C H A P T E R 8
Quadratic Equations, Functions, and Inequalities

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Section 8.1 Solving Quadratic Equations: Factoring and Special Forms

Solutions to Even-Numbered Exercises

2. $x^2 + 15x + 44 = 0$

$(x + 11)(x + 4) = 0$

$x = -11 \quad x = -4$

4. $x^2 + 2x - 63 = 0$

$(x + 9)(x - 7) = 0$

$x = -9 \qquad x = 7$

6. $x^2 - 7x = 18$

$x^2 - 7x - 18 = 0$

$(x - 9)(x + 2) = 0$

$x = 9 \quad x = -2$

8. $x^2 + 60x + 900 = 0$

$(x + 30)(x + 30) = 0$

$x = -30 \quad x = -30$

10. $8x^2 - 10x + 3 = 0$

$(4x - 3)(2x - 1) = 0$

$4x - 3 = 0 \qquad 2x - 1 = 0$

$4x = 3 \qquad 2x = 1$

$x = \frac{3}{4} \qquad x = \frac{1}{2}$

12. $25y^2 - 75y = 0$

$25y(y - 3) = 0$

$25y = 0 \qquad y - 3 = 0$

$y = 0 \qquad y = 3$

14. $16x(x - 8) - 12(x - 8) = 0$

$(x - 8)(16x - 12) = 0$

$4(x - 8)(4x - 3) = 0$

$x - 8 = 0 \qquad 4x - 3 = 0$

$x = 8 \qquad 4x = 3$

$x = \frac{3}{4}$

16. $3(4 - x) - 2x(4 - x) = 0$

$(4 - x)(3 - 2x) = 0$

$4 - x = 0 \qquad 3 - 2x = 0$

$4 = x \qquad 3 = 2x$

$\frac{3}{2} = x$

18. $(6 + u)(1 - u) = 10$

$6 - 6u + u - u^2 = 10$

$6 - 5u - u^2 = 10$

$0 = u^2 + 5u + 4$

$0 = (u + 4)(u + 1)$

$u + 4 = 0 \qquad u + 1 = 0$

$u = -4 \qquad u = -1$

20. $(2z + 1)(2z - 1) = -4z^2 - 5z + 2$

$4z^2 - 1 = -4z^2 - 5z + 2$

$8z^2 + 5z - 3 = 0$

$(8z - 3)(z + 1) = 0$

$8z - 3 = 0 \qquad z + 1 = 0$

$8z = 3 \qquad z = -1$

$z = \frac{3}{8}$

22. $z^2 = 144$

$z = \pm\sqrt{144}$

$z = \pm 12$

24. $5t^2 = 5$

$t^2 = 1$

$t = \pm 1$

26. $9z^2 = 121$

$z^2 = \frac{121}{9}$

$z = \pm\sqrt{\frac{121}{9}}$

$z = \pm\frac{11}{3}$

28. $\frac{x^2}{6} = 24$

$x^2 = 24(6)$

$x^2 = 144$

$x = \pm\sqrt{144}$

$x = \pm 12$

30. $16y^2 - 121 = 0$

$16y^2 = 121$

$y^2 = \frac{121}{16}$

$y = \pm\sqrt{\frac{121}{16}}$

$y = \pm\frac{11}{4}$

32. $16x^2 - 1 = 0$

$16x^2 = 1$

$x^2 = \frac{1}{16}$

$x = \pm\sqrt{\frac{1}{16}}$

$x = \pm\frac{1}{4}$

34. $(y - 20)^2 = 25$

$y - 20 = \pm\sqrt{25}$

$y - 20 = \pm 5$

$y = 20 \pm 5$

$y = 25 \quad y = 15$

36. $(x + 2)^2 = 0.81$

$x + 2 = \pm\sqrt{0.81}$

$x + 2 = \pm 0.9$

$x = -2 \pm 0.9$

$x = -2.9 \quad x = -1.1$

38. $(x + 8)^2 = 28$

$x + 8 = \pm\sqrt{28}$

$x + 8 = \pm 2\sqrt{7}$

$x = -8 \pm 2\sqrt{7}$

40. $(3x - 5)^2 = 48$

$3x - 5 = \pm\sqrt{48}$

$3x - 5 = \pm 4\sqrt{3}$

$3x = 5 \pm 4\sqrt{3}$

$x = \dfrac{5 \pm 4\sqrt{3}}{3}$

42. $(5x + 11)^2 - 300 = 0$

$(5x + 11)^2 = 300$

$5x + 11 = \pm\sqrt{300}$

$5x + 11 = \pm 10\sqrt{3}$

$5x = -11 \pm 10\sqrt{3}$

$x = \dfrac{-11 \pm 10\sqrt{3}}{5}$

44. $x^2 = -16$

$x = \pm\sqrt{-16}$

$x = \pm 4i$

46. $y^2 + 16 = 0$

$y^2 = -16$

$y = \pm\sqrt{-16}$

$y = \pm 4i$

48. $4v^2 + 9 = 0$

$4v^2 = -9$

$v^2 = -\frac{9}{4}$

$v = \pm\sqrt{-\frac{9}{4}}$

$v = \pm\frac{3}{2}i$

50. $(x + 5)^2 = -81$

$(x + 5)^2 + 81 = 0$

$(x + 5)^2 = -81$

$x + 5 = \pm\sqrt{-81}$

$x + 5 = \pm 9i$

$x = -5 \pm 9i$

52. $(2y - 3)^2 + 25 = 0$

$(2y - 3)^2 = -25$

$2y - 3 = \pm\sqrt{-25}$

$2y = 3 \pm 5i$

$y = \dfrac{3}{2} \pm \dfrac{5}{2}i$

54. $(6y - 5)^2 = -8$

$6y - 5 = \pm\sqrt{-8}$

$6y = 5 \pm 2i\sqrt{2}$

$y = \dfrac{5 \pm 2i\sqrt{2}}{6}$

$y = \dfrac{5}{6} \pm \dfrac{\sqrt{2}}{3}i$

56. $4(x - 4)^2 = -169$

$(x - 4)^2 = \dfrac{-169}{4}$

$x - 4 = \pm\sqrt{\dfrac{-169}{4}}$

$x - 4 = \pm\dfrac{13}{2}i$

$x = 4 \pm \dfrac{13}{2}i$

58. $(2x + 3)^2 = -54$

$2x + 3 = \pm\sqrt{-54}$

$2x + 3 = \pm 3\sqrt{6}i$

$2x = -3 \pm 3\sqrt{6}i$

$x = \dfrac{-3 \pm 3\sqrt{6}i}{2}$

$= \dfrac{-3}{2} \pm \dfrac{3\sqrt{6}}{2}i$

60. $(x - 3)^2 + 2.25 = 0$

$(x - 3)^2 = -2.25$

$x - 3 = \pm\sqrt{-2.25}$

$x = 3 \pm 1.5i$

62. $\left(u+\frac{5}{8}\right)^2+\frac{49}{16}=0$

$\left(u+\frac{5}{8}\right)^2=-\frac{49}{16}$

$u+\frac{5}{8}=\pm\sqrt{-\frac{49}{16}}$

$u+\frac{5}{8}=\pm\frac{7}{4}i$

$u=-\frac{5}{8}\pm\frac{7}{4}i$

64. $\left(y-\frac{5}{6}\right)^2=-\frac{4}{5}$

$y-\frac{5}{6}=\pm\sqrt{-\frac{4}{5}}$

$y-\frac{5}{6}=\pm\frac{2}{\sqrt{5}}i$

$y=\frac{5}{6}\pm\frac{2}{\sqrt{5}}i$

$y=\frac{5}{6}\pm\frac{2}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}i$

