}{x+2} = \dfrac{1(x+2)}{x+4(x+2)} - \dfrac{1(x+4)}{x+2(x+4)}$

$= \dfrac{x+2-x-4}{(x+4)(x+2)}$

$= \dfrac{-2}{(x+4)(x+2)}$

$= -\dfrac{2}{(x+4)(x+2)}$

60. $\dfrac{12}{x^2-4}-\dfrac{3}{x+2}=\dfrac{12}{(x-2)(x+2)}-\dfrac{3(x-2)}{(x-2)(x+2)}$

$=\dfrac{12-3(x-2)}{(x-2)(x+2)}$

$=\dfrac{12-3x+6}{(x-2)(x+2)}$

$=\dfrac{18-3x}{(x-2)(x+2)}$

$=\dfrac{-3(x-6)}{(x-2)(x+2)}$

$=-\dfrac{3(x-6)}{(x-2)(x+2)}$

62. $\dfrac{7}{2x-3}+\dfrac{3}{2x+3}=\dfrac{7(2x+3)}{(2x-3)(2x+3)}+\dfrac{3(2x-3)}{(2x+3)(2x-3)}$

$=\dfrac{7(2x+3)+3(2x-3)}{(2x-3)(2x+3)}$

$=\dfrac{14x+21+6x-9}{(2x-3)(2x+3)}$

$=\dfrac{20x+12}{(2x-3)(2x+3)}$

$=\dfrac{4(5x+3)}{(2x-3)(2x+3)}$

64. $\dfrac{2}{y^2+2}+\dfrac{1}{2y^2}=\dfrac{2(2y^2)}{(y^2+2)(2y^2)}+\dfrac{1(y^2+2)}{2y^2(y^2+2)}$

$=\dfrac{4y^2+y^2+2}{(y^2+2)(2y^2)}$

$=\dfrac{5y^2+2}{(2y^2)(y^2+2)}$

66. $\dfrac{x}{x^2-x-30}-\dfrac{1}{x+5}=\dfrac{x}{(x+5)(x-6)}-\dfrac{(x-6)}{(x+5)(x-6)}$

$=\dfrac{x-(x-6)}{(x+5)(x-6)}$

$=\dfrac{x-x+6}{(x+5)(x-6)}$

$=\dfrac{6}{(x+5)(x-6)}$

68. $\dfrac{3}{x-2}-\dfrac{1}{(x-2)^2}=\dfrac{3(x-2)}{(x-2)(x-2)}-\dfrac{1}{(x-2)^2}$

$=\dfrac{3(x-2)}{(x-2)^2}-\dfrac{1}{(x-2)^2}$

$=\dfrac{3(x-2)-1}{(x-2)^2}$

$=\dfrac{3x-6-1}{(x-2)^2}$

$=\dfrac{3x-7}{(x-2)^2}$

70. $\dfrac{5}{x+y}+\dfrac{5}{x^2-y^2}=\dfrac{5(x-y)}{(x+y)(x-y)}+\dfrac{5}{(x+y)(x-y)}$

$=\dfrac{5(x-y)+5}{(x+y)(x-y)}$

$=\dfrac{5x-5y+5}{(x+y)(x-y)}$

$=\dfrac{5(x-y+1)}{(x+y)(x-y)}$

72. $\dfrac{5}{2}-\dfrac{1}{2x}-\dfrac{3}{(x+1)}=\dfrac{5(x+1)(x)}{2(x+1)(x)}-\dfrac{1(x+1)}{2x(x+1)}-\dfrac{3(2x)}{2(x+1)(x)}$

$=\dfrac{5x(x+1)}{2x(x+1)}-\dfrac{(x+1)}{2x(x+1)}-\dfrac{6x}{2x(x+1)}$

$=\dfrac{5x(x+1)-(x+1)-6x}{2x(x+1)}$

$=\dfrac{5x^2+5x-x-1-6x}{2x(x+1)}$

$=\dfrac{5x^2-2x-1}{2x(x+1)}$

74. $\dfrac{1}{x-y}-\dfrac{3}{x+y}+\dfrac{3x-y}{x^2-y^2}=\dfrac{x+y}{(x-y)(x+y)}-\dfrac{3(x-y)}{(x+y)(x-y)}+\dfrac{3x-y}{(x+y)(x-y)}$

$$=\frac{x+y-3(x-y)+3x-y}{(x-y)(x+y)}$$

$$=\frac{x+y-3x+3y+3x-y}{(x-y)(x+y)}$$

$$=\frac{x+3y}{(x-y)(x+y)}$$

76. $\dfrac{x}{x^2+15x+50}+\dfrac{7}{x+10}-\dfrac{x-1}{x+5}=\dfrac{x}{(x+10)(x+5)}+\dfrac{7(x+5)}{(x+10)(x+5)}-\dfrac{(x-1)(x+10)}{(x+10)(x+5)}$

$$=\frac{x+7(x+5)-(x+10)(x-1)}{(x+10)(x+5)}$$

$$=\frac{x+7x+35-(x^2+9x-10)}{(x+10)(x+5)}$$

$$=\frac{x+7x+35-x^2-9x+10}{(x+10)(x+5)}$$

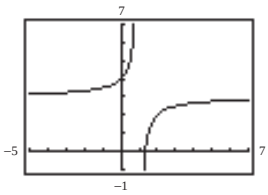
$$=\frac{-x^2-x+45}{(x+10)(x+5)}$$

$$=-\frac{x^2+x-45}{(x+10)(x+5)}$$

78. *Keystrokes:*

y_1: 3 [−] [(] 1 [÷] [(] [X,T,θ] [−] 1 [)] [)] [ENTER]

y_2: [(] 3 [X,T,θ] [−] 4 [)] [÷] [(] [X,T,θ] [−] 1 [)] [GRAPH]



$$3-\frac{1}{x-1}=\frac{3(x-1)}{x-1}-\frac{1}{x-1}$$

$$=\frac{3(x-1)-1}{x-1}$$

$$=\frac{3x-3-1}{x-1}$$

$$=\frac{3x-4}{x-1}$$

$y_1 = y_2$

80. $\dfrac{t}{3}+\dfrac{t}{5}=\dfrac{t(5)}{3(5)}+\dfrac{t(3)}{5(3)}$

$$=\frac{5t}{15}+\frac{3t}{15}$$

$$=\frac{5t+3t}{15}$$

$$=\frac{8t}{15}$$

82.
$$\begin{aligned} A \quad\quad + C &= 0 \\ -A + B \quad\quad &= 1 \\ -B \quad\quad &= 1 \end{aligned}$$
$$-B = 1$$
$$B = -1$$
$$-A + (-1) = 1$$
$$-A = 2$$
$$A = -2$$
$$-2 + C = 0$$
$$C = 2$$

$$\frac{x+1}{x^3-x^2} = \frac{-2}{x} + \frac{-1}{x^2} + \frac{2}{x-1}$$
$$= \frac{-2x(x-1)}{x^2(x-1)} + \frac{-(x-1)}{x^2(x-1)} + \frac{2x^2}{x^2(x-1)}$$
$$= \frac{-2x(x-1) - (x-1) + 2x^2}{x^2(x-1)}$$
$$= \frac{-2x^2 + 2x - x + 1 + 2x^2}{x^3-x^2} = \frac{x+1}{x^3-x^2}$$

84.

Year	1995	1996	1997	1998	1999	2000	2001
Participants	5825.01	6076.26	6271.56	6429.72	6561.57	6673.86	6771.10

(a) 1995 let $t = 5$ $\dfrac{750.27(5)^2 + 5660.36(5) - 4827.2}{5[0.09(5) + 1.0]}$

(b) 1996 let $t = 6$ $\dfrac{750.27(6)^2 + 5660.36(6) - 4827.2}{6[0.09(6) + 1.0]}$

Continue in the same manner for the remaining years.

86. Add or subtract the numerators and place the result over the common denominator.

88. Yes. $\dfrac{3}{2}(x+2) + \dfrac{x}{x+2}$

90. The simplification is not correct.

$$\frac{2}{x} - \frac{3}{x+1} + \frac{x+1}{x^2} = \frac{2x(x+1) - 3x^2 + (x+1)^2}{x^2(x+1)}$$
$$= \frac{2x^2 + 2x - 3x^2 + x^2 + 2x + 1}{x^2(x+1)}$$
$$= \frac{4x+1}{x^2(x+1)}$$

Two errors occurred, both in step 2. The $2x$ was not distributed properly, $2x(x+1) \neq 2x^2 + x$. The product $(x+1)^2$ was not multiplied correctly, $(x+1)^2 \neq x^2 + 1$.

Section 6.4 Complex Fractions

2. $\dfrac{\left(\frac{y^4}{12}\right)}{\left(\frac{y}{16}\right)} = \dfrac{y^4}{12} \div \dfrac{y}{16} = \dfrac{y^4}{12} \cdot \dfrac{16}{y} = \dfrac{(4)(4)y^4}{(4)(3)y} = \dfrac{4}{3}y^3, \quad y \neq 0$

4. $\dfrac{\left(\frac{3u^2}{6v^3}\right)}{\left(\frac{u}{3v}\right)} = \dfrac{3u^2}{6v^3} \div \dfrac{u}{3v} = \dfrac{3u^2}{6v^3} \cdot \dfrac{3v}{u} = \dfrac{3u}{2v^2}, \quad u \neq 0$

6. $\dfrac{\left(\frac{36x^4}{5y^4z^5}\right)}{\left(\frac{9xy^2}{20z^5}\right)} = \dfrac{36x^4}{5y^4z^5} \div \dfrac{9xy^2}{20z^5} = \dfrac{36x^4}{5y^4z^5} \cdot \dfrac{20z^5}{9xy^2}$
$$= \frac{(9)(4)(4)(5)x^4z^5}{(5)(9)xy^6z^5} = \frac{16x^3}{y^6}, \quad x \neq 0, \quad z \neq 0$$

8. $\dfrac{\left(\frac{(3r)^3}{10t^4}\right)}{\left(\frac{9r}{(2t)^2}\right)} = \dfrac{27r^3}{10t^4} \div \dfrac{9r}{4t^2} = \dfrac{27r^3}{10t^4} \cdot \dfrac{4t^2}{9r}$
$$= \frac{(9)(3)(2)(2)r^3t^2}{(5)(2)(3)(3)rt^4} = \frac{6r^2}{5t^2}, \quad r \neq 0$$

10. $$\frac{\left(\frac{x}{x-4}\right)}{\left(\frac{x}{4-x}\right)} = \frac{x}{x-4} \div \frac{x}{4-x} = \frac{x}{x-4} \cdot \frac{4-x}{x}$$

$$= \frac{x(-1)(x-4)}{(x-4)x} = -1, \quad x \neq 4, \quad x \neq 0$$

12. $$\frac{\left(\frac{5x}{x+7}\right)}{\left(\frac{10}{x^2+8x+7}\right)} = \frac{5x}{x+7} \cdot \frac{x^2+8x+7}{10}$$

$$= \frac{5x}{x+7} \cdot \frac{(x+7)(x+1)}{5 \cdot 2}$$

$$= \frac{x(x+1)}{2}, \quad x \neq -7, -1$$

14. $$\frac{\left(\frac{a+5}{6a-15}\right)}{\left(\frac{a+5}{2a-5}\right)} = \frac{a+5}{6a-15} \div \frac{a+5}{2a-5}$$

$$= \frac{a+5}{3(2a-5)} \cdot \frac{2a-5}{a+5}$$

$$= \frac{(a+5)(2a-5)}{3(2a-5)(a+5)}$$

$$= \frac{1}{3}, \quad a \neq -5, \quad a \neq \frac{5}{2}$$

16. $$\frac{\left(\frac{x^2-2x-8}{x-1}\right)}{5x-20} = \frac{x^2-2x-8}{x-1} \div \frac{5x-20}{1}$$

$$= \frac{(x-4)(x+2)}{x-1} \cdot \frac{1}{5(x-4)}$$

$$= \frac{(x-4)(x+2)}{(x-1)5(x-4)} = \frac{x+2}{5(x-1)}, \quad x \neq 4$$

18. $$\frac{4x+16}{\left(\frac{x^2+9x+20}{x-1}\right)} = \frac{4x+16}{1} \div \frac{x^2+9x+20}{x-1}$$

$$= \frac{4(x+4)}{1} \cdot \frac{x-1}{(x+5)(x+4)}$$

$$= \frac{4(x+4)(x-1)}{(x+5)(x+4)}$$

$$= \frac{4(x-1)}{x+5}, \quad x \neq -4, \quad 1$$

20. $$\frac{\left(\frac{6x^2-13x-5}{5x^2+5x}\right)}{\left(\frac{2x-5}{5x+1}\right)} = \frac{6x^2-13x-5}{5x^2+5x} \div \frac{2x-5}{5x+1}$$

$$= \frac{(3x+1)(2x-5)}{5x(x+1)} \cdot \frac{5x+1}{2x-5}$$

$$= \frac{(3x+1)(2x-5)(5x+1)}{5x(x+1)(2x-5)}$$

$$= \frac{(3x+1)(5x+1)}{5x(x+1)}, x \neq \frac{5}{2}, \quad x \neq -\frac{1}{5}$$

22. $$\frac{9x^2-24x+16}{x^2+10x+25} \div \frac{6x^2-5x-4}{2x^2+3x-35} = \frac{9x^2-24x+16}{x^2+10x+25} \cdot \frac{2x^2+3x-35}{6x^2-5x-4}$$

$$= \frac{(3x-4)(3x-4)}{(x+5)(x+5)} \cdot \frac{(2x-7)(x+5)}{(3x-4)(2x+1)}$$

$$= \frac{(3x-4)(2x-7)}{(x+5)(2x+1)}, \quad x \neq \frac{4}{3}, \frac{7}{2}$$

24. $$\frac{t^3+t^2-9t-9}{t^2-5t+6} \div \frac{t^2+6t+9}{t-2} = \frac{t^3+t^2-9t-9}{t^2-5t+6} \cdot \frac{t-2}{t^2+6t+9}$$

$$= \frac{t^2(t+1)-9(t+1)}{(t-3)(t-2)} \cdot \frac{t-2}{(t+3)(t+3)}$$

$$= \frac{(t^2-9)(t+1)(t-2)}{(t-3)(t+3)(t+3)(t-2)}$$

$$= \frac{(t-3)(t+3)(t+1)}{(t-3)(t+3)(t+3)}$$

$$= \frac{t+1}{t+3}, \quad t \neq 3, 2$$

26. $$\frac{\left(\dfrac{x^2+5x+6}{4x^2-20x+25}\right)}{\left(\dfrac{x^2-5x-24}{4x^2-25}\right)} = \frac{x^2+5x+6}{4x^2-20x+25} \div \frac{x^2-5x-24}{4x^2-25}$$
$$= \frac{(x+3)(x+2)}{(2x-5)(2x-5)} \cdot \frac{(2x-5)(2x+5)}{(x-8)(x+3)}$$
$$= \frac{(x+2)(2x+5)}{(2x-5)(x-8)}, \quad x \neq -3, -\frac{5}{2}$$