$y=\frac{5}{6}\pm\frac{2\sqrt{5}}{5}i$

66. $3t^2+6t=0$

$3t(t+2)=0$

$3t=0 \qquad t+2=0$

$t=0 \qquad t=-2$

68. $3x^2+8x-16=0$

$(3x-4)(x+4)=0$

$3x-4=0 \qquad x+4=0$

$3x=4 \qquad x=-4$

$x=\frac{4}{3}$

70. $y^2-225=0$

$y^2=225$

$y=\pm\sqrt{225}$

$y=\pm15$

72. $y^2+225=0$

$y^2=-225$

$y=\pm\sqrt{-225}$

$y=\pm15i$

74. $\frac{1}{3}x^2=4$

$x^2=12$

$x=\pm\sqrt{12}$

$x=\pm2\sqrt{3}$

76. $(y+12)^2-400=0$

$(y+12)^2=400$

$y+12=\pm20$

$y=-12\pm20$

$y=-32 \qquad y=8$

78. $(y+12)^2+400=0$

$(y+12)^2=-400$

$y+12=\pm20i$

$y=-12\pm20i$

80. $(x+2)^2-18=0$

$(x+2)^2=18$

$x+2=\pm\sqrt{18}$

$x=-2\pm3\sqrt{2}$

82. $y=5x-x^2$

Keystrokes:

Y= 5 X,T,θ − X,T,θ x^2 GRAPH

$0=5x-x^2$

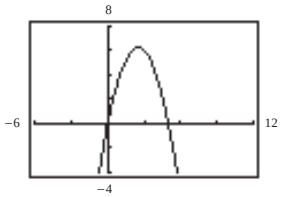
$0=x(5-x)$

$0=x \qquad 5-x=0$

$5=x$

x-intercepts are 0 and 5.

(0, 0), (5, 0)



84. $y=9-4(x-3)^2$

Keystrokes:

Y= 9 − 4 (X,T,θ − 3) x^2 GRAPH

x-intercepts are $\frac{3}{2}$ and $\frac{9}{2}$.

$0=9-4(x-3)^2$

$4(x-3)^2=9$

$(x-3)^2=\frac{9}{4}$

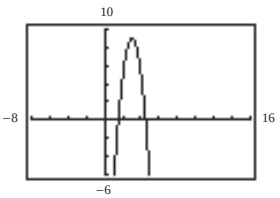
$x-3=\pm\sqrt{\frac{9}{4}}$

$x-3=\pm\frac{3}{2}$

$x=3\pm\frac{3}{2}$

$x=\frac{9}{2} \qquad x=\frac{3}{2}$

$\left(\frac{3}{2},0\right), \left(\frac{9}{2},0\right)$



86. $y = 4(x + 1)^2 - 9$

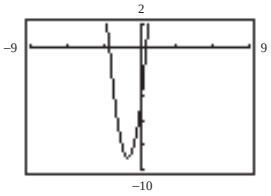
Keystrokes:

Y= 4 (X,T,θ) 1) x^2 − 9 GRAPH

x-intercepts are $-\frac{5}{2}$ and $\frac{1}{2}$.

$$0 = 4(x + 1)^2 - 9$$
$$9 = 4(x + 1)^2$$
$$\frac{9}{4} = (x + 1)^2$$
$$\pm\sqrt{\frac{9}{4}} = x + 1$$
$$\pm\frac{3}{2} = x + 1$$
$$-1 \pm \frac{3}{2} = x$$
$$x = -\frac{5}{2} \qquad x = \frac{1}{2}$$

$\left(-\frac{5}{2}, 0\right), \left(\frac{1}{2}, 0\right)$



88. $y = 4x^2 - x - 14$

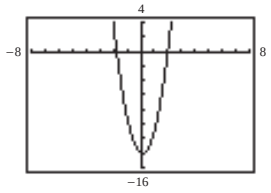
Keystrokes:

Y= 4 X,T,θ x^2 − X,T,θ − 14 GRAPH

x-intercepts are $-\frac{7}{4}$ and 2.

$$0 = 4x^2 - x - 14$$
$$0 = (4x + 7)(x - 2)$$
$$x = -\frac{7}{4} \qquad x = 2$$

$\left(-\frac{7}{4}, 0\right), (2, 0)$



90. $y = 5x^2 + 9x - 18$

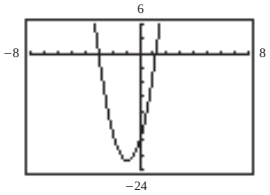
Keystrokes:

Y= 5 X,T,θ x^2 + 9 X,T,θ − 18 GRAPH

x-intercepts are $\frac{6}{5}$ and -3.

$$0 = 5x^2 + 9x - 18$$
$$0 = (5x - 6)(x + 3)$$
$$x = \frac{6}{5} \qquad x = -3$$

$\left(-3, 0\right), \left(\frac{6}{5}, 0\right)$



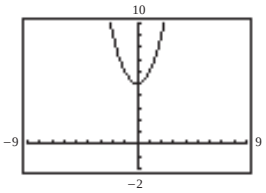
92. $y = x^2 + 5$

Keystrokes:

Y= X,T,θ x^2 + 5 GRAPH

$$0 = x^2 + 5$$
$$-5 = x^2$$
$$\pm\sqrt{-5} = x$$
$$\pm\sqrt{5}i = x$$

Not real, therefore, there are no x-intercepts.



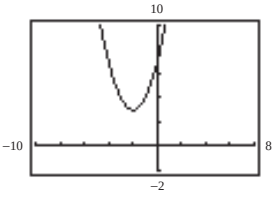
94. $y = (x + 2)^2 + 3$

Keystrokes:

Y= (X,T,θ + 2) x^2 + 3 GRAPH

$$0 = (x + 2)^2 + 3$$
$$-3 = (x + 2)^2$$
$$\pm\sqrt{-3} = x + 2$$
$$\pm\sqrt{3}i = x + 2$$
$$-2 \pm \sqrt{3}i = x$$

Not real, therefore, there are no x-intercepts.

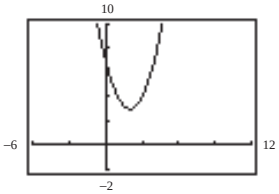


96. *Keystrokes:*

Y= (X,T,θ − 2) x^2 + 3 GRAPH

$$0 = (x - 2)^2 + 3$$
$$-3 = (x - 2)^2$$
$$\pm\sqrt{-3} = x - 2$$
$$2 \pm \sqrt{3}i = x$$

Not real, therefore, there are no x-intercepts.



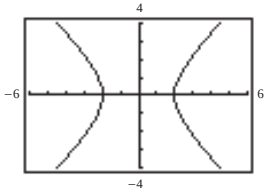
98. $x^2 - y^2 = 4$

$$-y^2 = 4 - x^2$$
$$y^2 = x^2 - 4$$
$$y = \pm\sqrt{x^2 - 4}$$

Keystrokes:

y_1: Y= √ (X,T,θ x^2 − 4) ENTER

y_2: (−) √ (X,T,θ x^2 − 4) GRAPH



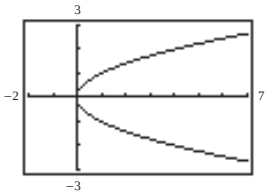
100. $x - y^2 = 0$

$$-y^2 = -x$$
$$y^2 = x$$
$$y = \pm\sqrt{x}$$

Keystrokes:

y_1: Y= √ X,T,θ ENTER

y_2: (−) √ X,T,θ GRAPH



102. $x^4 - 10x^2 + 25 = 0$

Let $u = x^2$.

$(x^2)^2 - 10x^2 + 25 = 0$

$u^2 - 10u + 25 = 0$

$(u - 5)(u - 5) = 0$

$u - 5 = 0$	$u - 5 = 0$
$u = 5$	$u = 5$
$x^2 = 5$	$x^2 = 5$
$x = \pm\sqrt{5}$	$x = \pm\sqrt{5}$

104. $x^4 - 11x^2 + 30 = 0$

Let $u = x^2$.

$(x^2)^2 - 11x^2 + 30 = 0$

$u^2 - 11u + 30 = 0$

$(u - 5)(u - 6) = 0$

$u - 5 = 0$	$u - 6 = 0$
$u = 5$	$u = 6$
$x^2 = 5$	$x^2 = 6$
$x = \pm\sqrt{5}$	$x = \pm\sqrt{6}$

106. $(x^2 - 1)^2 + (x^2 - 1) - 6 = 0$

Let $u = x^2 - 1$.

$(x^2 - 1)^2 + (x^2 - 1) - 6 = 0$

$u^2 + u - 6 = 0$

$(u + 3)(u - 2) = 0$

$u + 3 = 0$	$u - 2 = 0$
$u = -3$	$u = 2$
$x^2 - 1 = -3$	$x^2 - 1 = 2$
$x^2 = -2$	$x^2 = 3$
$x = \pm\sqrt{-2}$	$x = \pm\sqrt{3}$
$x = \pm\sqrt{2}i$	

108. $x - \sqrt{x} - 6 = 0$

Let $u = \sqrt{x}$.

$\left(\sqrt{x}\right)^2 - \sqrt{x} - 6 = 0$

$u^2 - u - 6 = 0$

$(u - 3)(u + 2) = 0$

$u = 3$	$u = -2$
$\sqrt{x} = 3$	$\sqrt{x} = -2$
$\left(\sqrt{x}\right)^2 = 3^2$	$\left(\sqrt{x}\right)^2 = (-2)^2$
$x = 9$	$x = 4$
Check:	**Check:**
$9 - \sqrt{9} - 6 \stackrel{?}{=} 0$	$4 - \sqrt{4} - 6 \stackrel{?}{=} 0$
$9 - 3 - 6 \stackrel{?}{=} 0$	$4 - 2 - 6 \stackrel{?}{=} 0$
$0 = 0$	$-4 \neq 0$

110. $x - 11\sqrt{x} + 24 = 0$

Let $u = \sqrt{x}$.

$\left(\sqrt{x}\right)^2 - 11\sqrt{x} + 24 = 0$

$u^2 - 11u + 24 = 0$

$(u - 8)(u - 3) = 0$

$u = 8$	$u = 3$
$\sqrt{x} = 8$	$\sqrt{x} = 3$
$\left(\sqrt{x}\right)^2 = 8^2$	$\left(\sqrt{x}\right)^2 = 3^2$
$x = 64$	$x = 9$
Check:	**Check:**
$64 - 11\sqrt{64} + 24 \stackrel{?}{=} 0$	$9 - 11\sqrt{9} + 24 \stackrel{?}{=} 0$
$64 - 88 + 24 \stackrel{?}{=} 0$	$9 - 33 + 24 \stackrel{?}{=} 0$
$0 = 0$	$0 = 0$

112. $x^{2/3} + 3x^{1/3} - 10 = 0$

$(x^{1/3} + 5)(x^{1/3} - 2) = 0$

$x^{1/3} = -5$	$x^{1/3} = 2$
$x = (-5)^3$	$x = 2^3$
$x = -125$	$x = 8$

114. $3x^{2/3} + 8x^{1/3} + 5 = 0$

$(3x^{1/3} + 5)(x^{1/3} + 1) = 0$

$3x^{1/3} + 5 = 0$	$x^{1/3} = -1$
$x^{1/3} = -\frac{5}{3}$	$x = (-1)^3$
$x = \left(-\frac{5}{3}\right)^3$	$x = -1$
$= -\frac{125}{27}$	

116. $x^{2/5} + 5x^{1/5} + 6 = 0$

$(x^{1/5} + 3)(x^{1/5} + 2) = 0$

$x^{1/5} = -3 \qquad x^{1/5} = -2$

$x = (-3)^5 \qquad x = (-2)^5$

$x = -243 \qquad x = -32$

118. $2x^{2/5} + 3x^{1/5} + 1 = 0$

$(2x^{1/5} + 1)(x^{1/5} + 1) = 0$

$x^{1/5} = -\frac{1}{2} \qquad x^{1/5} = -1$

$x = \left(-\frac{1}{2}\right)^5 \qquad x = (-1)^5$

$x = -\frac{1}{32} \qquad x = -1$

120. $x^{1/3} + 2x^{1/6} - 3 = 0$

$(x^{1/6} + 3)(x^{1/6} - 1) = 0$

$x^{1/6} = -3 \qquad x^{1/6} = 1$

$(x^{1/6})^6 = (-3)^6 \qquad (x^{1/6})^6 = 1^6$

$x = -729 \qquad x = 1$

Check: $(-729)^{1/3} + 2(-729)^{1/6} - 3 \stackrel{?}{=} 0$

$-9 + 2(3i) - 3 \neq 0$

Check: $(1)^{1/3} + 2(1)^{1/6} - 3 \stackrel{?}{=} 0$

$1 + 2 - 3 \stackrel{?}{=} 0$

$0 = 0$

122. $x^{1/2} - 5x^{1/4} + 6 = 0$

$(x^{1/4} - 3)(x^{1/4} - 2) = 0$

$x^{1/4} = 3 \qquad x^{1/4} = 2$

$(x^{1/4})^4 = 3^4 \qquad (x^{1/4})^4 = 2^4$

$x = 81 \qquad x = 16$

Check: $(81)^{1/2} - 5(81)^{1/4} + 6 \stackrel{?}{=} 0$

$9 - 5(3) + 6 \stackrel{?}{=} 0$

$0 = 0$

Check: $(16)^{1/2} - 5(16)^{1/4} + 6 \stackrel{?}{=} 0$

$4 - 5(2) + 6 \stackrel{?}{=} 0$

$0 = 0$

124. $\frac{1}{x^2} - \frac{1}{x} - 6 = 0$

$1 - x - 6x^2 = 0$

$6x^2 + x - 1 = 0$

$(3x - 1)(2x + 1) = 0$

$x = \frac{1}{3} \qquad x = -\frac{1}{2}$

126. $2x^{-2} - x^{-1} - 1 = 0$

Let $u = x^{-1}$.

$2u^2 - u - 1 = 0$

$(2u + 1)(u - 1) = 0$

$u = -\frac{1}{2} \qquad u = 1$

$x^{-1} = -\frac{1}{2} \qquad x^{-1} = 1$

$x = -2 \qquad x = 1$

128. $(x^2 - 6x)^2 - 2(x^2 - 6x) - 35 = 0$

Let $u = (x^2 - 6x)$.

$u^2 - 2u - 35 = 0$

$(u - 7)(u + 5) = 0$

$u = 7$	$u = -5$
$x^2 - 6x = 7$	$x^2 - 6x = -5$
$x^2 - 6x - 7 = 0$	$x^2 - 6x + 5 = 0$
$(x - 7)(x + 1) = 0$	$(x - 5)(x - 1) = 0$
$x = 7 \quad x = -1$	$x = 5 \quad x = 1$

130. $9\left(\frac{x+2}{x+3}\right)^2 - 6\left(\frac{x+2}{x+3}\right) + 1 = 0$

Let $u = \left(\frac{x+2}{x+3}\right)$.