28. $$\frac{x}{\left(\dfrac{5}{x}+2\right)} = \frac{\dfrac{x}{1}}{\dfrac{5}{x}+2} \cdot \frac{x}{x} = \frac{x^2}{5+2x}, \quad x \neq 0$$

30. $$\frac{\left(1-\dfrac{2}{x}\right)}{\left(\dfrac{x}{2}\right)} = \frac{1-\dfrac{2}{x}}{\dfrac{x}{2}} \cdot \frac{2x}{2x} = \frac{2x-4}{x^2} = \frac{2(x-2)}{x^2}$$

32. $$\frac{\left(\dfrac{1}{t}-1\right)}{\left(\dfrac{1}{t}+1\right)} = \frac{\left(\dfrac{1}{t}-1\right)}{\left(\dfrac{1}{t}+1\right)} \cdot \frac{t}{t} = \frac{1-t}{1+t}, \quad t \neq 0$$

34. $$\frac{\left(x+\dfrac{2}{x-3}\right)}{\left(x+\dfrac{6}{x-3}\right)} = \frac{\left(x+\dfrac{2}{x-3}\right)}{\left(x+\dfrac{6}{x-3}\right)} \cdot \frac{x-3}{x-3}$$
$$= \frac{x(x-3)+2}{x(x-3)+6}$$
$$= \frac{x^2-3x+2}{x^2-3x+6}$$
$$= \frac{(x-2)(x-1)}{x^2-3x+6}, \quad x \neq 3$$

36. $$\frac{\left(16-\dfrac{1}{x^2}\right)}{\left(\dfrac{1}{4x^2}-4\right)} = \frac{\left(16-\dfrac{1}{x^2}\right)}{\left(\dfrac{1}{4x^2}-4\right)} \cdot \frac{4x^2}{4x^2}$$
$$= \frac{64x^2-4}{1-16x^2}$$
$$= \frac{4(16x^2-1)}{1-16x^2}$$
$$= \frac{4(16x^2-1)}{-1(16x^2-1)}$$
$$= \frac{4}{-1}$$
$$= -4, \quad x \neq 0, -\frac{1}{4}, \frac{1}{4}$$

38. $$\frac{\left(x-\dfrac{2y^2}{x-y}\right)}{x-2y} = \frac{\left(x-\dfrac{2y^2}{x-y}\right)}{x-2y} \cdot \frac{(x-y)}{(x-y)}$$
$$= \frac{x(x-y)-2y^2}{(x-2y)(x-y)}$$
$$= \frac{x^2-xy-2y^2}{(x-2y)(x-y)}$$
$$= \frac{(x-2y)(x+y)}{(x-2y)(x-y)}$$
$$= \frac{x+y}{x-y}, \quad x \neq 2y$$

40. $$\frac{\left(\dfrac{x+1}{x+2}-\dfrac{1}{x}\right)}{\left(\dfrac{2}{x+2}\right)} = \frac{\left(\dfrac{x+1}{x+2}-\dfrac{1}{x}\right)}{\left(\dfrac{2}{x+2}\right)} \cdot \frac{x(x+2)}{x(x+2)}$$
$$= \frac{(x+1)x-1(x+2)}{2(x)}$$
$$= \frac{x^2+x-x-2}{2x}$$
$$= \frac{x^2-2}{2x}, \quad x \neq -2$$

42. $$\frac{\left(\dfrac{2}{x+5}\right)}{\left(\dfrac{2}{x+5}+\dfrac{1}{4x+20}\right)} = \frac{\left(\dfrac{2}{x+5}\right)}{\left(\dfrac{2}{x+5}+\dfrac{1}{4x+20}\right)} \cdot \frac{4(x+5)}{4(x+5)}$$
$$= \frac{2(4)}{(4)2+1} = \frac{8}{9}, \quad x \neq -5$$

44. $$\frac{\left(\dfrac{5}{y}-\dfrac{6}{2y+1}\right)}{\left(\dfrac{5}{2y+1}\right)} = \frac{\dfrac{5}{y}-\dfrac{6}{2y+1}}{\dfrac{5}{2y+1}} \cdot \frac{y(2y+1)}{y(2y+1)}$$
$$= \frac{5(2y+1)-6y}{5y} = \frac{10y+5-6y}{5y}$$
$$= \frac{4y+5}{5y}, \quad y \neq -\frac{1}{2}$$

46. $$\frac{\left(\frac{1}{2x}-\frac{6}{x+5}\right)}{\left(\frac{x}{x-5}+\frac{1}{x}\right)}=\frac{\left(\frac{1}{2x}-\frac{6}{x+5}\right)}{\left(\frac{x}{x-5}+\frac{1}{x}\right)}\cdot\frac{2x(x+5)(x-5)}{2x(x+5)(x-5)}$$

$$=\frac{(x+5)(x-5)-6(2x)(x-5)}{x(2x)(x+5)+1(2)(x+5)(x-5)}$$

$$=\frac{x^2-25-12x(x-5)}{2x^2(x+5)+2(x+5)(x-5)}$$

$$=\frac{x^2-25-12x^2+60x}{(x+5)(2x^2+2(x-5))}$$

$$=\frac{-11x^2+60x-25}{(x+5)(2x^2+2x-10)}$$

$$=-\frac{(11x^2-60x+25)}{(x+5)(2x^2+2x-10)}$$

$$=-\frac{(11x-5)(x-5)}{(x+5)(2x^2+2x-10)}$$

$$=\frac{(5-x)(11x-5)}{2(x+5)(x^2+x-5)},\quad x\neq 0,5$$

48. $$\frac{9x-x^{-1}}{3+x^{-1}}=\frac{9x-\frac{1}{x}}{3+\frac{1}{x}}\cdot\frac{x}{x}=\frac{9x^2-1}{3x+1}$$

$$=\frac{(3x-1)(3x+1)}{3x+1}$$

$$=3x-1,\quad x\neq -\frac{1}{3},\quad 0$$

50. $$\frac{3x^{-2}-x}{4x^{-1}+6x}=\frac{\frac{3}{x^2}-x}{\frac{4}{x}+6x}\cdot\frac{x^2}{x^2}$$

$$=\frac{3-x^3}{4x+6x^3}=\frac{3-x^3}{2x(2+3x^2)}$$

52. $$\frac{x^{-1}-y^{-1}}{x^{-2}-y^{-2}}=\frac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{x^2}-\frac{1}{y^2}}\cdot\frac{x^2y^2}{x^2y^2}$$

$$=\frac{xy^2-x^2y}{y^2-x^2}=\frac{xy(y-x)}{(y-x)(y+x)}$$

$$=\frac{xy}{y+x},\quad x\neq 0,\quad y\neq 0,\quad x\neq y$$

54. $$\frac{x-y}{x^{-2}-y^{-2}}=\frac{\frac{x-y}{1}}{\frac{1}{x^2}-\frac{1}{y^2}}\cdot\frac{x^2y^2}{x^2y^2}$$

$$=\frac{x^2y^2(x-y)}{y^2-x^2}=\frac{-x^2y^2(y-x)}{(y-x)(y+x)}$$

$$=-\frac{x^2y^2}{y+x},\quad x\neq 0,\quad y\neq 0,\quad x\neq y$$

56. $$\frac{f(2+h)-f(2)}{h}=\frac{\frac{2+h}{2+h-1}-\frac{2}{2-1}}{h}$$

$$=\frac{\frac{2+h}{1+h}-\frac{2}{1}}{h}$$

$$=\frac{\frac{2+h}{1+h}-2}{h}\cdot\frac{1+h}{1+h}$$

$$=\frac{2+h-2-2h}{h(1+h)}$$

$$=\frac{-h}{h(1+h)}$$

$$=-\frac{1}{1+h}$$

58. $$\frac{\frac{2x}{3}+\frac{3x}{5}}{2}=\frac{\frac{2x}{3}+\frac{3x}{5}}{\frac{2}{1}}\cdot\frac{15}{15}=\frac{10x+9x}{30}=\frac{19x}{30}$$

60. $$\frac{\frac{4}{a^2}+\frac{2}{a}}{2}=\frac{\frac{4}{a^2}+\frac{2}{a}}{2}\cdot\frac{a^2}{a^2}$$

$$=\frac{\frac{4}{a^2}(a^2)+\frac{2}{a}(a^2)}{2(a^2)}$$

$$=\frac{4+2a}{2a^2}$$

$$=\frac{2(2+a)}{2a^2}$$

$$=\frac{2+a}{a^2}$$

62. $$\frac{\frac{5}{2s}+\frac{s+1}{5}}{2}=\frac{\frac{5}{2s}+\frac{s+1}{5}}{2}\cdot\frac{10s}{10s}$$

$$=\frac{\frac{5}{2s}(10s)+\frac{s+1}{5}(10s)}{2(10s)}$$

$$=\frac{25+2s(s+1)}{20s}$$

$$=\frac{25+2s^2+2s}{20s}$$

$$=\frac{2s^2+2s+25}{20s}$$

64. $$\frac{\frac{5x}{4}-\frac{x}{3}}{3}=\frac{\frac{5x}{4}-\frac{x}{3}}{\frac{3}{1}}\cdot\frac{12}{12}=\frac{15x-4x}{36}=\frac{11x}{36}$$

$$x_1=\frac{x}{3}+\frac{11x}{36}=\frac{12x}{36}+\frac{11x}{36}=\frac{23x}{36}$$

$$x_2=\frac{23x}{36}+\frac{11x}{36}=\frac{34x}{36}=\frac{17x}{18}$$

66. (a) $$r=\frac{\frac{24(MN-P)}{N}}{P+\frac{MN}{12}}\cdot\frac{12N}{12N}$$

$$=\frac{288(MN-P)}{12NP+MN^2}$$

$$=\frac{288(MN-P)}{N(MN+12P)}$$

(b) $$r=\frac{288(350(48)-15{,}000)}{48(350(48)+12(15{,}000))}$$

$$r=5.49\%$$

68. (a)

Year, t	4	6	8	10
Monthly bill	\$49.08	\$41.42	\$37.48	\$35.08

1994: let $t=4$ Monthly bill $=\frac{250(1382.16(4)+5847.9)}{3(4568.33(4)+1042.7)}=\$49.08$

1996: let $t=6$ Monthly bill $=\frac{250(1382.16(6)+5847.9)}{3(4568.33(6)+1042.7)}=\$41.42$

1998: let $t=8$ Monthly bill $=\frac{250(1382.16(8)+5847.9)}{3(4568.33(8)+1042.7)}=\$37.48$

2000: let $t=10$ Monthly bill $=\frac{250(1382.16(10)+5847.9)}{3(4568.33(10)+1042.7)}=\$35.08$

(b) The number of subscribers was increasing at a higher rate than the revenue.

70. (a) $\dfrac{5}{\left(\dfrac{3}{x^2+5x+6}\right)}$ Numerator: 5; Denominator: $\dfrac{3}{x^2+5x+6}$

(b) $\dfrac{\dfrac{5}{3}}{x^2+5x+6}$ Numerator: $\dfrac{5}{3}$; Denominator: x^2+5x+6

72. Method 1: Invert the denominator and multiply by the numerator to simplify the fraction.

Method 2: Multiply the numerator and denominator by the least common denominator for all fractions in the numerator and the denominator.

Section 6.5 Dividing Polynomials and Synthetic Division

2. $(6a^2+7a)\div a = \dfrac{6a^2+7a}{a}$

$= \dfrac{6a^2}{a}+\dfrac{7a}{a}$

$= 6a+7, \quad a\neq 0$

4. $(5y^3+6y^2-3y)\div(-y) = \dfrac{5y^3+6y^2-3y}{-y}$

$= \dfrac{5y^3}{-y}+\dfrac{6y^2}{-y}-\dfrac{3y}{-y}$

$= -5y^2-6y+3, \quad y\neq 0$

6. $(x^3+x-2)\div x = \dfrac{x^3+x-2}{x}$

$= \dfrac{x^3}{x}+\dfrac{x}{x}-\dfrac{2}{x}$

$= x^2+1-\dfrac{2}{x}, \quad x\neq 0$

8. $\dfrac{18c^4-24c^2}{-6c} = \dfrac{18c^4}{-6c}-\dfrac{24c^2}{-6c}$

$= -3c^3+4c, \quad c\neq 0$

10. $\dfrac{6x^4+8x^3-18x^2}{3x^2} = \dfrac{6x^4}{3x^2}+\dfrac{8x^3}{3x^2}-\dfrac{18x^3}{3x^2}$

$= 2x^2+\dfrac{8x}{3}-6, \quad x\neq 0$

12. $\dfrac{15x^{12}-5x^9+30x^6}{5x^6} = \dfrac{15x^{12}}{5x^6}-\dfrac{5x^9}{5x^6}+\dfrac{30x^6}{5x^6}$

$= 3x^6-x^3+6, \quad x\neq 0$

14. $(-14s^4t^2+7s^2t^2-18t)\div 2s^2t = \dfrac{-14s^4t^2+7s^2t^2-18t}{2s^2t}$

$= \dfrac{-14s^4t^2}{2s^2t}+\dfrac{7s^2t^2}{2s^2t}-\dfrac{18t}{2s^2t}$

$= -7s^2t+\dfrac{7t}{2}-\dfrac{9}{s^2}, \quad t\neq 0$

16.
$$\begin{array}{r} t-12, \quad t\neq 6 \\ t-6\,\overline{)\,t^2-18t+72} \\ \underline{t^2-6t} \\ -12t+72 \\ \underline{-12t+72} \end{array}$$

18.
$$\begin{array}{r} y-8, \quad y\neq -2 \\ y+2\,\overline{)\,y^2-6y-16} \\ \underline{y^2+2y} \\ -8y-16 \\ \underline{-8y-16} \end{array}$$

20. $x - 3 \overline{)\, x^2 + 10x - 9}$

$$\begin{array}{r} x + 13 + \dfrac{30}{x-3} \\ \underline{x^2 - 3x} \\ 13x - 9 \\ \underline{13x - 39} \\ 30 \end{array}$$

22. $x + 1 \overline{)\, -x^2 + 4x + 5}$

$$\begin{array}{r} -x + 5, \quad x \neq -1 \\ \underline{-x^2 - x} \\ 5x + 5 \\ \underline{5x + 5} \end{array}$$

24. $x + 5 \overline{)\, 2x^2 + 13x + 15}$

$$\begin{array}{r} 2x + 3, \quad x \neq -5 \\ \underline{2x^2 + 10x} \\ 3x + 15 \\ \underline{3x + 15} \end{array}$$