$9u^2 - 6u + 1 = 0$

$(3u - 1)(3u - 1) = 0$

$u = \frac{1}{3}$

$\frac{x+2}{x+3} = \frac{1}{3}$

$3x + 6 = x + 3$

$2x = -3$

$x = -\frac{3}{2}$

132.
$$S = 4\pi r^2$$
$$\frac{900}{\pi} = 4\pi r^2$$
$$\frac{900}{\pi(4\pi)} = r^2$$
$$\frac{225}{\pi^2} = r^2$$
$$\sqrt{\frac{225}{\pi^2}} = r$$
$$\frac{15}{\pi} = r \approx 4.77 \text{ inches}$$

134.
$$s_0 = 48$$
$$0 = -16t^2 + 48$$
$$16t^2 = 48$$
$$t^2 = 3$$
$$t = \pm\sqrt{3}$$
$$t = \sqrt{3} \approx 1.732 \text{ seconds}$$

136.
$$s_0 = 500$$
$$0 = -16t^2 + 500$$
$$16t^2 = 500$$
$$t^2 = 31.25$$
$$t = \sqrt{31.25} \approx 5.590 \text{ sec}$$

138.
$$7000 = x\left(120 - \frac{1}{2}x\right)$$
$$7000 = 120x - \frac{1}{2}x^2$$
$$2(7000) = \left(120x - \frac{1}{2}x^2\right)2$$
$$14{,}000 = 240x - x^2$$
$$x^2 - 240x + 14{,}000 = 0$$
$$(x - 100)(x - 140) = 0$$
$$x - 100 = 0 \qquad x - 140 = 0$$
$$x = 100 \qquad x = 140$$

You must sell 100 units to produce a revenue of \$7000. If you sell 140 units, you will also have a revenue of \$7000.

140. $P = \$5000$, $A = \$5724.50$
$$A = P(1 + r)^2$$
$$5724.50 = 5000(1 + r)^2$$
$$\frac{5724.50}{5000} = (1 + r)^2$$
$$1.1449 = (1 + r)^2$$
$$\sqrt{1.1449} = 1 + r$$
$$1.07 - 1 = r$$
$$0.7 = r$$
$$7\% = r$$

142.
$$1200 = 4.43t^2 + 872$$
$$328 = 4.43t^2$$
$$\frac{328}{4.43} = t^2$$
$$\sqrt{\frac{328}{4.43}} = t \approx 9;\ 1999$$

144. If $a = 0$, the equation would not be quadratic.

146. Yes. For the quadratic equation $(x - 1)^2 = 0$, the only solution is $x = 1$.

148. Write the equation in the form $u^2 = d$, where u is an algebraic expression and d is a positive constant. Take the square roots of each side to obtain the solutions $u = \pm\sqrt{d}$.

Section 8.2 Completing the Square

2. $x^2 + 12x + 36$
$$\left[36 = \left(\frac{12}{2}\right)^2\right]$$

4. $y^2 - 2y + 1$
$$\left[1 = \left(\frac{-2}{2}\right)^2\right]$$

6. $x^2 + 18x + 81$
$$\left[81 = \left(\frac{18}{2}\right)^2\right]$$

8. $u^2 + 7u + \frac{49}{4}$
$$\left[\frac{49}{4} = \left(\frac{7}{2}\right)^2\right]$$

10. $y^2 - 11y + \frac{121}{4}$
$$\left[\frac{121}{4} = \left(\frac{-11}{2}\right)^2\right]$$

12. $y^2 + \frac{4}{3}y + \frac{4}{9}$
$$\left[\frac{4}{9} = \left[\left(\frac{4}{3}\right)\frac{1}{2}\right]^2\right]$$

14. $x^2 - \frac{6}{5}x + \frac{9}{25}$
$$\left[\frac{9}{25} = \left[\left(\frac{-6}{5}\right)\frac{1}{2}\right]^2\right]$$

16. $s^2 + 4.6s + 5.29$
$$\left[5.29 = \left(\frac{4.6}{2}\right)^2\right]$$

18. $x^2 + 32x = 0$

(a) $x^2 + 32x = 0$

$x^2 + 32x + 256 = 0 + 256$

$(x + 16)^2 = 256$

$x + 16 = \pm 16$

$x = -16 \pm 16$

$x = 0, -32$

(b) $x^2 + 32x = 0$

$x(x + 32) = 0$

$x = 0 \qquad x + 32 = 0$

$x = -32$

20. $t^2 - 10t = 0$

(a) $t^2 - 10t + 25 = 25$

$(t - 5)^2 = 25$

$t - 5 = \pm 5$

$t = 5 \pm 5$

$t = 10, 0$

(b) $t^2 - 10t = 0$

$t(t - 10) = 0$

$t = 0, 10$

22. $t^2 - 9t = 0$

(a) $t^2 - 9t + \frac{81}{4} = 0 + \frac{81}{4}$

$\left(t - \frac{9}{2}\right)^2 = \frac{81}{4}$

$t - \frac{9}{2} = \pm\sqrt{\frac{81}{4}}$

$t - \frac{9}{2} = \pm\frac{9}{2}$

$t = \frac{9}{2} \pm \frac{9}{2}$

$t = 9, 0$

(b) $t^2 - 9t = 0$

$t(t - 9) = 0$

$t = 0 \qquad t - 9 = 0$

$t = 9$

24. $y^2 - 8y + 12 = 0$

(a) $y^2 - 8y + 12 = 0$

$y^2 - 8y = -12$

$y^2 - 8y + 16 = -12 + 16$

$(y - 4)^2 = 4$

$y - 4 = \pm\sqrt{4}$

$y - 4 = \pm 2$

$y = 4 \pm 2$

$y = 6, 2$

(b) $y^2 - 8y + 12 = 0$

$(y - 6)(y - 2) = 0$

$y - 6 = 0 \qquad y - 2 = 0$

$y = 6 \qquad y = 2$

26. $z^2 + 3z - 10 = 0$

(a) $z^2 + 3z - 10 = 0$

$z^2 + 3z = 10$

$z^2 + 3z + \frac{9}{4} = 10 + \frac{9}{4}$

$\left(z + \frac{3}{2}\right)^2 = \frac{49}{4}$

$z + \frac{3}{2} = \pm\sqrt{\frac{49}{4}}$

$z + \frac{3}{2} = \pm\frac{7}{2}$

$z = -\frac{3}{2} \pm \frac{7}{2}$

$z = 2, -5$

(b) $z^2 + 3z - 10 = 0$

$(z + 5)(z - 2) = 0$

$z + 5 = 0 \qquad z - 2 = 0$

$z = -5 \qquad z = 2$

28. $t^2 - 5t - 36 = 0$

(a) $t^2 - 5t - 36 = 0$

$t^2 - 5t = 36$

$t^2 - 5t + \frac{25}{4} = 36 + \frac{25}{4}$

$\left(t - \frac{5}{2}\right)^2 = \frac{169}{4}$

$t - \frac{5}{2} = \pm\sqrt{\frac{169}{4}}$

$t - \frac{5}{2} = \pm\frac{13}{2}$

$t = \frac{5}{2} \pm \frac{13}{2}$

$t = 9, -4$

(b) $t^2 - 5t - 36 = 0$

$(t - 9)(t + 4) = 0$

$t - 9 = 0 \qquad t + 4 = 0$

$t = 9 \qquad t = -4$

30. $3x^2 - 3x - 6 = 0$

(a) $3x^2 - 3x - 6 = 0$

$x^2 - 1x - 2 = 0$

$x^2 - 1x + \frac{1}{4} = 2 + \frac{1}{4}$

$\left(x - \frac{1}{2}\right)^2 = \frac{9}{4}$

$x - \frac{1}{2} = \pm\frac{3}{2}$

$x = \frac{1}{2} \pm \frac{3}{2}$

$x = 2, -1$

(b) $3x^2 - 3x - 6 = 0$

$3(x^2 - x - 2) = 0$

$(x - 2)(x + 1) = 0$

$x = 2 \qquad x = -1$

32. $3x^2 - 13x + 12 = 0$

(a) $3x^2 - 13x = -12$

$x^2 - \frac{13}{3}x + \frac{169}{36} = -4 + \frac{169}{36}$

$\left(x - \frac{13}{6}\right)^2 = \frac{25}{36}$

$x - \frac{13}{6} = \pm\frac{5}{6}$

$x = \frac{13}{6} \pm \frac{5}{6} = \frac{18}{6}, \frac{8}{6} = 3, \frac{4}{3}$

(b) $3x^2 - 13x + 12 = 0$

$(3x - 4)(x - 3) = 0$

$x = \frac{4}{3} \qquad x = 3$

34. $x^2 - 6x + 7 = 0$

$x^2 - 6x = -7$

$x^2 - 6x + 9 = -7 + 9$

$(x - 3)^2 = 2$

$x - 3 = \pm\sqrt{2}$

$x = 3 \pm \sqrt{2}$

$x \approx 4.41, 1.59$

36. $x^2 + 6x + 7 = 0$

$x^2 + 6x + 9 = -7 + 9$

$(x + 3)^2 = 2$

$x + 3 = \pm\sqrt{2}$

$x = -3 \pm \sqrt{2}$

$x \approx -1.59, -4.41$

38. $x^2 + 8x = 9$

$x^2 + 8x + 16 = 9 + 16$

$(x + 4)^2 = 25$

$x + 4 = \pm 5$

$x = -4 \pm 5$

$x = 1, -9$

40. $x^2 - 4x = -9$

$x^2 - 4x + 4 = -9 + 4$

$(x - 2)^2 = -5$

$x - 2 = \pm\sqrt{-5}$

$x = 2 \pm \sqrt{5}i$

$x \approx 2 + 2.24i, 2 - 2.24i$

42. $x^2 + 10x + 9 = 0$

$x^2 + 10x + 25 = -9 + 25$

$(x + 5)^2 = 16$

$x + 5 = \pm 4$

$x = -5 \pm 4$

$x = -1, -9$

44. $x^2 - 10x + 24 = 0$

$x^2 - 10x + 25 = -24 + 25$

$(x - 5)^2 = 1$

$x - 5 = \pm 1$

$x = 5 \pm 1$

$x = 6, 4$

46. $y^2 + 6y + 7 = 0$

$y^2 + 6y + 9 = -7 + 9$

$(y + 3)^2 = 2$

$y + 3 = \pm\sqrt{2}$

$y = -3 \pm \sqrt{2}$

$y \approx -1.59, -4.41$

48. $x^2 + 23 = 10x$

$x^2 - 10x + 25 = -23 + 25$

$(x - 5)^2 = 2$

$x - 5 = \pm\sqrt{2}$

$x = 5 \pm \sqrt{2}$

$x \approx 6.41, 3.59$

50. $z^2 + 12z + 25 = 0$

$z^2 + 12z + 36 = -25 + 36$

$(z + 6)^2 = 11$

$z + 6 = \pm\sqrt{11}$

$z = -6 \pm \sqrt{11}$

$z \approx -2.68, -9.32$

52. $1 - x - x^2 = 0$

$$0 = x^2 + x - 1$$
$$1 = x^2 + x$$
$$1 + \frac{1}{4} = x^2 + x + \frac{1}{4}$$
$$\frac{5}{4} = \left(x + \frac{1}{2}\right)^2$$
$$\pm\sqrt{\frac{5}{4}} = x + \frac{1}{2}$$
$$\pm\frac{\sqrt{5}}{2} - \frac{1}{2} = x$$
$$x \approx 0.62, -1.62$$

54. $y^2 + 5y + 9 = 0$

$$y^2 + 5y + \frac{25}{4} = -9 + \frac{25}{4}$$
$$\left(y + \frac{5}{2}\right)^2 = \frac{-36}{4} + \frac{25}{4}$$
$$\left(y + \frac{5}{2}\right)^2 = \frac{-11}{4}$$
$$y + \frac{5}{2} = \pm\sqrt{\frac{-11}{4}}$$
$$y = \frac{5}{2} \pm \frac{\sqrt{11}}{2}i$$
$$y \approx -2.50 + 1.66i$$
$$y \approx -2.50 - 1.66i$$

56. $x^2 + \frac{4}{5}x - 1 = 0$

$$x^2 + \frac{4}{5}x = 1$$
$$x^2 + \frac{4}{5}x + \frac{4}{25} = 1 + \frac{4}{25}$$
$$\left(x + \frac{2}{5}\right)^2 = \frac{29}{25}$$
$$x + \frac{2}{5} = \pm\sqrt{\frac{29}{25}}$$
$$x + \frac{2}{5} = \pm\frac{\sqrt{29}}{5}$$
$$x = -\frac{2}{5} \pm \frac{\sqrt{29}}{5}$$
$$x \approx 0.68, -1.48$$

58. $u^2 - \frac{2}{3}u + 5 = 0$

$$u^2 - \frac{2}{3}u + \frac{1}{9} = -5 + \frac{1}{9}$$
$$\left(u - \frac{1}{3}\right)^2 = \frac{-45}{9} + \frac{1}{9}$$
$$\left(u - \frac{1}{3}\right)^2 = -\frac{44}{9}$$
$$u - \frac{1}{3} = \pm\sqrt{-\frac{44}{9}}$$
$$u = \frac{1}{3} \pm \frac{2\sqrt{11}}{3}i$$
$$u \approx 0.33 + 2.21i$$
$$u \approx 0.33 - 2.21i$$

60. $3x^2 - 24x - 5 = 0$

$$x^2 - 8x - \frac{5}{3} = 0$$
$$x^2 - 8x = \frac{5}{3}$$
$$x^2 - 8x + 16 = \frac{5}{3} + 16$$
$$(x - 4)^2 = \frac{53}{3}$$
$$x - 4 = \pm\sqrt{\frac{53}{3}}$$
$$x = 4 \pm \frac{\sqrt{53}}{\sqrt{3}}$$
$$x = 4 \pm \frac{\sqrt{159}}{3}$$
$$x \approx 8.20, -0.20$$

62. $5x^2 - 15x + 7 = 0$

$$x^2 - 3x = -\frac{7}{5}$$
$$x^2 - 3x + \frac{9}{4} = -\frac{7}{5} + \frac{9}{4}$$
$$\left(x - \frac{3}{2}\right)^2 = \frac{17}{20}$$
$$x - \frac{3}{2} = \pm\sqrt{\frac{17}{20}}$$
$$x = \frac{3}{2} \pm \frac{\sqrt{17}}{2\sqrt{5}}$$
$$x = \frac{3}{2} \pm \frac{\sqrt{85}}{10}$$
$$x = \frac{15 \pm \sqrt{85}}{10}$$
$$x \approx 2.42, 0.58$$

64. $4z^2 - 3z + 2 = 0$

$$z^2 - \frac{3}{4}z = -\frac{2}{4}$$
$$z^2 - \frac{3}{4}z + \frac{9}{64} = -\frac{1}{2} + \frac{9}{64}$$
$$\left(z - \frac{3}{8}\right)^2 = -\frac{23}{64}$$
$$z - \frac{3}{8} = \pm\sqrt{-\frac{23}{64}}$$
$$z = \frac{3}{8} \pm \frac{\sqrt{-23}}{8}$$
$$z = \frac{3}{8} \pm \frac{\sqrt{23}i}{8}$$
$$z \approx 0.38 + 0.60i$$
$$z \approx 0.38 - 0.60i$$