26. $4x - 1 \overline{)\, 8x^2 + 2x + 3}$

$$\begin{array}{r} 2x + 1 + \dfrac{4}{4x-1} \\ \underline{8x^2 - 2x} \\ 4x + 3 \\ \underline{4x - 1} \\ 4 \end{array}$$

28. $2u + 5 \overline{)\, -8u^2 - 14u + 15}$

$$\begin{array}{r} -4u + 3, \quad u \neq -\frac{5}{2} \\ \underline{-8u^2 - 20u} \\ 6u + 15 \\ \underline{6u + 15} \end{array}$$

30. $2t - 3 \overline{)\, 10t^2 - 7t - 12}$

$$\begin{array}{r} 5t + 4, \quad t \neq \frac{3}{2} \\ \underline{10t^2 - 15t} \\ 8t - 12 \\ \underline{8t - 12} \end{array}$$

32. $x + 4 \overline{)\, x^3 + 4x^2 + 7x + 28}$

$$\begin{array}{r} x^2 + 7, \quad x \neq -4 \\ \underline{x^3 + 4x^2} \\ 7x + 28 \\ \underline{7x + 28} \end{array}$$

34. $2y + 3 \overline{)\, 4y^3 + 12y^2 + 7y - 3}$

$$\begin{array}{r} 2y^2 + 3y - 1, \quad y \neq -\dfrac{3}{2} \\ \underline{4y^3 + 6y^2} \\ 6y^2 + 7y \\ \underline{6y^2 + 9y} \\ -2y - 3 \\ \underline{-2y - 3} \end{array}$$

36. $2x + 3 \overline{)\, 12x - 5}$

$$\begin{array}{r} 6 - \dfrac{23}{2x+3} \\ \underline{12x + 18} \\ -23 \end{array}$$

38. $y + 2 \overline{)\, y^2 + 0y + 8}$

$$\begin{array}{r} y - 2 + \dfrac{12}{y+2} \\ \underline{y^2 + 2y} \\ -2y + 8 \\ \underline{-2y - 4} \\ 12 \end{array}$$

40. $3y + 5 \overline{)\, 8y^2 - 2y + 0}$

$$\begin{array}{r} \frac{8}{3}y - \frac{46}{9} + \dfrac{230}{9(3y+5)} \\ \underline{8y^2 + \frac{40}{3}y} \\ -\frac{46}{3}y + 0 \\ \underline{-\frac{46}{3}y - \frac{230}{9}} \\ \frac{230}{9} \end{array}$$

42. $9y - 5 \overline{)\, 81y^2 + 0y - 25}$

$$\begin{array}{r} 9y + 5, \quad y \neq \frac{5}{9} \\ \underline{81y^2 - 45y} \\ 45y - 25 \\ \underline{45y - 25} \end{array}$$

44.
$$\begin{array}{r} x^2 + 3x + 9, \quad x \neq 3 \\ x - 3 \overline{)\, x^3 + 0x^2 + 0x - 27} \\ \underline{x^3 - 3x^2} \\ 3x^2 + 0x \\ \underline{3x^2 - 9x} \\ 9x - 27 \\ \underline{9x - 27} \end{array}$$

46.
$$\begin{array}{r} x - 1 - \dfrac{3x + 10}{2x^2 + 4x + 5} \\ 2x^2 + 4x + 5 \overline{)\, 2x^3 + 2x^2 - 2x - 15} \\ \underline{2x^3 + 4x^2 + 5x} \\ -2x^2 - 7x - 15 \\ \underline{-2x^2 - 4x - 5} \\ -3x - 10 \end{array}$$

48.
$$\begin{array}{r} 4x^2 + 5x + 2 + \dfrac{14x^2 + 15x + 7}{2x^3 - x^2 - 3} \\ 2x^3 - x^2 - 3 \overline{)\, 8x^5 + 6x^4 - x^3 + 0x^2 + 0x + 1} \\ \underline{8x^5 - 4x^4 \qquad - 12x^2} \\ 10x^4 - x^3 + 12x^2 + 0x \\ \underline{10x^4 - 5x^3 \qquad - 15x} \\ 4x^3 + 12x^2 + 15x + 1 \\ \underline{4x^3 - 2x^2 \qquad - 6} \\ 14x^2 + 15x + 7 \end{array}$$

50.
$$\begin{array}{r} x^2 + x + 1 + \dfrac{1}{x - 1} \\ x - 1 \overline{)\, x^3 + 0x^2 + 0x + 0} \\ \underline{x^3 - x^2} \\ x^2 + 0x \\ \underline{x^2 - x} \\ x + 0 \\ \underline{x - 1} \\ 1 \end{array}$$

52.
$$\begin{array}{r} x^3 + 2x^2 + 4x + 8 + \dfrac{16}{x - 2} \\ x - 2 \overline{)\, x^4 + 0x^3 + 0x^2 + 0x + 0} \\ \underline{x^4 - 2x^3} \\ 2x^3 + 0x^2 \\ \underline{2x^3 - 4x^2} \\ 4x^2 + 0x \\ \underline{4x^2 - 8x} \\ 8x + 0 \\ \underline{8x - 16} \\ 16 \end{array}$$

54.
$$\begin{aligned} \frac{15x^3y}{10x^2} + \frac{3xy^2}{2y} &= \frac{3xy}{2} + \frac{3}{2}xy \\ &= \frac{6}{2}xy \\ &= 3xy, \quad x \neq 0, y \neq 0 \end{aligned}$$

56.
$$\begin{aligned} \frac{x^2 + 2x - 3}{x - 1} - (3x - 4) &= \frac{(x + 3)\cancel{(x - 1)}}{\cancel{x - 1}} - (3x - 4) \\ &= x + 3 - 3x + 4 \\ &= -2x + 7, \quad x \neq 1 \end{aligned}$$

58. $(x^2 + 5x - 6) \div (x + 6)$

$$\begin{array}{r|rrr} -6 & 1 & 5 & -6 \\ & & -6 & 6 \\ \hline & 1 & -1 & 0 \end{array}$$

$(x^2 + 5x - 6) \div (x + 6) = x - 1, \quad x \neq -6$

60. $\dfrac{x^4 - 4x^2 + 6}{x - 4}$

$$\begin{array}{r|rrrrr} 4 & 1 & 0 & -4 & 0 & 6 \\ & & 4 & 16 & 48 & 192 \\ \hline & 1 & 4 & 12 & 48 & 198 \end{array}$$

$$\frac{x^4 - 4x^2 + 6}{x - 4} = x^3 + 4x^2 + 12x + 48 + \frac{198}{x - 4}$$

62. $\dfrac{2x^5 - 3x^3 + x}{x - 3}$

$$\begin{array}{r|cccccc} 3 & 2 & 0 & -3 & 0 & 1 & 0 \\ & & 6 & 18 & 45 & 135 & 408 \\ \hline & 2 & 6 & 15 & 45 & 136 & 408 \end{array}$$

$$\frac{2x^5 - 3x^3 + x}{x - 3} = 2x^4 + 6x^3 + 15x^2 + 45x + 136 + \frac{408}{x - 3}$$

64. $\dfrac{5x^3 + 6x + 8}{x + 2}$

$$\begin{array}{r|cccc} -2 & 5 & 0 & 6 & 8 \\ & & -10 & 20 & -52 \\ \hline & 5 & -10 & 26 & -44 \end{array}$$

$$\frac{5x^3 + 6x + 8}{x + 2} = 5x^2 - 10x + 26 - \frac{44}{x + 2}$$

66. $\dfrac{x^5 - 13x^4 - 120x + 80}{x + 3}$

$$\begin{array}{r|cccccc} -3 & 1 & -13 & 0 & 0 & -120 & 80 \\ & & -3 & 48 & -144 & 432 & -936 \\ \hline & 1 & -16 & 48 & -144 & 312 & -856 \end{array}$$

$$\frac{x^5 - 13x^4 - 120x + 80}{x + 3} = x^4 - 16x^3 + 48x^2 - 144x + 312 - \frac{856}{x + 3}$$

68. $\dfrac{x^3 - 0.8x + 2.4}{x + 0.1}$

$$\begin{array}{r|cccc} -0.1 & 1 & 0 & -0.8 & 2.4 \\ & & -0.1 & 0.01 & 0.079 \\ \hline & 1 & -0.1 & -0.79 & 2.479 \end{array}$$

$$\frac{x^3 - 0.8x + 2.4}{x + 0.1} = x^2 - 0.1x - 0.79 + \frac{2.479}{x + 0.1}$$

70.

$$\begin{array}{r|cccc} -5 & 1 & 1 & -32 & -60 \\ & & -5 & 20 & 60 \\ \hline & 1 & -4 & -12 & 0 \end{array}$$

$$x^2 - 4x - 12 = (x - 6)(x + 2)$$

$$x^3 + x^2 - 32x - 60 = (x + 5)(x - 6)(x + 2)$$

72.

$$\begin{array}{r|cccc} 3 & 9 & -3 & -56 & -48 \\ & & 27 & 72 & 48 \\ \hline & 9 & 24 & 16 & 0 \end{array}$$

$$9x^2 + 24x + 16 = (3x + 4)^2$$

$$9x^3 - 3x^2 - 56x - 48 = (x - 3)(3x + 4)^2$$

74.

$$\begin{array}{r|ccccc} 4 & 1 & -6 & -8 & 96 & -128 \\ & & 4 & -8 & -64 & 128 \\ \hline & 1 & -2 & -16 & 32 & 0 \end{array}$$

$$x^3 - 2x^2 - 16x + 32 = x^2(x - 2) - 16(x - 2)$$

$$= (x - 2)(x^2 - 16)$$

$$= (x - 2)(x - 4)(x + 4)$$

$$x^4 + 7x^3 + 3x^2 - 63x - 108 = (x - 4)^2(x - 2)(x + 4)$$

76. $\dfrac{18x^2 - 9x - 20}{x + \frac{5}{6}}$

$$\begin{array}{r|ccc} -\frac{5}{6} & 18 & -9 & -20 \\ & & -15 & 20 \\ \hline & 18 & -24 & 0 \end{array}$$

$$18x - 24 = 6(3x - 4)$$

$$(18x^2 - 9x - 20) = 6\left(x + \frac{5}{6}\right)(3x - 4)$$

78. $\dfrac{x^4 - 3x^2 + c}{x + 6}$

$$\begin{array}{r|ccccc} -6 & 1 & 0 & -3 & 0 & c \\ & & -6 & 36 & -198 & 1188 \\ \hline & 1 & -6 & 33 & -198 & 0 \end{array}$$

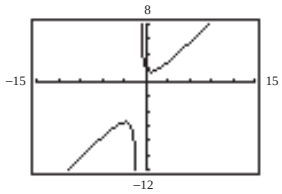
$$c + 1188 = 0$$

$$c = -1188$$

80. *Keystrokes:*

y_1 [Y=] [(] [X,T,θ] [x^2] [+] 2 [)] [÷] [(] [X,T,θ] [+] 1 [)] [ENTER]

y_2 [X,T,θ] [−] 1 [+] 3 [÷] [(] [X,T,θ] 1 [+] [)] [GRAPH]



$$\begin{array}{r|rrr} -1 & 1 & 0 & 2 \\ & & -1 & 1 \\ \hline & 1 & -1 & 3 \end{array}$$

Thus, $\dfrac{x^2+2}{x+1} = x - 1 + \dfrac{3}{x+1}$.

82.
$$\begin{array}{r} x^{2n} + 5, \quad x^n \neq 1 \\ x^n - 1 \overline{)\, x^{3n} - x^{2n} + 5x^n - 5} \\ \underline{x^{3n} - x^{2n}} \\ 0x^{2n} + 5x^n - 5 \\ \underline{5x^n - 5} \end{array}$$

84. Dividend = Divisor · Quotient + Remainder

$= (x+3)(x^3 + x^2 - 4) + 8$

$= x^4 + x^3 - 4x + 3x^3 + 3x^2 - 12 + 8$

$= x^4 + 4x^3 + 3x^2 - 4x - 4$

86.

k	$f(k)$	Divisors, $(x-k)$	Remainder
-2	-15	$x+2$	-15
-1	0	$x+1$	0
0	1	x	1
$\frac{1}{2}$	0	$x-\frac{1}{2}$	0
1	0	$x-1$	0
2	9	$x-2$	9

$f(-2) = 2(-2)^3 - (-2)^2 - 2(-2) + 1$
$= -16 - 4 + 4 + 1$
$= -15$

$$\begin{array}{r|rrrr} -2 & 2 & -1 & -2 & 1 \\ & & -4 & 10 & -16 \\ \hline & 2 & -5 & 8 & -15 \end{array}$$

$f(-1) = 2(-1)^3 - (-1)^2 - 2(-1) + 1$
$= -2 - 1 + 2 + 1$
$= 0$

$$\begin{array}{r|rrrr} -1 & 2 & -1 & -2 & 1 \\ & & -2 & 3 & -1 \\ \hline & 2 & -3 & 1 & 0 \end{array}$$

$f(0) = 2(0)^3 - (0)^2 - 2(0) + 1$
$= 0 - 0 - 0 + 1$
$= 1$

$$\begin{array}{r|rrrr} 0 & 2 & -1 & -2 & 1 \\ & & 0 & 0 & 0 \\ \hline & 2 & -1 & -2 & 1 \end{array}$$

$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right) + 1$
$= \frac{1}{4} - \frac{1}{4} - 1 + 1$
$= 0$

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -1 & -2 & 1 \\ & & 1 & 0 & -1 \\ \hline & 2 & 0 & -2 & 0 \end{array}$$

$f(1) = 2(1)^3 - (1)^2 - 2(1) + 1$
$= 2 - 1 - 2 + 1$
$= 0$

$$\begin{array}{r|rrrr} 1 & 2 & -1 & -2 & 1 \\ & & 2 & 1 & -1 \\ \hline & 2 & 1 & -1 & 0 \end{array}$$

$f(2) = 2(2)^3 - (2)^2 - 2(2) + 1$
$= 16 - 4 - 4 + 1$
$= 9$

$$\begin{array}{r|rrrr} 2 & 2 & -1 & -2 & 1 \\ & & 4 & 6 & 8 \\ \hline & 2 & 3 & 4 & 9 \end{array}$$

The polynomial values equal the remainders.