66. $7x^2 + 4x + 3 = 0$

$$x^2 + \frac{4}{7}x + \frac{4}{49} = -\frac{3}{7} + \frac{4}{49}$$
$$\left(x + \frac{2}{7}\right)^2 = -\frac{21}{49} + \frac{4}{49}$$
$$\left(x + \frac{2}{7}\right)^2 = -\frac{17}{49}$$
$$x + \frac{2}{7} = \pm\sqrt{-\frac{17}{49}}$$
$$x = \frac{-2}{7} \pm \frac{\sqrt{17}i}{7}$$
$$x \approx -0.29 + 0.59i$$
$$x \approx -0.29 - 0.59i$$

68. $2x\left(x+\frac{4}{3}\right)=5$

$$2x^2+\frac{8}{3}x=5$$
$$x^2+\frac{4}{3}x=\frac{5}{2}$$
$$x^2+\frac{4}{3}x+\frac{4}{9}=\frac{5}{2}+\frac{4}{9}$$
$$\left(x+\frac{2}{3}\right)^2=\frac{53}{18}$$
$$x+\frac{2}{3}=\pm\sqrt{\frac{53}{18}}$$
$$x=-\frac{2}{3}\pm\frac{\sqrt{53}}{3\sqrt{2}}$$
$$x=-\frac{2}{3}\pm\frac{\sqrt{106}}{6}$$
$$x=-\frac{4}{6}\pm\frac{\sqrt{106}}{6}$$
$$x=\frac{-4\pm\sqrt{106}}{6}$$
$$x\approx 1.05,\ -2.38$$

70. $0.1x^2+0.5x=-0.2$

$$0.1x^2+0.5x+0.2=0$$
$$x^2+5x+2=0$$
$$x^2+5x+\frac{25}{4}=-2+\frac{25}{4}$$
$$\left(x+\frac{5}{2}\right)^2=\frac{-8+25}{4}$$
$$\left(x+\frac{5}{2}\right)^2=\frac{17}{4}$$
$$x+\frac{5}{2}=\pm\sqrt{\frac{17}{4}}$$
$$x=-\frac{5}{2}\pm\frac{\sqrt{17}}{2}$$
$$x=\frac{-5\pm\sqrt{17}}{2}$$
$$x\approx -0.44,\ -4.56$$

72. $0.02x^2+0.10x-0.05=0$

$$2x^2+10x-5=0$$
$$x^2+5x-\frac{5}{2}=0$$
$$x^2+5x=\frac{5}{2}$$
$$x^2+5x+\frac{25}{4}=\frac{5}{2}+\frac{25}{4}$$
$$\left(x+\frac{5}{2}\right)^2=\frac{35}{4}$$
$$x+\frac{5}{2}=\pm\sqrt{\frac{35}{4}}$$
$$x=-\frac{5}{2}\pm\frac{\sqrt{35}}{2}$$
$$x\approx 0.46,\ -5.46$$

74. $\frac{x}{2}+\frac{5}{x}=4$

$$2x\left[\frac{x}{2}+\frac{5}{x}\right]=(4)2x$$
$$x^2+10=8x$$
$$x^2-8x=-10$$
$$x^2-8x+16=-10+16$$
$$(x-4)^2=6$$
$$x-4=\pm\sqrt{6}$$
$$x=4\pm\sqrt{6}$$

76. $\frac{x^2+2}{24}=\frac{x-1}{3}$

$$3x^2+6=24x-24$$
$$3x^2-24x+30=0$$
$$x^2-8x+10=0$$
$$x^2-8x=-10$$
$$x^2-8x+16=-10+16$$
$$(x-4)^2=6$$
$$x-4=\pm\sqrt{6}$$
$$x=4\pm\sqrt{6}$$

78. $\sqrt{3x-2}=x-2$

$$\left(\sqrt{3x-2}\right)^2=(x-2)^2$$
$$3x-2=x^2-2x(2)+4$$
$$3x-2=x^2-4x+4$$
$$0=x^2-7x+6$$
$$0=(x-6)(x-1)$$

$x-6=0$ $\qquad$ $x-1=0$

$x=6$ $\qquad$ $x=1$

Not a solution

Check: $\sqrt{3\cdot 6-2}\stackrel{?}{=}6-2$

$$\sqrt{18-2}\stackrel{?}{=}4$$
$$\sqrt{16}\stackrel{?}{=}4$$
$$4=4$$

Check: $\sqrt{3\cdot 1-2}\stackrel{?}{=}1-2$

$$\sqrt{3-2}\stackrel{?}{=}-1$$
$$\sqrt{1}\stackrel{?}{=}-1$$
$$1\neq -1$$

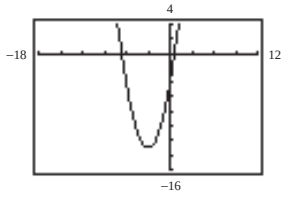
80. $y = x^2 + 6x - 4$

Keystrokes:

[Y=] [X,T,θ] [x^2] [+] 6 [X,T,θ] [−] 4 [GRAPH]

$$0 = x^2 + 6x - 4$$
$$4 = x^2 + 6x$$
$$4 + 9 = x^2 + 6x + 9$$
$$13 = (x + 3)^2$$
$$\pm\sqrt{13} = x + 3$$
$$-3 \pm \sqrt{13} = x$$
$$x \approx 0.61, -6.61$$

$\left(-3 \pm \sqrt{13}, 0\right)$



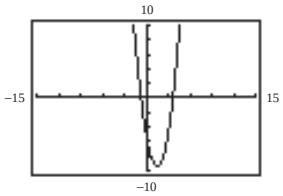
82. $y = 2x^2 - 6x - 5$

Keystrokes:

[Y=] 2 [X,T,θ] [x^2] [−] 6 [X,T,θ] [−] 5 [GRAPH]

$$0 = 2x^2 - 6x - 5$$
$$0 = x^2 - 3x - \frac{5}{2}$$
$$\frac{5}{2} = x^2 - 3x$$
$$\frac{5}{2} + \frac{9}{4} = x^2 - 3x + \frac{9}{4}$$
$$\frac{19}{4} = \left(x - \frac{3}{2}\right)^2$$
$$\pm\sqrt{\frac{19}{4}} = x - \frac{3}{2}$$
$$\frac{3}{2} \pm \frac{\sqrt{19}}{2} = x$$
$$x \approx 3.68, -0.68$$

$\left(\frac{3 \pm \sqrt{19}}{2}, 0\right)$



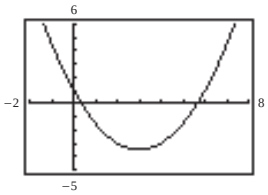
84. $y = \frac{1}{2}x^2 - 3x + 1$

Keystrokes:

[Y=] [(] 1 [÷] 2 [)] [X,T,θ] [x^2] [−] 3 [X,T,θ] [+] 1 [GRAPH]

$$0 = \tfrac{1}{2}x^2 - 3x + 1$$
$$0 = x^2 - 6x + 2$$
$$-2 = x^2 - 6x$$
$$-2 + 9 = x^2 - 6x + 9$$
$$7 = (x - 3)^2$$
$$\pm\sqrt{7} = x - 3$$
$$3 \pm \sqrt{7} = x$$
$$x \approx 5.65, 0.35$$

$\left(3 \pm \sqrt{7}, 0\right)$



86. $y = \sqrt{x} - x + 2$

Keystrokes:

[Y=] [√] [X,T,θ] [−] [X,T,θ] [+] 2 [GRAPH]

$$0 = \sqrt{x} - x + 2$$
$$x - 2 = \sqrt{x}$$
$$(x - 2)^2 = \left(\sqrt{x}\right)^2$$
$$x^2 - 4x + 4 = x$$
$$x^2 - 5x = -4$$
$$x^2 - 5x + \tfrac{25}{4} = -4 + \tfrac{25}{4}$$
$$\left(x - \tfrac{5}{2}\right)^2 = \tfrac{9}{4}$$
$$x - \tfrac{5}{2} = \pm\sqrt{\tfrac{9}{4}}$$
$$x - \tfrac{5}{2} = \pm\tfrac{3}{2}$$
$$x = \tfrac{5}{2} \pm \tfrac{3}{2}$$
$$x = 4 \qquad x = 1$$

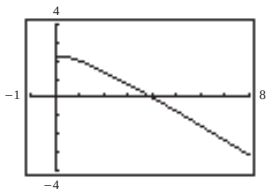
Not a solution

Check: $0 \stackrel{?}{=} \sqrt{4} - 4 + 2$

$$0 \stackrel{?}{=} 2 - 4 + 2$$
$$0 = 0$$

Check: $0 \stackrel{?}{=} \sqrt{1} - 1 + 2$

$$0 \stackrel{?}{=} 1 - 1 + 2$$
$$0 \neq 2$$



88. (a) Area of square $= 3 \cdot 3 = 9$

Area of vertical rectangle $= 3x$

Area of horizontal rectangle $= 3x$

Total area $= 9 + 3x + 3x = 9 + 6x$

(b) Area of small square $= x \cdot x = x^2$

Total area $= x^2 + 6x + 9$

(c) $(x + 3)(x + 3) = x^2 + 3x + 3x + 9$

$(x + 3)^2 = x^2 + 6x + 9$

90. *Verbal Model:* $\boxed{\text{Area}} = \boxed{\text{Length}} \cdot \boxed{\text{Width}}$

Labels: Length $= x$

Width $= \frac{1}{4}x + 3$

Equations: $160 = x\left(\frac{1}{4}x + 3\right)$

$160 = \frac{1}{4}x^2 + 3x$

$0 = x^2 + 12x - 640$

$0 = (x + 32)(x - 20)$

$x + 32 = 0 \qquad x - 20 = 0$

$x = 20$ feet length

$\frac{1}{4}x + 3 = 8$ feet width

92. *Verbal Model:* $\boxed{\text{Volume}} = \boxed{\text{Length}} \cdot \boxed{\text{Width}} \cdot \boxed{\text{Height}}$

Labels: Length $= x + 4$

Width $= x$

Height $= 6$

Equation: $840 = (x + 4)(x)(6)$

$140 = x(x + 4)$

$140 = x^2 + 4x$

$0 = x^2 + 4x - 140$

$0 = (x + 14)(x - 10)$

$x + 14 = 0 \qquad x - 10 = 0$

$x = -14 \qquad x = 10$

Not a solution

Thus, the dimensions are: length $= 10 + 4 = 14$ inches, width $= x = 10$ inches, height $= 6$ inches

94. *Equation:* $11{,}967.90 = x\left(100 - \frac{1}{10}x\right)$

$11{,}967.90 = 100x - \frac{1}{10}x^2$

$119{,}679 = 1000x - x^2$

$x^2 - 1000x + 119{,}679 = 0$

$x^2 - 1000x = -119{,}679$

$x^2 - 1000x + 250{,}000 = -119{,}679 + 250{,}000$

$(x - 500)^2 = 130{,}321$

$x - 500 = \pm\sqrt{130{,}321}$

$x - 500 = \pm 361$

$x = 500 \pm 361$

$x = 861, 139$

Thus, 139 or 861 golf clubs must be sold.

96. $\frac{25}{4}$. Divide the coefficient of the first-degree term by 2, and square the result to obtain $\left(\frac{5}{2}\right)^2 = \frac{25}{4}$.

98. Yes. $x^2 + 1 = 0$

100. True. Given the solutions $x = r_1$ and $x = r_2$, the quadratic equation can be written as $(x - r_1)(x - r_2) = 0$.

102. The student forgot to add 3^2 to the right side of the equation. Correct solution:

$x^2 + 6x = 13$

$x^2 + 6x + \left(\frac{6}{2}\right)^2 = 13 + \left(\frac{6}{2}\right)^2$

$(x + 3)^2 = 22$

$x + 3 = \pm\sqrt{22}$

$x = -3 \pm \sqrt{22}$

Section 8.3 The Quadratic Formula

2. $7x^2 + 15x = 5$

$7x^2 + 15x - 5 = 0$

4. $x(3x + 8) = 15$

$3x^2 + 8x = 15$

$3x^2 + 8x - 15 = 0$

6. (a) $x^2 - 12x + 27 = 0$

$$x = \frac{12 \pm \sqrt{12^2 - 4(1)(27)}}{2(1)}$$

$$x = \frac{12 \pm \sqrt{144 - 108}}{2}$$

$$x = \frac{12 \pm \sqrt{36}}{2}$$

$$x = \frac{12 \pm 6}{2}$$

$$x = 9, 3$$

(b) $(x - 9)(x - 3) = 0$

$x - 9 = 0$ $\quad$ $x - 3 = 0$

$x = 9$ $\quad$ $x = 3$

8. (a) $x^2 + 9x + 14 = 0$

$$x = \frac{-9 \pm \sqrt{9^2 - 4(1)(14)}}{2(1)}$$

$$x = \frac{-9 \pm \sqrt{81 - 56}}{2}$$

$$x = \frac{-9 \pm \sqrt{25}}{2}$$

$$x = \frac{-9 \pm 5}{2}$$

$$x = -2, -7$$

(b) $(x + 7)(x + 2) = 0$

$x + 7 = 0$ $\quad$ $x + 2 = 0$

$x = -7$ $\quad$ $x = -2$

10. (a) $9x^2 + 12x + 4 = 0$

$$x = \frac{-12 \pm \sqrt{(12)^2 - 4(9)(4)}}{2(9)}$$

$$x = \frac{-12 \pm \sqrt{144 - 144}}{18}$$

$$x = \frac{-12 \pm 0}{18}$$

$$x = -\frac{12}{18} = -\frac{2}{3}$$

(b) $(3x + 2)(3x + 2) = 0$

$3x + 2 = 0$ $\quad$ $3x + 2 = 0$

$x = -\frac{2}{3}$ $\quad$ $x = -\frac{2}{3}$

12. (a) $9x^2 - 30x + 25 = 0$

$$x = \frac{30 \pm \sqrt{(-30)^2 - 4(9)(25)}}{2(9)}$$

$$x = \frac{30 \pm \sqrt{900 - 900}}{18}$$

$$x = \frac{30 \pm \sqrt{0}}{18}$$

$$x = \frac{30}{18}$$

$$x = \frac{5}{3}$$

(b) $(3x - 5)(3x - 5) = 0$

$3x - 5 = 0$ $\quad$ $3x - 5 = 0$

$3x = 5$ $\quad$ $3x = 5$

$x = \frac{5}{3}$ $\quad$ $x = \frac{5}{3}$

14. (a) $10x^2 - 11x + 3 = 0$

$$x = \frac{11 \pm \sqrt{(-11)^2 - 4(10)(3)}}{2(10)}$$

$$x = \frac{11 \pm \sqrt{121 - 120}}{20}$$

$$x = \frac{11 \pm \sqrt{1}}{20}$$

$$x = \frac{11 \pm 1}{20}$$

$$x = \frac{3}{5}, \frac{1}{2}$$

(b) $(5x - 3)(2x - 1) = 0$

$5x - 3 = 0$ $\quad$ $2x - 1 = 0$

$5x = 3$ $\quad$ $2x = 1$

$x = \frac{3}{5}$ $\quad$ $x = \frac{1}{2}$

16. (a) $x^2 + 20x - 300 = 0$

$$x = \frac{-20 \pm \sqrt{20^2 - 4(1)(-300)}}{2(1)}$$

$$x = \frac{-20 \pm \sqrt{400 + 1200}}{2}$$

$$x = \frac{-20 \pm \sqrt{1600}}{2}$$

$$x = \frac{-20 \pm 40}{2} = 10, -30$$

(b) $(x + 30)(x - 10) = 0$

$x + 30 = 0$ $\quad$ $x - 10 = 0$

$x = -30$ $\quad$ $x = 10$

18. $x^2 - 2x - 6 = 0$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-6)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4 + 24}}{2}$$