88. Volume = Length · Width · Height

Volume = Area of first floor · Height

$$\text{Area of first floor} = \frac{\text{Volume}}{\text{Height}} = \frac{x^3 + 55x^2 + 650x + 2000}{x + 5}$$

$$\begin{array}{r|cccc} -5 & 1 & 55 & 650 & 2000 \\ & & -5 & -250 & -2000 \\ \hline & 1 & 50 & 400 & 0 \end{array}$$

Area of first floor = $x^2 + 50x + 400$ (square feet)

90. Volume = Length · Width · Height

$$\begin{aligned} \text{Length} &= \frac{\text{Volume}}{\text{Width} \cdot \text{Height}} \\ &= \frac{h^4 + 3h^3 + 2h^2}{h(h+1)} \\ &= \frac{h^2(h^2 + 3h + 2)}{h(h+1)} \\ &= \frac{h^2(h+2)(h+1)}{h(h+1)} \\ &= h(h+2) \\ &= h^2 + 2h \end{aligned}$$

92. $\dfrac{x^2 + 4}{x + 1} = x - 1 + \dfrac{5}{x + 1}$

Divisor: $x + 1$

Dividend: $x^2 + 4$

Quotient: $x - 1$

Remainder: 5

94. Check polynomial division by multiplication. Using Exercise 92 as an example:

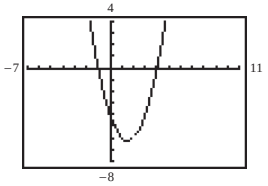
$$\begin{aligned} (x+1)\left(x - 1 + \frac{5}{x+1}\right) &= (x+1)(x-1) + (x-1)\frac{5}{x+1} \\ &= x^2 - 1 + 5 \\ &= x^2 + 4 \end{aligned}$$

96. For synthetic division, the divisor must be of the form $x - k$.

98. $f(x) = \dfrac{x^3 - 5x^2 + 2x + 8}{x - 2}$

Keystrokes:

Y= (X,T,θ ^ 3 − 5 X,T,θ x² + 2 X,T,θ + 8) ÷
(X,T,θ − 2) GRAPH



The function has x-intercepts $(-1, 0)$ and $(4, 0)$. It has only two x-intercepts because the numerator factors and it is possible to divide out the same factor in the numerator and denominator, leaving a second-degree polynomial having two factors. The function appears to be equivalent to $f(x) = (x - 4)(x + 1)$. The difference is at $x = 2$.

Section 6.6 Solving Rational Equations

2. (a) $x = 0$

$$0 \stackrel{?}{=} 4 + \frac{21}{0}$$

$\frac{21}{0}$ is undefined.

Not a solution

(b) $x = -3$

$$-3 \stackrel{?}{=} 4 + \frac{21}{-3}$$
$$-3 \stackrel{?}{=} 4 + (-7)$$
$$-3 = -3$$

Solution

(c) $x = 7$

$$7 \stackrel{?}{=} 4 + \frac{21}{7}$$
$$7 \stackrel{?}{=} 4 + 3$$
$$7 = 7$$

Solution

(d) $x = -1$

$$-1 \stackrel{?}{=} 4 + \frac{21}{-1}$$
$$-1 \stackrel{?}{=} 4 - 21$$
$$-1 \neq -17$$

Not a solution

4. (a) $x = \frac{10}{3}$

$$5 - \frac{1}{\frac{10}{3} - 3} \stackrel{?}{=} 2$$
$$5 - \frac{1}{\frac{10}{3} - \frac{9}{3}} \stackrel{?}{=} 2$$
$$5 - \frac{1}{\frac{1}{3}} \stackrel{?}{=} 2$$
$$5 - 3 \stackrel{?}{=} 2$$
$$2 = 2$$

Solution

(b) $x = -\frac{1}{3}$

$$5 - \frac{1}{-\frac{1}{3} - 3} \stackrel{?}{=} 2$$
$$5 - \frac{1}{-\frac{1}{3} - \frac{9}{3}} \stackrel{?}{=} 2$$
$$5 - \frac{1}{-\frac{10}{3}} \stackrel{?}{=} 2$$
$$5 - \left(-\frac{3}{10}\right) \stackrel{?}{=} 2$$
$$\frac{50}{10} + \frac{3}{10} \stackrel{?}{=} 2$$
$$\frac{53}{10} \neq 2$$

Not a solution

(c) $x = 0$

$$5 - \frac{1}{0 - 3} \stackrel{?}{=} 2$$
$$5 - \left(-\frac{1}{3}\right) \stackrel{?}{=} 2$$
$$\frac{15}{3} + \frac{1}{3} \stackrel{?}{=} 2$$
$$\frac{16}{3} \neq 2$$

Not a solution

(d) $x = 1$

$$5 - \frac{1}{1 - 3} \stackrel{?}{=} 2$$
$$5 - \frac{1}{-2} \stackrel{?}{=} 2$$
$$\frac{10}{2} + \frac{1}{2} \stackrel{?}{=} 2$$
$$\frac{11}{2} \neq 2$$

Not a solution

6.
$$\frac{y}{8} + 7 = -\frac{1}{2}$$
$$8\left(\frac{y}{8} + 7\right) = \left(-\frac{1}{2}\right)8$$
$$y + 56 = -4$$
$$y = -60$$

Check:
$$\frac{-60}{8} + 7 \stackrel{?}{=} -\frac{1}{2}$$
$$\frac{-15}{2} + \frac{14}{2} \stackrel{?}{=} -\frac{1}{2}$$
$$-\frac{1}{2} = -\frac{1}{2}$$

8.
$$\frac{a}{5} = \frac{a - 3}{2}$$
$$2a = 5(a - 3)$$
$$2a = 5a - 15$$
$$-3a = -15$$
$$a = 5$$

Check:
$$\frac{5}{5} \stackrel{?}{=} \frac{5 - 3}{2}$$
$$1 \stackrel{?}{=} \frac{2}{2}$$
$$1 = 1$$

10.
$$\frac{x}{4} - \frac{x}{6} = \frac{1}{4}$$
$$12\left(\frac{x}{4} - \frac{x}{6}\right) = \left(\frac{1}{4}\right)12$$
$$3x - 2x = 3$$
$$x = 3$$

Check:
$$\frac{3}{4} - \frac{3}{6} \stackrel{?}{=} \frac{1}{4}$$
$$\frac{3}{4} - \frac{1}{2} \stackrel{?}{=} \frac{1}{4}$$
$$\frac{3}{4} - \frac{2}{4} \stackrel{?}{=} \frac{1}{4}$$
$$\frac{1}{4} = \frac{1}{4}$$

12.
$$\frac{x - 5}{5} + 3 = -\frac{x}{4}$$
$$20\left(\frac{x - 5}{5} + 3\right) = \left(-\frac{x}{4}\right)20$$
$$4(x - 5) + 60 = -5x$$
$$4x - 20 + 60 = -5x$$
$$4x + 40 = -5x$$
$$40 = -9x$$
$$-\frac{40}{9} = x$$

Check:
$$\frac{-\frac{40}{9} - 5}{5} + 3 \stackrel{?}{=} -\frac{-\frac{40}{9}}{4}$$
$$\frac{-\frac{40}{9} - \frac{45}{9}}{5} + 3 \stackrel{?}{=} \frac{40}{9} \cdot \frac{1}{4}$$
$$\frac{1}{5}\left(-\frac{85}{9}\right) + 3 \stackrel{?}{=} \frac{10}{9}$$
$$-\frac{17}{9} + \frac{27}{9} \stackrel{?}{=} \frac{10}{9}$$
$$\frac{10}{9} = \frac{10}{9}$$

14. $\dfrac{4x-2}{7}-\dfrac{5}{14}=2x$

$14\left(\dfrac{4x-2}{7}-\dfrac{5}{14}\right)=(2x)14$

$2(4x-2)-5=28x$

$8x-4-5=28x$

$-9=20x$

$-\dfrac{9}{20}=x$

Check:

$\dfrac{4\left(\frac{-9}{20}\right)-2}{7}-\dfrac{5}{14}\stackrel{?}{=}2\left(\dfrac{-9}{20}\right)$

$\dfrac{\frac{-9}{5}-\frac{10}{5}}{7}-\dfrac{5}{14}\stackrel{?}{=}\dfrac{-9}{10}$

$\dfrac{-19}{5}\cdot\dfrac{1}{7}-\dfrac{5}{14}\stackrel{?}{=}\dfrac{-9}{10}$

$\dfrac{-19}{35}-\dfrac{5}{14}\stackrel{?}{=}\dfrac{-9}{10}$

$\dfrac{-38}{70}-\dfrac{25}{70}\stackrel{?}{=}\dfrac{-9}{10}$

$\dfrac{-63}{70}\stackrel{?}{=}\dfrac{-9}{10}$

$\dfrac{-9}{10}=\dfrac{-9}{10}$

16. $\dfrac{x^2}{2}-\dfrac{3x}{5}=\dfrac{-1}{10}$

$10\left(\dfrac{x^2}{2}-\dfrac{3x}{5}\right)=\left(\dfrac{-1}{10}\right)10$

$5x^2-6x=-1$

$5x^2-6x+1=0$

$(5x-1)(x-1)=0$

$5x-1=0 \qquad x-1=0$

$x=\dfrac{1}{5} \qquad x=1$

Check: $\dfrac{\left(\frac{1}{5}\right)^2-3\left(\frac{1}{5}\right)}{2}\stackrel{?}{=}\dfrac{-1}{10}$

$\dfrac{\frac{1}{25}}{2}-\dfrac{\frac{3}{5}}{5}\stackrel{?}{=}\dfrac{-1}{10}$

$\dfrac{1}{50}-\dfrac{3}{25}\stackrel{?}{=}\dfrac{-1}{10}$

$\dfrac{1}{50}-\dfrac{6}{50}\stackrel{?}{=}\dfrac{-1}{10}$

$-\dfrac{5}{50}\stackrel{?}{=}\dfrac{-1}{10}$

$\dfrac{-1}{10}=\dfrac{-1}{10}$

Check: $\dfrac{1^2}{2}-\dfrac{3(1)}{5}\stackrel{?}{=}\dfrac{-1}{10}$

$\dfrac{1}{2}-\dfrac{3}{5}\stackrel{?}{=}\dfrac{-1}{10}$

$\dfrac{5}{10}-\dfrac{6}{10}\stackrel{?}{=}\dfrac{-1}{10}$

$-\dfrac{1}{10}=\dfrac{-1}{10}$

18. $\dfrac{z-4}{9}-\dfrac{3z+1}{18}=\dfrac{3}{2}$

$18\left(\dfrac{z-4}{9}-\dfrac{3z+1}{18}\right)=\left(\dfrac{3}{2}\right)18$

$2(z-4)-(3z+1)=27$

$2z-8-3z-1=27$

$-z-9=27$

$-z=36$

$z=-36$

Check: $\dfrac{-36-4}{9}-\dfrac{3(-36)+1}{18}\stackrel{?}{=}\dfrac{3}{2}$

$\dfrac{-40}{9}-\dfrac{-107}{18}\stackrel{?}{=}\dfrac{3}{2}$

$\dfrac{-80}{18}+\dfrac{107}{18}\stackrel{?}{=}\dfrac{3}{2}$

$\dfrac{27}{18}\stackrel{?}{=}\dfrac{3}{2}$

$\dfrac{3}{2}=\dfrac{3}{2}$

20. $\dfrac{u-2}{6}+\dfrac{2u+5}{15}=3$

$30\left(\dfrac{u-2}{6}+\dfrac{2u+5}{15}\right)=(3)30$

$5(u-2)+2(2u+5)=90$

$5u-10+4u+10=90$

$9u=90$

$u=10$

Check: $\dfrac{10-2}{6}+\dfrac{2(10)+5}{15}\stackrel{?}{=}3$

$\dfrac{8}{6}+\dfrac{25}{15}\stackrel{?}{=}3$

$\dfrac{4}{3}+\dfrac{5}{3}\stackrel{?}{=}3$

$\dfrac{9}{3}\stackrel{?}{=}3$

$3=3$

22. $\dfrac{2x-7}{10}-\dfrac{3x+1}{5}=\dfrac{6-x}{5}$

$10\left(\dfrac{2x-7}{10}-\dfrac{3x+1}{5}\right)=\left(\dfrac{6-x}{5}\right)10$

$2x-7-2(3x+1)=2(6-x)$

$2x-7-6x-2=12-2x$

$-4x-9=12-2x$

$-2x=21$

$x=\dfrac{-21}{2}$

Check: $\dfrac{2\left(\frac{-21}{2}\right)-7}{10}-\dfrac{3\left(\frac{-21}{2}\right)+1}{5}\stackrel{?}{=}\dfrac{6-\left(\frac{-21}{2}\right)}{5}$

$\dfrac{-21-7}{10}-\dfrac{\frac{-63}{2}+\frac{2}{2}}{5}\stackrel{?}{=}\dfrac{\frac{12}{2}+\frac{21}{2}}{5}$

$\dfrac{-28}{10}-\dfrac{\frac{-61}{2}}{5}\stackrel{?}{=}\dfrac{\frac{33}{2}}{5}$

$\dfrac{-28}{10}+\dfrac{61}{10}\stackrel{?}{=}\dfrac{33}{10}$

$\dfrac{33}{10}=\dfrac{33}{10}$

24. $\dfrac{2}{u+4}=\dfrac{5}{8}$

$8(u+4)\left(\dfrac{2}{u+4}\right)=\left(\dfrac{5}{8}\right)8(u+4)$

$8(2)=5(u+4)$

$16=5u+20$

$-4=5u$

$-\dfrac{4}{5}=u$

Check: $\dfrac{2}{-\frac{4}{5}+4}\stackrel{?}{=}\dfrac{5}{8}$

$\dfrac{2}{-\frac{4}{5}+\frac{20}{5}}\stackrel{?}{=}\dfrac{5}{8}$

$\dfrac{2}{\frac{16}{5}}\stackrel{?}{=}\dfrac{5}{8}$

$\dfrac{10}{16}\stackrel{?}{=}\dfrac{5}{8}$

$\dfrac{5}{8}=\dfrac{5}{8}$

26. $\dfrac{6}{b}+22=24$

$\dfrac{6}{b}=2$

$b\left(\dfrac{6}{b}\right)=(2)b$

$6=2b$

$3=b$

Check: $\dfrac{6}{3}+22\stackrel{?}{=}24$

$2+22\stackrel{?}{=}24$

$24=24$

28. $\dfrac{5}{3}=\dfrac{6}{7x}+\dfrac{2}{x}$

$21x\left(\dfrac{5}{3}\right)=\left(\dfrac{6}{7x}+\dfrac{2}{x}\right)21x$

$35x=18+42$

$35x=60$

$x=\dfrac{60}{35}$

$x=\dfrac{12}{7}$

Check: $\dfrac{5}{3}\stackrel{?}{=}\dfrac{6}{7\left(\frac{12}{7}\right)}+\dfrac{2}{\frac{12}{7}}$

$\dfrac{5}{3}\stackrel{?}{=}\dfrac{6}{12}+\dfrac{14}{12}$

$\dfrac{5}{3}\stackrel{?}{=}\dfrac{20}{12}$

$\dfrac{5}{3}=\dfrac{5}{3}$

30. $\dfrac{7}{8}-\dfrac{16}{t-2}=\dfrac{3}{4}$

$8(t-2)\left(\dfrac{7}{8}-\dfrac{16}{t-2}\right)=\left(\dfrac{3}{4}\right)8(t-2)$

$7(t-2)-16(8)=6(t-2)$

$7t-14-128=6t-12$

$7t-142=6t-12$

$7t=6t+130$

$t=130$

Check: $\dfrac{7}{8}-\dfrac{16}{130-2}\stackrel{?}{=}\dfrac{3}{4}$

$\dfrac{7}{8}-\dfrac{16}{128}\stackrel{?}{=}\dfrac{3}{4}$

$\dfrac{7}{8}-\dfrac{1}{8}\stackrel{?}{=}\dfrac{3}{4}$

$\dfrac{6}{8}\stackrel{?}{=}\dfrac{3}{4}$

$\dfrac{3}{4}=\dfrac{3}{4}$

32.
$$\frac{10}{x+4}=\frac{15}{4(x+1)}$$
$$4(x+1)(x+4)\left(\frac{10}{x+4}\right)=\left(\frac{15}{4(x+1)}\right)4(x+1)(x+4)$$
$$4\cdot 10(x+1)=15(x+4)$$
$$40(x+1)=15(x+4)$$
$$40x+40=15x+60$$
$$25x+40=60$$
$$25x=20$$
$$x=\frac{20}{25}$$
$$x=\frac{4}{5}$$

Check:
$$\frac{10}{\frac{4}{5}+4}\stackrel{?}{=}\frac{15}{4\left(\frac{4}{5}+1\right)}$$
$$\frac{10}{\frac{4}{5}+\frac{20}{5}}\stackrel{?}{=}\frac{15}{4\left(\frac{4}{5}+\frac{5}{5}\right)}$$
$$\frac{10}{\frac{24}{5}}\stackrel{?}{=}\frac{15}{4\left(\frac{9}{5}\right)}$$
$$\frac{50}{24}\stackrel{?}{=}\frac{15}{\frac{36}{5}}$$
$$\frac{50}{24}\stackrel{?}{=}\frac{75}{36}$$
$$\frac{25}{12}=\frac{25}{12}$$

34.
$$\frac{500}{3x+5}=\frac{50}{x-3}$$
$$(x-3)(3x+5)\left(\frac{500}{3x+5}\right)=\left(\frac{50}{x-3}\right)(x-3)(3x+5)$$
$$500(x-3)=50(3x+5)$$
$$500x-1500=150x+250$$
$$350x-1500=250$$
$$350x=1750$$
$$x=\frac{1750}{350}$$
$$x=5$$

Check:
$$\frac{500}{3(5)+5}\stackrel{?}{=}\frac{50}{5-3}$$
$$\frac{500}{15+5}\stackrel{?}{=}\frac{50}{2}$$
$$\frac{500}{20}\stackrel{?}{=}25$$
$$25=25$$

36.
$$\frac{12}{x+5}+\frac{5}{x}=\frac{20}{x}$$
$$x(x+5)\left(\frac{12}{x+5}+\frac{5}{x}\right)=\left(\frac{20}{x}\right)x(x+5)$$
$$12x+5(x+5)=20(x+5)$$
$$12x+5x+25=20x+100$$
$$17x+25=20x+100$$
$$25=3x+100$$
$$-75=3x$$
$$-25=x$$

Check:
$$\frac{12}{-25+5}+\frac{5}{-25}\stackrel{?}{=}\frac{20}{-25}$$
$$\frac{12}{-20}+\frac{5}{-25}\stackrel{?}{=}\frac{20}{-25}$$
$$\frac{3}{-5}+\frac{1}{-5}\stackrel{?}{=}-\frac{4}{5}$$
$$-\frac{4}{5}=-\frac{4}{5}$$

38.
$$\frac{1}{4} = \frac{16}{z^2}$$
$$4\left(\frac{1}{4}\right) = \left(\frac{16}{z^2}\right)4$$
$$z^2 = 64$$
$$z^2 - 64 = 0$$
$$(z-8)(z+8) = 0$$
$$z - 8 = 0 \qquad z + 8 = 0$$
$$z = 8 \qquad z = -8$$

Check: $\frac{1}{4} \stackrel{?}{=} \frac{16}{(8)^2}$

$\frac{1}{4} \stackrel{?}{=} \frac{16}{64}$

$\frac{1}{4} = \frac{1}{4}$

Check: $\frac{1}{4} \stackrel{?}{=} \frac{16}{(-8)^2}$

$\frac{1}{4} \stackrel{?}{=} \frac{16}{64}$

$\frac{1}{4} = \frac{1}{4}$

40.
$$\frac{20}{u} = \frac{u}{5}$$
$$5u\left(\frac{20}{u}\right) = \left(\frac{u}{5}\right)5u$$
$$100 = u^2$$
$$0 = u^2 - 100$$
$$0 = (u-10)(u+10)$$
$$u - 10 = 0 \qquad u + 10 = 0$$
$$u = 10 \qquad u = -10$$

Check: $\frac{20}{10} \stackrel{?}{=} \frac{10}{5}$

$2 = 2$

Check: $\frac{20}{-10} \stackrel{?}{=} \frac{-10}{5}$

$-2 = -2$

42.
$$\frac{48}{x} = x - 2$$
$$x\left(\frac{48}{x}\right) = (x-2)x$$
$$48 = x^2 - 2x$$
$$0 = x^2 - 2x - 48$$
$$0 = (x-8)(x+6)$$
$$x = 8 \qquad x = -6$$

Check: $\frac{48}{8} \stackrel{?}{=} 8 - 2$

$6 = 6$

Check: $\frac{48}{-6} \stackrel{?}{=} -6 - 2$

$-8 = -8$

44.
$$x - \frac{24}{x} = 5$$
$$x\left(x - \frac{24}{x}\right) = (5)x$$
$$x^2 - 24 = 5x$$
$$x^2 - 5x - 24 = 0$$
$$(x-8)(x+3) = 0$$
$$x = 8$$
$$x = -3$$

Check: $8 - \frac{24}{8} \stackrel{?}{=} 5$

$8 - 3 = 5$

$5 = 5$

Check: $-3 - \frac{24}{-3} \stackrel{?}{=} 5$

$-3 + 8 = 5$

$5 = 5$

46.
$$\frac{x+42}{x} = x$$
$$x\left(\frac{x+42}{x}\right) = (x)x$$
$$x + 42 = x^2$$
$$0 = x^2 - x - 42$$
$$0 = (x-7)(x+6)$$
$$x - 7 = 0 \qquad x + 6 = 0$$
$$x = 7 \qquad x = -6$$

Check: $\frac{7+42}{7} \stackrel{?}{=} 7$

$\frac{49}{7} \stackrel{?}{=} 7$

$7 = 7$

Check: $\frac{-6+42}{-6} \stackrel{?}{=} -6$

$\frac{36}{-6} \stackrel{?}{=} -6$

$-6 = -6$

48. $\dfrac{3x}{4} = \dfrac{x^2 + 3x}{8x}$

$8x\left(\dfrac{3x}{4}\right) = \left(\dfrac{x^2 + 3x}{8x}\right)8x$

$6x^2 = x^2 + 3x$

$5x^2 - 3x = 0$

$x(5x - 3) = 0$

$x = 0 \qquad 5x - 3 = 0$

$x = \dfrac{3}{5}$

Check: $\dfrac{3(0)}{4} \stackrel{?}{=} \dfrac{0^2 + 3(0)}{8(0)}$

Division by zero is undefined.

Solution $x = 0$ is extraneous.

Check: $\dfrac{3\left(\frac{3}{5}\right)}{4} \stackrel{?}{=} \dfrac{\left(\frac{3}{5}\right)^2 + 3\left(\frac{3}{5}\right)}{8\left(\frac{3}{5}\right)}$

$\dfrac{9}{5} \cdot \dfrac{1}{4} \stackrel{?}{=} \dfrac{\frac{9}{25} + \frac{9}{5}}{\frac{24}{5}}$

$\dfrac{9}{20} \stackrel{?}{=} \dfrac{54}{25} \cdot \dfrac{5}{24}$

$\dfrac{9}{20} \stackrel{?}{=} \dfrac{18}{5} \cdot \dfrac{1}{8}$

$\dfrac{9}{20} = \dfrac{9}{20}$

50. $\dfrac{x - 3}{x + 1} = \dfrac{x - 6}{x + 5}$

$(x - 3)(x + 5) = (x + 1)(x - 6)$

$x^2 + 2x - 15 = x^2 - 5x - 6$

$2x - 15 = -5x - 6$

$7x - 15 = -6$

$7x = 9$

$x = \dfrac{9}{7}$

Check: $\dfrac{\frac{9}{7} - 3}{\frac{9}{7} + 1} \stackrel{?}{=} \dfrac{\frac{9}{7} - 6}{\frac{9}{7} + 5}$

$\dfrac{\frac{9}{7} - \frac{21}{7}}{\frac{9}{7} + \frac{7}{7}} \stackrel{?}{=} \dfrac{\frac{9}{7} - \frac{42}{7}}{\frac{9}{7} + \frac{35}{7}}$

$\dfrac{\frac{-12}{7}}{\frac{16}{7}} \stackrel{?}{=} \dfrac{\frac{-33}{7}}{\frac{44}{7}}$

$\dfrac{-12}{16} \stackrel{?}{=} \dfrac{-33}{44}$

$\dfrac{-3}{4} = \dfrac{-3}{4}$

52. $\dfrac{3z - 2}{z + 1} = 4 - \dfrac{z + 2}{z - 1}$

$(z + 1)(z - 1)\left(\dfrac{3z - 2}{z + 1}\right) = \left(4 - \dfrac{z + 2}{z - 1}\right)(z + 1)(z - 1)$

$(3z - 2)(z - 1) = 4(z + 1)(z - 1) - (z + 2)(z + 1)$

$3z^2 - 5z + 2 = 4(z^2 - 1) - (z^2 + 3z + 2)$

$3z^2 - 5z + 2 = 4z^2 - 4 - z^2 - 3z - 2$

$3z^2 - 5z + 2 = 3z^2 - 3z - 6$

$-5z + 2 = -3z - 6$

$-2z = -8$

$z = 4$

Check: $\dfrac{3(4) - 2}{4 + 1} \stackrel{?}{=} 4 - \dfrac{4 + 2}{4 - 1}$

$\dfrac{12 - 2}{5} \stackrel{?}{=} 4 - \dfrac{6}{3}$

$\dfrac{10}{5} \stackrel{?}{=} 4 - 2$

$2 = 2$

54. $\dfrac{10}{x(x-2)} + \dfrac{4}{x} = \dfrac{5}{x-2}$

$$x(x-2)\left(\frac{10}{x(x-2)} + \frac{4}{x}\right) = \left(\frac{5}{x-2}\right)x(x-2)$$
$$10 + 4(x-2) = 5x$$
$$10 + 4x - 8 = 5x$$
$$2 = x$$

Check: $\dfrac{10}{2(2-2)} + \dfrac{4}{2} \stackrel{?}{=} \dfrac{5}{2-2}$

$$\frac{10}{0} + \frac{4}{2} \neq \frac{5}{0}$$

Division by zero is undefined. Solution is extraneous, so equation has no solution.

56. $\dfrac{1}{x-1} + \dfrac{3}{x+1} = 2$

$$(x-1)(x+1)\left(\frac{1}{x-1} + \frac{3}{x+1}\right) = (2)(x-1)(x+1)$$
$$x + 1 + 3(x-1) = 2(x^2-1)$$
$$x + 1 + 3x - 3 = 2x^2 - 2$$
$$4x - 2 = 2x^2 - 2$$
$$0 = 2x^2 - 4x$$
$$0 = 2x(x-2)$$
$$x = 0 \qquad x = 2$$

Check: $\dfrac{1}{0-1} + \dfrac{3}{0+1} \stackrel{?}{=} 2$

$$-1 + 3 = 2$$
$$2 = 2$$

Check: $\dfrac{1}{2-1} + \dfrac{3}{2+1} \stackrel{?}{=} 2$

$$1 + 1 = 2$$
$$2 = 2$$

58. $\dfrac{5}{x+2} + \dfrac{2}{x^2-6x-16} = \dfrac{-4}{x-8}$

$$(x+2)(x-8)\left(\frac{5}{x+2} + \frac{2}{(x+2)(x-8)}\right) = \left(\frac{-4}{x-8}\right)(x+2)(x-8)$$
$$5(x-8) + 2 = -4(x+2)$$
$$5x - 40 + 2 = -4x - 8$$
$$5x - 38 = -4x - 8$$
$$9x = 30$$
$$x = \frac{30}{9}$$
$$x = \frac{10}{3}$$

Check: $\dfrac{5}{\left(\frac{10}{3}\right)+2} + \dfrac{2}{\left(\frac{10}{3}\right)^2 - 6\left(\frac{10}{3}\right) - 16} \stackrel{?}{=} \dfrac{-4}{\left(\frac{10}{3}\right)-8}$

$$\frac{5}{\frac{10}{3}+\frac{6}{3}} + \frac{2}{\frac{100}{9} - \frac{60}{3} - 16} \stackrel{?}{=} \frac{-4}{\frac{10}{3} - \frac{24}{3}}$$
$$\frac{5}{\frac{16}{3}} + \frac{2}{\frac{100}{9} - \frac{180}{9} - \frac{144}{9}} \stackrel{?}{=} \frac{-4}{-\frac{14}{3}}$$
$$\frac{15}{16} + \frac{2}{-\frac{224}{9}} \stackrel{?}{=} \frac{12}{14}$$
$$\frac{15}{16} - \frac{18}{224} \stackrel{?}{=} \frac{12}{14}$$
$$\frac{210}{224} - \frac{18}{224} \stackrel{?}{=} \frac{192}{224}$$
$$\frac{192}{224} = \frac{192}{224}$$

60.
$$1 - \frac{6}{4-x} = \frac{x+2}{x^2-16}$$
$$1 + \frac{6}{x-4} = \frac{x+2}{(x-4)(x+4)}$$
$$(x-4)(x+4)\left(1 + \frac{6}{x-4}\right) = \left(\frac{x+2}{(x-4)(x+4)}\right)(x-4)(x+4)$$
$$(x^2-16) + 6(x+4) = x+2$$
$$x^2 - 16 + 6x + 24 = x + 2$$
$$x^2 + 5x + 6 = 0$$
$$(x+3)(x+2) = 0$$
$$x+3 = 0 \qquad x+2 = 0$$
$$x = -3 \qquad x = -2$$

Check: $1 - \frac{6}{4-(-3)} \stackrel{?}{=} \frac{-3+2}{(-3)^2-16}$

$$1 - \frac{6}{7} \stackrel{?}{=} \frac{-1}{9-16}$$
$$\frac{1}{7} = \frac{1}{7}$$

Check: $1 - \frac{6}{4-(-2)} \stackrel{?}{=} \frac{-2+2}{(-2)^2-16}$

$$1 - \frac{6}{6} \stackrel{?}{=} \frac{0}{4-16}$$
$$0 = 0$$

62.
$$\frac{2(x+1)}{x^2-4x+3} + \frac{6x}{x-3} = \frac{3x}{x-1}$$
$$(x-3)(x-1)\left(\frac{2(x+1)}{(x-3)(x-1)}\right) + (x-3)(x-1)\left(\frac{6x}{x-3}\right) = \left(\frac{3x}{x-1}\right)(x-3)(x-1)$$
$$2(x+1) + 6x(x-1) = 3x(x-3)$$
$$2x + 2 + 6x^2 - 6x = 3x^2 - 9x$$
$$3x^2 + 5x + 2 = 0$$
$$(3x+2)(x+1) = 0$$
$$3x+2 = 0 \qquad x+1 = 0$$
$$x = -\frac{2}{3} \qquad x = -1$$

Check: $\frac{2\left(-\frac{2}{3}+1\right)}{\left(-\frac{2}{3}\right)^2 - 4\left(-\frac{2}{3}\right) + 3} + \frac{6\left(-\frac{2}{3}\right)}{-\frac{2}{3}-3} \stackrel{?}{=} \frac{3\left(-\frac{2}{3}\right)}{-\frac{2}{3}-1}$

$$\frac{\frac{2}{3}}{\frac{4}{9}+\frac{8}{3}+3} + \frac{-4}{-\frac{11}{3}} \stackrel{?}{=} \frac{-2}{-\frac{5}{3}}$$
$$\frac{2}{3}\cdot\frac{9}{55} + 4\cdot\frac{3}{11} \stackrel{?}{=} 2\cdot\frac{3}{5}$$
$$\frac{6}{55} + \frac{12}{11} \stackrel{?}{=} \frac{6}{5}$$
$$\frac{66}{55} \stackrel{?}{=} \frac{6}{5}$$
$$\frac{6}{5} = \frac{6}{5}$$

Check: $\frac{2(-1+1)}{(-1)^2 - 4(-1) + 3} + \frac{6(-1)}{-1-3} \stackrel{?}{=} \frac{3(-1)}{-1-1}$

$$0 + \frac{6}{4} \stackrel{?}{=} \frac{-3}{-2}$$
$$\frac{3}{2} = \frac{3}{2}$$

64. $$\frac{2}{x^2+2x-8}-\frac{1}{x^2+9x+20}=\frac{4}{x^2+3x-10}$$

$$(x+4)(x-2)(x+5)\left[\frac{2}{(x+4)(x-2)}-\frac{1}{(x+5)(x+4)}=\frac{4}{(x+5)(x-2)}\right](x+4)(x-2)(x+5)$$

$$2(x+5)-(x-2)=4(x+4)$$

$$2x+10-x+2=4x+16$$

$$x+12=4x+16$$

$$-3x+12=16$$

$$-3x=4$$

$$x=-\frac{4}{3}$$

Check: $$\frac{2}{\left(-\frac{4}{3}\right)^2+2\left(-\frac{4}{3}\right)-8}-\frac{1}{\left(-\frac{4}{3}\right)^2+9\left(-\frac{4}{3}\right)+20}\stackrel{?}{=}\frac{4}{\left(-\frac{4}{3}\right)^2+3\left(-\frac{4}{3}\right)-10}$$

$$\frac{2}{\frac{16}{9}-\frac{8}{3}-8}-\frac{1}{\frac{16}{9}-12+20}\stackrel{?}{=}\frac{4}{\frac{16}{9}-4-10}$$

$$\frac{2}{\frac{16}{9}-\frac{24}{9}-\frac{72}{9}}-\frac{1}{\frac{16}{9}+\frac{72}{9}}\stackrel{?}{=}\frac{4}{\frac{16}{9}-\frac{126}{9}}$$

$$\frac{2}{-\frac{80}{9}}-\frac{1}{\frac{88}{9}}\stackrel{?}{=}\frac{4}{-\frac{110}{9}}$$

$$-\frac{9}{40}-\frac{9}{88}\stackrel{?}{=}-\frac{18}{55}$$

$$-\frac{99}{440}-\frac{45}{440}\stackrel{?}{=}-\frac{18}{55}$$

$$-\frac{144}{440}\stackrel{?}{=}-\frac{18}{55}$$

$$-\frac{18}{55}=-\frac{18}{55}$$

66.
$$\frac{2x}{3} = \frac{1+\frac{2}{x}}{1+\frac{1}{x}}$$
$$\frac{2x}{3} = \frac{\left(1+\frac{2}{x}\right)x}{\left(1+\frac{1}{x}\right)x}$$
$$\frac{2x}{3} = \frac{(x+2)}{(x+1)}$$
$$3(x+1)\frac{2x}{3} = 3(x+1)\frac{(x+2)}{(x+1)}$$
$$2x^2 + 2x = 3x + 6$$
$$2x^2 - x - 6 = 0$$
$$(2x+3)(x-2) = 0$$
$$2x + 3 = 0 \qquad x - 2 = 0$$
$$x = -\frac{3}{2} \qquad x = 2$$

Check: $\dfrac{2\left(-\frac{3}{2}\right)}{3} \stackrel{?}{=} \dfrac{1+\frac{2}{(-3/2)}}{1+\frac{1}{(-3/2)}}$

$$\frac{-3}{3} \stackrel{?}{=} \frac{1-\frac{4}{3}}{1-\frac{2}{3}}$$
$$-1 \stackrel{?}{=} \frac{-\frac{1}{3}}{\frac{1}{3}}$$
$$-1 = -1$$

Check: $\dfrac{2(2)}{3} \stackrel{?}{=} \dfrac{1+\frac{2}{2}}{1+\frac{1}{2}}$

$$\frac{4}{3} \stackrel{?}{=} \frac{2}{\frac{3}{2}}$$
$$\frac{4}{3} = \frac{4}{3}$$

68. (a) x-intercept: $(0, 0)$

(b)
$$0 = \frac{2x}{x+4}$$
$$(x+4)(0) = \left(\frac{2x}{x+4}\right)(x+4)$$
$$0 = 2x$$
$$0 = x$$

(a) and (b) $(0, 0)$

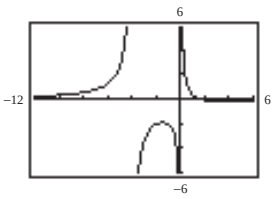
70. (a) x-intercepts: $(-1, 0)$ and $(2, 0)$

(b)
$$0 = x - \frac{2}{x} - 1$$
$$x(0) = \left(x - \frac{2}{x} - 1\right)x$$
$$0 = x^2 - 2 - x$$
$$0 = x^2 - x - 2$$
$$0 = (x-2)(x+1)$$
$$x - 2 = 0 \qquad x + 1 = 0$$
$$x = 2, \qquad x = -1$$

(a) and (b) $(-1, 0), (2, 0)$

72. (a) *Keystrokes:*

Y= 1 ÷ X,T,θ − 3 ÷ (X,T,θ + 4) GRAPH



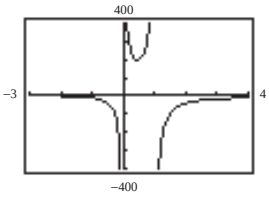
x-intercept: $(2, 0)$

(b)
$$0 = \frac{1}{x} - \frac{3}{x+4}$$
$$x(x+4)(0) = \left(\frac{1}{x} - \frac{3}{x+4}\right)x(x+4)$$
$$0 = x + 4 - 3x$$
$$0 = -2x + 4$$
$$2x = 4$$
$$x = 2$$

$(2, 0)$

74. (a) *Keystrokes:*

[Y=] 20 [(] 2 [÷] [X,T,θ] [−] 3 [÷] [(] [X,T,θ] [−] 1 [)] [)] [GRAPH]



x-intercept: $(-2, 0)$

(b) $$0 = 20\left(\frac{2}{x} - \frac{3}{x-1}\right)$$

$$x(x-1)(0) = \left[20\left(\frac{2}{x} - \frac{3}{x-1}\right)\right]x(x-1)$$

$$0 = 40(x-1) - 60x$$

$$0 = 40x - 40 - 60x$$

$$0 = -20x - 40$$

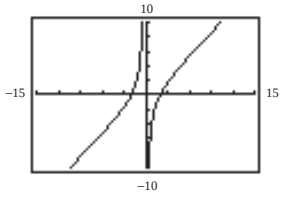
$$20x = -40$$

$$x = -2$$

$(-2, 0)$

76. (a) *Keystrokes:*

[=] [(] [X,T,θ] [x²] [−] 4 [)] [÷] [X,T,θ] [GRAPH]



x-intercepts: $(-2, 0)$ and $(2, 0)$

(b) $$0 = \frac{x^2 - 4}{x}$$

$$x(0) = \left(\frac{x^2 - 4}{x}\right)x$$

$$0 = x^2 - 4$$

$$0 = (x-2)(x+2)$$

$x - 2 = 0$ $\qquad$ $x + 2 = 0$

$x = 2$ $\qquad$ $x = -2$

$(2, 0)$ $\qquad$ $(-2, 0)$

78. $\dfrac{5}{x+3} + \dfrac{5}{3} + 3 \rightarrow$ expression

$$\frac{3(5)}{3(x+3)} + \frac{5(x+3)}{3(x+3)} + \frac{3(3)(x+3)}{3(x+3)} = \frac{15 + 5x + 15 + 9x + 27}{3(x+3)} = \frac{14x + 57}{3(x+3)}$$

80. $$\frac{5}{x+3} + \frac{5}{3} = 3 \rightarrow \text{ equation}$$

$$3(x+3)\left(\frac{5}{x+3} + \frac{5}{3}\right) = (3)3(x+3)$$

$$15 + 5(x+3) = 9(x+3)$$

$$15 + 5x + 15 = 9x + 27$$

$$3 = 4x$$

$$\frac{3}{4} = x$$

82. *Verbal Model:* $\boxed{\text{Twice a number}} + \boxed{\text{3 times the reciprocal}} = \frac{97}{4}$

Labels: $x =$ a number

$\frac{1}{x} =$ reciprocal of number

Equation:

$$2x + \frac{3}{x} = \frac{97}{4}$$

$$4x\left(2x + \frac{3}{x}\right) = \left(\frac{97}{4}\right)4x$$

$$8x^2 + 12 = 97x$$

$$8x^2 - 97x + 12 = 0$$

$$(8x - 1)(x - 12) = 0$$

$$8x - 1 = 0 \qquad x - 12 = 0$$

$$8x = 1 \qquad x = 12$$

$$x = \frac{1}{8}$$

84. *Verbal Model:* $\boxed{\frac{\text{Distance first part}}{\text{Speed first part}}} + \boxed{\frac{\text{Distance second part}}{\text{Speed second part}}} = \boxed{\text{Total time}}$

Labels: $r =$ speed first part

$r + 10 =$ speed second part

Equation:

$$\frac{240}{r} + \frac{72}{r + 10} = 6$$

$$r(r + 10)\left(\frac{240}{r}\right) + r(r + 10)\left(\frac{72}{r + 10}\right) = r(r + 10)6$$

$$(r + 10)240 + r(72) = 6r^2 + 60r$$

$$312r + 2400 = 6r^2 + 60r$$

$$6r^2 - 252r - 2400 = 0$$

$$3r^2 - 126r - 1200 = 0$$

$$(r - 50)(3r + 24) = 0$$

$$r - 50 = 0 \qquad 3r + 24 = 0$$

$$r = 50 \qquad r = -8$$

Choose the positive value of r. The two average speeds are 50 miles per hour and 60 miles per hour.

86. *Verbal Model:* $\boxed{\frac{\text{Distance traveled by commuter plane}}{\text{Rate of commuter plane}}} = \boxed{\frac{\text{Distance traveled by jet}}{\text{Rate of jet}}}$

Labels: $x =$ rate of jet

$x - 150 =$ rate of commuter plane

Equation:

$$\frac{450}{x - 150} = \frac{1150}{x}$$

$$x(x - 150)\left(\frac{450}{x - 150}\right) = \left(\frac{1150}{x}\right)x(x - 150)$$

$$450x = 1150(x - 150)$$

$$450x = 1150x - 172{,}500$$

$$-700x = -172{,}500$$

$$x \approx 246.43$$

Thus, the speed of the jet is approximately 246 miles per hour and the speed of the commuter plane is approximately $246 - 150 \approx 96$ miles per hour.

88. Solve a rational equation by multiplying both sides of the equation by the lowest common denominator. Then solve the resulting equation, checking for any extraneous solutions.

90. Graph the rational equation and approximate any x-intercepts of the graph.

Section 6.7 Applications and Variation

2. $C = kr$

4. $s = kt^3$

6. $V = k\sqrt[3]{x}$

8. $S = \dfrac{k}{v^2}$

10. $P = \dfrac{k}{\sqrt{1+r}}$

12. $V = khr^2$

14. $F = \dfrac{km_1m_2}{r^2}$

16. The area of a rectangle varies jointly as the length and the width.

18. The volume of a sphere varies directly as the cube of the radius.

20. The height of a cylinder varies directly as the volume and inversely as the square of the radius.

22.
$$\begin{aligned} h &= kr \\ 28 &= k(12) \\ \tfrac{28}{12} &= k \\ \tfrac{7}{3} &= k \\ h &= \tfrac{7}{3}r \end{aligned}$$

24.
$$\begin{aligned} M &= kn^3 \\ 0.012 &= k(0.2)^3 \\ 0.012 &= k(0.008) \\ \frac{0.012}{0.008} &= k \\ 1.5 &= k \\ M &= 1.5n^3 \end{aligned}$$

26.
$$\begin{aligned} q &= \frac{k}{P} \\ \frac{3}{2} &= \frac{k}{50} \\ 150 &= 2k \\ 75 &= k \\ q &= \frac{75}{P} \end{aligned}$$

28.
$$\begin{aligned} u &= \frac{k}{v^2} \\ 40 &= \frac{k}{\left(\frac{1}{2}\right)^2} \\ 40 &= \frac{k}{\frac{1}{4}} \\ 40 &= 4k \\ 10 &= k \\ u &= \frac{10}{v^2} \end{aligned}$$

30.
$$\begin{aligned} V &= khb^2 \\ 288 &= k(6)(12)^2 \\ 288 &= k(6)(144) \\ 288 &= 864k \\ \frac{288}{864} &= k \\ \frac{1}{3} &= k \\ V &= \frac{1}{3}hb^2 \end{aligned}$$

32.
$$\begin{aligned} z &= \frac{kx}{\sqrt{y}} \\ 720 &= \frac{k(48)}{\sqrt{81}} \\ 720 &= \frac{48k}{9} \\ 6480 &= 48k \\ 135 &= k \\ z &= \frac{135x}{\sqrt{y}} \end{aligned}$$

34. *Verbal Model:* $\boxed{\frac{\text{Distance}}{\text{Faster rate}}} = \boxed{\frac{\text{Distance}}{\text{Slower rate}}}$

Labels: Faster rate $= r$

Slower rate $= r - 6$

Equation: $$\frac{210}{r} = \frac{190}{r-6}$$

$$210(r-6) = 190r$$

$$210r - 1260 = 190r$$

$$20r = 1260$$

$$r = 63 \text{ mph}$$

$$r - 6 = 57 \text{ mph}$$

36. *Verbal Model:* $\boxed{\frac{\text{Total cost}}{\text{Current group}}} - \boxed{\frac{\text{Total cost}}{\text{New group}}} = 6250$

Labels: $x =$ number of persons in current group

$x + 4 =$ number of persons in new group

Equation: $$\frac{150{,}000}{x} - \frac{150{,}000}{x+4} = 6250$$

$$x(x+4)\left(\frac{150{,}000}{x} - \frac{150{,}000}{x+4}\right) = (6250)x(x+4)$$

$$150{,}000(x+4) - 150{,}000x = 6250x^2 + 25{,}000x$$

$$150{,}000x + 600{,}000 - 150{,}000x = 6250x^2 + 25{,}000x$$

$$0 = 6250x^2 + 25{,}000x - 600{,}000$$

$$0 = 6250(x^2 + 4x - 96)$$

$$0 = 6250(x+12)(x-8)$$

$x + 12 = 0$ $\qquad$ $x - 8 = 0$

$x = -12$ $\qquad$ $x = 8$

There are presently 8 people in the group.

38. *Verbal Model:* $\boxed{\text{Cost}} = \boxed{\frac{120{,}000p}{100 - p}}$

Equation: $$680{,}000 = \frac{120{,}000p}{100-p}$$

$$(100-p)(680{,}000) = \left(\frac{120{,}000p}{100-p}\right)(100-p)$$

$$68{,}000{,}000 - 680{,}000p = 120{,}000p$$

$$68{,}000{,}000 = 800{,}000p$$

$$85\% = p$$

40. *Verbal Model:* [Rate Pipe 1] + [Rate Pipe 2] = [Rate Together]

Labels: Second pipe's time $= x$

First pipe's time $= \frac{5}{4}x$

Equation: $\frac{1}{x} + \frac{1}{\frac{5}{4}x} = \frac{1}{5}$

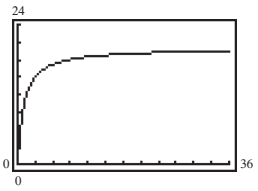
$5x\left(\frac{1}{x} + \frac{4}{5x}\right) = \left(\frac{1}{5}\right)5x$

$5 + 4 = x$

$9 \text{ hours} = x$

$11\frac{1}{4}\text{hours} = \frac{45}{4} = \frac{5}{4}x$

42. (a)



Keystrokes:

Y= 20 X,T,θ ÷ (X,T,θ + 1)

(b) 9 sessions

(c) $18 = \frac{20x}{x + 1}$

$18(x + 1) = 20x$

$18x + 18 = 20x$

$18 = 2x$

$9 = x$

44. $R = kx$

$300 = k(25)$

$12 = k$

$R = 12(42)$

$R = \$504$

Price per unit

46. (a) $d = kF$ $\qquad d = \frac{3}{50}F$

$3 = k(50)$ $\qquad d = \frac{3}{50}(20)$

$\frac{3}{50} = k$ $\qquad d = \frac{6}{5}$ inches or 1.2 inches

(b) $d = \frac{3}{50}F$

$1.5 = \frac{3}{50}F$

$1.5\left(\frac{50}{3}\right) = F$

$25 \text{ pounds} = F$

48. (a) $F = kx$

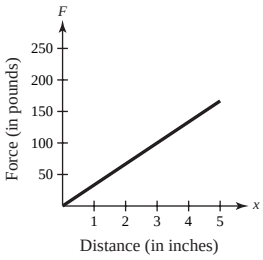
$50 = k(1.5)$

$\frac{100}{3} = k$

$F = \frac{100}{3}x$

The graph is a line with slope $\frac{100}{3}$ and a y-intercept at $(0, 0)$.

(b)



50. $d = kt^2$ $\qquad d = 16t^2$

$64 = k(2)^2$ $\qquad d = 16(6)^2$

$64 = 4k$ $\qquad d = 16(36)$

$16 = k$ $\qquad d = 576$ ft

52. $F = ks^2$

$F = k(2s)^2$

$F = 4ks^2$

$F = 4(ks^2)$

F will change by a factor of 4.

54. $x = \frac{k}{p}$

$800 = \frac{k}{5}$

$4000 = k$

$x = \frac{4000}{6}$

$x = 666.\overline{6} \approx 667$ boxes

56. $W_m = k \cdot W_e$ $\qquad$ $W_m = \frac{1}{6} \cdot W_e$

$60 = k \cdot 360$ $\qquad$ $54 = \frac{1}{6} \cdot x$

$k = \frac{1}{6}$ $\qquad$ $x = 324$ pounds

58. $p = \frac{k}{t}$

$38 = \frac{k}{3}$

$114 = k$

So, $p = \frac{114}{t}$.

$p = \frac{114}{6.5}$

$p = 17.5\%$

60. (a) $P = \frac{kWD^2}{L}$ (b) Unchanged

(c) Increases by a factor of 8.

(d) Increases by a factor of 4.

(e) Increases by a factor of $\frac{1}{4}$.

(f) $2000 = \frac{k(3)8^2}{120}$

$2000 = \frac{k(192)}{120}$

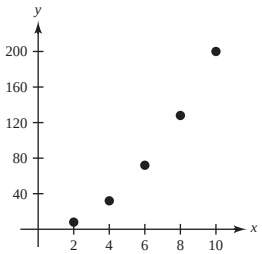
$1250 = k$

$L = \frac{1250(3)10^2}{120}$

$L = 3125$ pounds

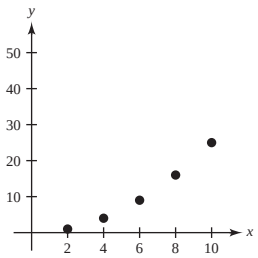
62.

x	2	4	6	8	10
$y = kx^2$	8	32	72	128	200



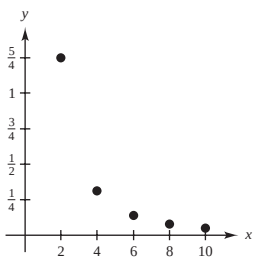
64.

x	2	4	6	8	10
$y = kx^2$	1	4	9	16	25



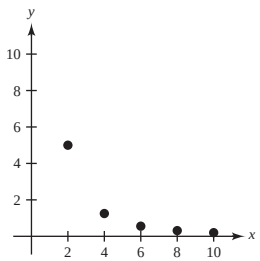
66.

x	2	4	6	8	10
$y = \frac{k}{x^2}$	$\frac{5}{4}$	$\frac{5}{16}$	$\frac{5}{36}$	$\frac{5}{64}$	$\frac{1}{20}$



68.

x	2	4	6	8	10
$y = \frac{k}{x^2}$	5	$\frac{5}{4}$	$\frac{5}{9}$	$\frac{5}{16}$	$\frac{1}{5}$



70.

x	10	20	30	40	50
y	-3	-6	-9	-12	-15

$-3 = k \cdot 10$ $\qquad$ $-6 = k \cdot 20$

$-\frac{3}{10} = k$ $\qquad$ $-\frac{6}{20} = k$

$-\frac{3}{10} = k$

Using any two pairs of numbers, k is $-\frac{3}{10}$.

72. Decrease. $y = k/x$ and $k > 0$ so if one variable increases the other decreases.

74. $y = \dfrac{k}{x^2}$

$y = \dfrac{k}{(2x)^2}$

$y = \dfrac{k}{4x^2}$

y will be $\frac{1}{4}$ as great.

Review Exercises for Chapter 6

2. $t + 12 \neq 0$

$t \neq -12$

$D = (-\infty, -12) \cup (-12, \infty)$

4. $x(x^2 - 16) \neq 0$

$x(x - 4)(x + 4) \neq 0$

$x \neq 0 \quad x - 4 \neq 0 \quad x + 4 \neq 0$

$x \neq 4 \qquad x \neq -4$

$D = (-\infty, -4) \cup (-4, 0) \cup (0, 4) \cup (4, \infty)$

6. Domain of $\overline{C} = \dfrac{15{,}000 + 0.75x}{x}$ is $\{1, 2, 3, 4, \ldots\}$.

8. $\dfrac{2(y^3z)^2}{28(yz^2)^2} = \dfrac{2y^6z^2}{28y^2z^4}$

$= \dfrac{2 \cdot y^2 \cdot y^4 \cdot z^2}{2 \cdot 14 \cdot y^2 \cdot z^2 \cdot z^2}$

$= \dfrac{y^4}{14z^2}, \quad y \neq 0$

10. $\dfrac{4a}{10a^2 + 26a} = \dfrac{4a}{2a(5a + 13)}$

$= \dfrac{2 \cdot 2 \cdot a}{2 \cdot a(5a + 13)}$

$= \dfrac{2}{5a + 13}, \quad a \neq 0$

12. $\dfrac{x + 3}{x^2 - x - 12} = \dfrac{x + 3}{(x - 4)(x + 3)}$

$= \dfrac{1}{x - 4}, \quad x \neq -3$

14. $\dfrac{x^2 + 3x + 9}{x^3 - 27} = \dfrac{x^2 + 3x + 9}{(x - 3)(x^2 + 3x + 9)}$

$= \dfrac{1}{x - 3}$

16. $2b(-3b)^3 = (2b)(-27b^3) = -54b^4$

18. $\dfrac{15(x^2y)^3}{3y^3} \cdot \dfrac{12y}{x} = \dfrac{3 \cdot 5 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot 2 \cdot 2 \cdot 3 \cdot y}{3 \cdot y \cdot y \cdot y \cdot x}$

$= 5x^5y \cdot 2 \cdot 2 \cdot 3$

$= 60x^5y, \quad x \neq 0, y \neq 0$

20. $\dfrac{x^2 - 16}{6} \cdot \dfrac{3}{x^2 - 8x + 16} = \dfrac{(x - 4)(x + 4) \cdot 3}{2 \cdot 3 \cdot (x - 4)(x - 4)}$

$= \dfrac{x + 4}{2(x - 4)}$

22. $x^2 \cdot \dfrac{x + 1}{x^2 - x} \cdot \dfrac{(5x - 5)^2}{x^2 + 6x + 5} = \dfrac{x^2 \cdot (x + 1)(5)^2(x - 1)^2}{1 \cdot x(x - 1)(x + 5)(x + 1)}$

$= \dfrac{25x(x - 1)}{x + 5}, \quad x \neq -1, 1, 0$

24. $\dfrac{8u^2v}{6v} = \dfrac{4u^2}{3}$

26. $\dfrac{6}{z^2} \div 4z^2 = \dfrac{6}{z^2} \cdot \dfrac{1}{4z^2}$

$= \dfrac{2 \cdot 3}{z^2 \cdot 2 \cdot 2 \cdot z^2}$

$= \dfrac{3}{2z^4}$

28. $\dfrac{x^2 - 14x + 48}{x^2 - 6x} \div (3x - 24) = \dfrac{(x - 8)(x - 6)}{x(x - 6)} \cdot \dfrac{1}{3(x - 8)}$

$= \dfrac{(x - 8)(x - 6)}{x(x - 6)3(x - 8)}$

$= \dfrac{1}{3x}, \quad x \neq 6, \quad x \neq 8$

30. $\dfrac{x^2 - x}{x + 1} \div \dfrac{5x - 5}{x^2 + 6x + 5} = \dfrac{x(x - 1)}{x + 1} \cdot \dfrac{(x + 1)(x + 5)}{5(x - 1)}$

$= \dfrac{x(x - 1)(x + 1)(x + 5)}{(x + 1)5(x - 1)}$

$= \dfrac{x(x + 5)}{5}, \quad x \neq -1, \quad x \neq 1, \quad x \neq -5$

32. $\dfrac{7y}{12} - \dfrac{4y}{12} = \dfrac{3y}{12} = \dfrac{y}{4}$

34. $\dfrac{4}{5x} + \dfrac{1}{5x} = \dfrac{5}{5x} = \dfrac{1}{x}$

36. $\dfrac{4x - 2}{3x + 1} - \dfrac{x + 1}{3x + 1} = \dfrac{4x - 2 - x - 1}{3x + 1}$

$= \dfrac{3x - 3}{3x + 1} = \dfrac{3(x - 1)}{3x + 1}$

38. $\dfrac{3}{2y - 3} - \dfrac{y - 10}{2y - 3} + \dfrac{5y}{2y - 3} = \dfrac{3 - y + 10 + 5y}{2y - 3}$

$= \dfrac{4y + 13}{2y - 3}$

40. $\dfrac{2}{x - 10} + \dfrac{3}{4 - x} = \dfrac{2(4 - x)}{(x - 10)(4 - x)} + \dfrac{3(x - 10)}{(x - 10)(4 - x)}$

$= \dfrac{2(4 - x) + 3(x - 10)}{(x - 10)(4 - x)}$

$= \dfrac{8 - 2x + 3x - 30}{(x - 10)(4 - x)}$

$= \dfrac{x - 22}{(x - 10)(4 - x)}$

42. $4 - \dfrac{4x}{x+6} + \dfrac{7}{x-5} = \dfrac{4(x+6)(x-5)}{(x+6)(x-5)} - \dfrac{4x(x-5)}{(x+6)(x-5)} + \dfrac{7(x+6)}{(x+6)(x-5)}$

$$= \frac{4(x+6)(x-5) - 4x(x-5) + 7(x+6)}{(x+6)(x-5)}$$

$$= \frac{4(x^2 + x - 30) - 4x^2 + 20x + 7x + 42}{(x+6)(x-5)}$$

$$= \frac{4x^2 + 4x - 120 - 4x^2 + 20x + 7x + 42}{(x+6)(x-5)}$$

$$= \frac{31x - 78}{(x+6)(x-5)}$$

44. $\dfrac{5}{x+2} + \dfrac{25-x}{x^2-3x-10} = \dfrac{5}{x+2} + \dfrac{25-x}{(x+2)(x-5)}$

$$= \frac{5(x-5)}{(x+2)(x-5)} + \frac{25-x}{(x+2)(x-5)}$$

$$= \frac{5(x-5) + 25 - x}{(x+2)(x-5)}$$

$$= \frac{5x - 25 + 25 - x}{(x+2)(x-5)}$$

$$= \frac{4x}{(x+2)(x-5)}$$

46. $\dfrac{8}{y} - \dfrac{3}{y+5} + \dfrac{4}{y-2} = \dfrac{8(y+5)(y-2)}{y(y+5)(y-2)} - \dfrac{3y(y-2)}{y(y+5)(y-2)} + \dfrac{4y(y+5)}{y(y+5)(y-2)}$

$$= \frac{8(y+5)(y-2) - 3y(y-2) + 4y(y+5)}{y(y+5)(y-2)}$$

$$= \frac{8(y^2 + 3y - 10) - 3y^2 + 6y + 4y^2 + 20y}{y(y+5)(y-2)}$$

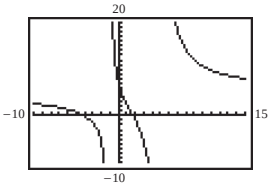
$$= \frac{8y^2 + 24y - 80 - 3y^2 + 6y + 4y^2 + 20y}{y(y+5)(y-2)}$$

$$= \frac{9y^2 + 50y - 80}{y(y+5)(y-2)}$$

48. *Keystrokes:*

y_1: [Y=] 5 [X,T,θ] [÷] [(] [X,T,θ] [−] 5 [)] [+] 7 [÷] [(] [X,T,θ] [+] 1 [ENTER]

y_2: [(] 5 [X,T,θ] [x^2] [+] 12 [X,T,θ] − 35 [)] [÷] [(] [X,T,θ] [x^2] [−] 4 [X,T,θ] [−] 5 [GRAPH]



$$\frac{5x}{x-5} + \frac{7}{x+1} = \frac{5x(x+1)}{(x-5)(x+1)} + \frac{7(x-5)}{(x-5)(x+1)}$$

$$= \frac{5x^2 + 5x + 7x - 35}{(x-5)(x+1)}$$

$$= \frac{5x^2 + 12x - 35}{x^2 - 4x - 5}$$

$$y_1 = y_2$$

50. $$\frac{xy}{\left(\frac{5x^2}{2y}\right)} = \frac{xy}{1} \div \frac{5x^2}{2y}$$
$$= \frac{xy}{1} \cdot \frac{2y}{5x^2}$$
$$= \frac{2xy^2}{5x^2}$$
$$= \frac{2y^2}{5x}, \quad y \neq 0$$

52. $$\frac{\left[\frac{24 - 18x}{(2-x)^2}\right]}{\left(\frac{60 - 45x}{x^2 - 4x - 4}\right)} = \frac{\left[\frac{6(4-3x)}{(2-x)^2}\right]}{\left[\frac{15(4-3x)}{(x-2)^2}\right]}$$
$$= \frac{6(4-3x)}{(2-x)^2} \cdot \frac{(x-2)^2}{15(4-3x)}$$
$$= \frac{2 \cdot 3(4-3x)(x-2)^2}{[-1(x-2)]^2 \cdot 3 \cdot 5 \cdot (4-3x)}$$
$$= \frac{2 \cdot 3(4-3x)(x-2)^2}{(-1)^2(x-2)^2 \cdot 3 \cdot 5(4-3x)}$$
$$= \frac{2 \cdot 3 \cdot (4-3x)(x-2)^2}{1 \cdot 3 \cdot 5 \cdot (x-2)^2(4-3x)}$$
$$= \frac{2}{5}, \quad x \neq 2, \frac{4}{3}$$

54. $$\frac{\left(\frac{1}{x} - \frac{1}{2}\right)}{2x} = \frac{\frac{1}{x} - \frac{1}{2}}{\frac{2x}{1}} \cdot \frac{2x}{2x} = \frac{2-x}{4x^2}$$

56. $$\frac{3x-1}{\left(\frac{2}{x^2} + \frac{5}{x}\right)} = \frac{\frac{3x-1}{1}}{\frac{2}{x^2} + \frac{5}{x}} \cdot \frac{x^2}{x^2} = \frac{x^2(3x-1)}{2+5x}, \; x \neq 0, \quad x \neq -\frac{2}{5}$$

58. $$\frac{\left(\frac{1}{x^2} - \frac{1}{y^2}\right)}{\left(\frac{1}{x} + \frac{1}{y}\right)} = \frac{\left(\frac{1}{x^2} - \frac{1}{y^2}\right)}{\left(\frac{1}{x} + \frac{1}{y}\right)} \cdot \frac{x^2y^2}{x^2y^2}$$
$$= \frac{y^2 - x^2}{xy^2 + x^2y}$$
$$= \frac{(y-x)(y+x)}{xy(y+x)}$$
$$= \frac{y-x}{xy}, \quad x \neq -y$$

60. $$\frac{10x + 15}{5x} = \frac{10x}{5x} + \frac{15}{5x}$$
$$= 2 + \frac{3}{x}$$

62. $$\frac{6a^3b^3 + 2a^2b - 4ab^2}{2ab} = \frac{6a^3b^3}{2ab} + \frac{2a^2b}{2ab} - \frac{4ab^2}{2ab}$$
$$= 3a^2b^2 + a - 2b, \quad a \neq 0, \quad b \neq 0$$

64.
$$\begin{array}{r} 4x^3 + 7x^2 + 7x + 32 + \dfrac{64}{x-2} \\ x-2\,\overline{)\,4x^4 - x^3 - 7x^2 + 18x + 0} \\ \underline{4x^4 - 8x^3} \\ 7x^3 - 7x^2 \\ \underline{7x^3 - 14x^2} \\ 7x^2 + 18x \\ \underline{7x^2 - 14x} \\ 32x + 0 \\ \underline{32x - 64} \\ 64 \end{array}$$

66.
$$\begin{array}{r} x^2 - 4x + 1 - \dfrac{1}{x+1}, \quad x \neq 1 \\ x^2-1\,\overline{)\,x^4 - 4x^3 + 0x^2 + 3x + 0} \\ \underline{x^4 \qquad - x^2} \\ -4x^3 + x^2 + 3x \\ \underline{-4x^3 \qquad + 4x} \\ x^2 - x + 0 \\ \underline{x^2 \qquad - 1} \\ -x + 1 \end{array}$$
$$\frac{-x+1}{x^2-1} = \frac{-1(\cancel{x-1})}{\cancel{(x-1)}(x+1)} = -\frac{1}{x+1}$$

68.
$$\begin{array}{r} x^3 + 3x^2 + x + 8 - \dfrac{16x^2 - 34x + 24}{x^3 + x^2 - 4x + 3} \\ x^3 + x^2 - 4x + 3 \overline{)\, x^6 + 4x^5 + 0x^4 + 0x^3 - 3x^2 + 5x + 0} \\ \underline{x^6 + x^5 - 4x^4 + 3x^3} \\ 3x^5 + 4x^4 - 3x^3 - 3x^2 \\ \underline{3x^5 + 3x^4 - 12x^3 + 9x^2} \\ x^4 + 9x^3 - 12x^2 + 5x \\ \underline{x^4 + x^3 - 4x^2 + 3x} \\ 8x^3 - 8x^2 + 2x + 0 \\ \underline{8x^3 + 8x^2 - 32x + 24} \\ -16x^2 + 34x - 24 \end{array}$$

70.
$$\begin{array}{r|rrrrr} 5 & 1 & -2 & -15 & -2 & 10 \\ & & 5 & 15 & 0 & -10 \\ \hline & 1 & 3 & 0 & -2 & 0 \end{array}$$

$$\frac{x^4 - 2x^3 - 15x^2 - 2x + 10}{x - 5} = x^3 + 3x^2 - 2, \; x \neq 5$$

72.
$$\begin{array}{r|rrrr} -\frac{1}{2} & 2 & 0 & 5 & -2 \\ & & -1 & \frac{1}{2} & -\frac{11}{4} \\ \hline & 2 & -1 & \frac{11}{2} & -\frac{19}{4} \end{array}$$

$$(2x^3 + 5x - 2) \div \left(x + \frac{1}{2}\right) = 2x^2 - x + \frac{11}{2} - \frac{\frac{19}{4}}{x + \frac{1}{2}}$$
$$= 2x^2 - x + \frac{11}{2} - \frac{19}{4x - 2}$$

74.
$$\begin{array}{r|rrrr} -1 & 2 & 1 & -2 & -1 \\ & & -2 & 1 & 1 \\ \hline & 2 & -1 & -1 & 0 \end{array}$$

$$2x^2 - x - 1 = (2x + 1)(x - 1)$$

$$2x^3 + x^2 - 2x - 1 = (x + 1)(2x + 1)(x - 1)$$

76.
$$\frac{t + 1}{6} = \frac{1}{2} - 2t$$
$$6\left(\frac{t + 1}{6}\right) = \left(\frac{1}{2} - 2t\right)6$$
$$t + 1 = 3 - 12t$$
$$13t = 2$$
$$t = \frac{2}{13}$$

78.
$$5 + \frac{2}{x} = \frac{1}{4}$$
$$4x\left(5 + \frac{2}{x}\right) = \left(\frac{1}{4}\right)4x$$
$$20x + 8 = x$$
$$19x = -8$$
$$x = -\frac{8}{19}$$

80.
$$\frac{7}{4x} - \frac{6}{8x} = 1$$
$$8x\left(\frac{7}{4x} - \frac{6}{8x}\right) = (1)8x$$
$$14 - 6 = 8x$$
$$8 = 8x$$
$$1 = x$$

82.
$$\frac{2}{x} - \frac{x}{6} = \frac{2}{3}$$
$$6x\left(\frac{2}{x} - \frac{x}{6}\right) = \left(\frac{2}{3}\right)6x$$
$$12 - x^2 = 4x$$
$$0 = x^2 + 4x - 12$$
$$0 = (x - 2)(x + 6)$$
$$x = 2, \quad x = -6$$

84.
$$\frac{3}{y+1}-\frac{8}{y}=1$$
$$y(y+1)\left(\frac{3}{y+1}-\frac{8}{y}\right)=(1)y(y+1)$$
$$3y-8(y+1)=y(y+1)$$
$$3y-8y-8=y^2+y$$
$$-5y-8=y^2+y$$
$$0=y^2+6y+8$$
$$0=(y+4)(y+2)$$
$$y+4=0 \qquad y+2=0$$
$$y=-4 \qquad y=-2$$

86.
$$\frac{2x}{x-3}-\frac{3}{x}=0$$
$$x(x-3)\left(\frac{2x}{x-3}-\frac{3}{x}\right)=(0)x(x-3)$$
$$2x(x)-3(x-3)=0$$
$$2x^2-3x+9=0$$
No real solution

88.
$$\frac{3}{x-1}+\frac{6}{x^2-3x+2}=2$$
$$\frac{3}{x-1}+\frac{6}{(x-1)(x-2)}=2$$
$$(x-1)(x-2)\left(\frac{3}{x-1}+\frac{6}{(x-1)(x-2)}\right)=2(x-1)(x-2)$$
$$3(x-2)+6=2(x-1)(x-2)$$
$$3x-6+6=2(x^2-3x+2)$$
$$3x=2x^2-6x+4$$
$$0=2x^2-9x+4$$
$$0=(2x-1)(x-4)$$
$$2x-1=0 \qquad x-4=0$$
$$2x=1 \qquad x=4$$
$$x=\frac{1}{2}$$

90.
$$\frac{3}{x^2-9}+\frac{4}{x+3}=1$$
$$\frac{3}{(x-3)(x+3)}+\frac{4}{x+3}=1$$
$$(x+3)(x-3)\left(\frac{3}{(x+3)(x-3)}+\frac{4}{x+3}\right)=(1)(x+3)(x-3)$$
$$3+4(x-3)=(x+3)(x-3)$$
$$3+4x-12=x^2-9$$
$$4x-9=x^2-9$$
$$0=x^2-4x$$
$$0=x(x-4)$$
$$x=0 \qquad x-4=0$$
$$x=4$$

92. *Verbal Model:* $\boxed{\text{Distance}} = \boxed{\text{Rate}} \cdot \boxed{\text{Time}}$

$$\boxed{\begin{matrix}\text{Original}\\ \text{trip time}\end{matrix}} = \boxed{\begin{matrix}\text{Return}\\ \text{trip time}\end{matrix}} + \frac{1}{3}$$

Labels: Speed of return trip $= x$

Speed of original trip $= x - 5$

Equation:
$$\frac{220}{x-5} = \frac{220}{x} + \frac{1}{3}$$
$$3x(x-5)\left(\frac{220}{x-5}\right) = \left(\frac{220}{x} + \frac{1}{3}\right)3x(x-5)$$
$$660x = 660(x-5) + x(x-5)$$
$$660x = 660x - 3300 + x^2 - 5x$$
$$0 = x^2 - 5x - 3300$$
$$0 = (x-60)(x+55)$$
$$x = 60, \quad x = -55$$
$$x = 60 \text{ miles per hour}$$

94. *Verbal Model:* $\boxed{\begin{matrix}\text{Rate of}\\ \text{Painter 1}\end{matrix}} = \boxed{\begin{matrix}\text{Rate of}\\ \text{Painter 2}\end{matrix}} = \boxed{\begin{matrix}\text{Rate}\\ \text{Together}\end{matrix}}$

Labels: Rate of Painter 1 $= \frac{1}{x}$

Rate of Painter 2 $= \frac{1}{\frac{3}{2}x}$

Rate Together $= \frac{1}{4}$

Equation:
$$\frac{1}{x} + \frac{1}{\frac{3}{2}x} = \frac{1}{4}$$
$$12x\left(\frac{1}{x} + \frac{2}{3x} = \frac{1}{4}\right)12x$$
$$12 + 8 = 3x$$
$$20 = 3x$$
$$\text{hours } \frac{20}{3} = x \qquad \text{Painter 1}$$
$$\text{hours } 10 = \frac{3}{2}\left(\frac{20}{3}\right) = \frac{3}{2}x \qquad \text{Painter 2}$$

96.
$$2.90 = 1.5 + \frac{4200}{x}$$
$$1.4 = \frac{4200}{x}$$
$$x = \frac{4200}{1.4}$$
$$x = 3000 \text{ units}$$

98.
$$d = ks^2$$
$$d = k(2s)^2$$
$$d = 4ks^2 = 4(ks)^2$$

The stopping distance, d, will increase by a factor of 4.

100.
$$x = \frac{k}{p}$$
$$1000 = \frac{k}{25}$$
$$25{,}000 = k$$
$$x = \frac{25{,}000}{28}$$
$$x \approx 893 \text{ telephones}$$

102.
$$C = khw^2$$
$$28.80 = k(16)(6)^2 \qquad C = 0.05(14)(8)^2$$
$$28.80 = k(576) \qquad C = \$44.80$$
$$\frac{28.80}{576} = k$$
$$0.05 = k$$