$$x = \frac{2 \pm \sqrt{28}}{2}$$

$$x = \frac{2 \pm 2\sqrt{7}}{2}$$

$$x = \frac{2(1 \pm \sqrt{7})}{2}$$

$$x = 1 \pm \sqrt{7}$$

20. $y^2 + 6y + 4 = 0$

$$y = \frac{-6 \pm \sqrt{6^2 - 4(1)(4)}}{2(1)}$$

$$y = \frac{-6 \pm \sqrt{36 - 16}}{2}$$

$$y = \frac{-6 \pm \sqrt{20}}{2}$$

$$y = \frac{-6 \pm 2\sqrt{5}}{2}$$

$$y = \frac{2(-3 \pm \sqrt{5})}{2}$$

$$y = -3 \pm \sqrt{5}$$

22. $x^2 + 8x - 4 = 0$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(-4)}}{2(1)}$$

$$x = \frac{-8 \pm \sqrt{64 + 16}}{2}$$

$$x = \frac{-8 \pm \sqrt{80}}{2}$$

$$x = \frac{-8 \pm 4\sqrt{5}}{2}$$

$$x = \frac{2(-4 \pm 2\sqrt{5})}{2}$$

$$x = -4 \pm 2\sqrt{5}$$

24. $u^2 - 12u + 29 = 0$

$$u = \frac{12 \pm \sqrt{(-12)^2 - 4(1)(29)}}{2(1)}$$

$$u = \frac{12 \pm \sqrt{144 - 116}}{2}$$

$$u = \frac{12 \pm \sqrt{28}}{2}$$

$$u = \frac{12 \pm 2\sqrt{7}}{2}$$

$$u = \frac{2(6 \pm \sqrt{7})}{2}$$

$$u = 6 \pm \sqrt{7}$$

26. $2x^2 - x + 1 = 0$

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(2)(1)}}{2(2)}$$

$$x = \frac{1 \pm \sqrt{1 - 8}}{4}$$

$$x = \frac{1 \pm \sqrt{-7}}{4}$$

$$x = \frac{1 \pm \sqrt{7}}{4}i$$

$$x = \frac{1}{4} \pm \frac{\sqrt{7}}{4}i$$

28. $4x^2 + 6x + 1 = 0$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(4)(1)}}{2(4)}$$

$$x = \frac{-6 \pm \sqrt{36 - 16}}{8}$$

$$x = \frac{-6 \pm \sqrt{20}}{8}$$

$$x = \frac{-6 \pm 2\sqrt{5}}{8}$$

$$x = \frac{2(-3 \pm \sqrt{5})}{8}$$

$$x = \frac{-3 \pm \sqrt{5}}{4}$$

30. $2x^2 + 3x + 3 = 0$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(3)}}{2(2)}$$

$$x = \frac{-3 \pm \sqrt{9 - 24}}{4}$$

$$x = \frac{-3 \pm \sqrt{-15}}{4}$$

$$x = \frac{-3 \pm \sqrt{15}}{4}i$$

$$x = \frac{-3}{4} \pm \frac{\sqrt{15}}{4}i$$

32. $8y^2 - 8y - 1 = 0$

$$y = \frac{8 \pm \sqrt{(-8)^2 - 4(8)(-1)}}{2(8)}$$

$$y = \frac{8 \pm \sqrt{64 + 32}}{16}$$

$$y = \frac{8 \pm \sqrt{96}}{16}$$

$$y = \frac{8 \pm 4\sqrt{6}}{16}$$

$$y = \frac{4(2 \pm \sqrt{6})}{16}$$

$$y = \frac{2 \pm \sqrt{6}}{4}$$

34. $-5x^2 - 15x + 10 = 0$

$$x = \frac{15 \pm \sqrt{(-15)^2 - 4(-5)(10)}}{2(-5)}$$

$$x = \frac{15 \pm \sqrt{225 + 200}}{-10}$$

$$x = \frac{-15 \pm \sqrt{425}}{10}$$

$$x = \frac{-15 \pm 5\sqrt{17}}{10}$$

$$x = \frac{-5(3 \pm \sqrt{17})}{10}$$

$$x = \frac{-3 \pm \sqrt{17}}{2}$$

36. $6x^2 + 3x - 9 = 0$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(6)(-9)}}{2(6)}$$

$$x = \frac{-3 \pm \sqrt{9 + 216}}{12}$$

$$x = \frac{-3 \pm \sqrt{225}}{12}$$

$$x = \frac{-3 \pm 15}{12}$$

$$x = 1, -\frac{18}{12}$$

$$x = 1, -\frac{3}{2}$$

38. $-15x^2 - 10x + 25 = 0$ (Divide by -5.)

$$3x^2 + 2x - 5 = 0$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(3)(-5)}}{2(3)}$$

$$x = \frac{-2 \pm \sqrt{4 + 60}}{6}$$

$$x = \frac{-2 \pm \sqrt{64}}{6}$$

$$x = \frac{-2 \pm 8}{6}$$

$$x = 1, -\frac{10}{6}$$

$$x = 1, -\frac{5}{3}$$

40. $7x^2 = 3 - 5x$

$$7x^2 + 5x - 3 = 0$$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(7)(-3)}}{2(7)}$$

$$x = \frac{-5 \pm \sqrt{25 + 84}}{14}$$

$$x = \frac{-5 \pm \sqrt{109}}{14}$$

42. $x - x^2 = 1 - 6x^2$

$$5x^2 + x - 1 = 0$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(5)(-1)}}{2(5)}$$

$$x = \frac{-1 \pm \sqrt{1 + 20}}{10}$$

$$x = \frac{-1 \pm \sqrt{21}}{10}$$

44. $x^2 + 0.6x - 0.41 = 0$

$$x = \frac{-0.6 \pm \sqrt{0.6^2 - 4(1)(-0.41)}}{2(1)}$$

$$x = \frac{-0.6 \pm \sqrt{0.36 + 1.64}}{2}$$

$$x = \frac{-0.6 \pm \sqrt{2.00}}{2}$$

$$x = \frac{-0.6 \pm \sqrt{2}}{2}$$

46. $0.09x^2 - 0.12x - 0.26 = 0$

$$9x^2 - 12x - 26 = 0$$

$$x = \frac{12 \pm \sqrt{(-12)^2 - 4(9)(-26)}}{2(9)}$$

$$x = \frac{12 \pm \sqrt{144 + 936}}{18}$$

$$x = \frac{12 \pm \sqrt{1080}}{18}$$

$$x = \frac{12 \pm 6\sqrt{30}}{18}$$

$$= \frac{6\left(2 \pm \sqrt{30}\right)}{18} = \frac{2 \pm \sqrt{30}}{3} = \frac{0.02 \pm \sqrt{0.003}}{0.03}$$

48. $x^2 + x - 1 = 0$

$b^2 - 4ac = 1^2 - 4(1)(-1)$

$= 1 + 4$

$= 5$

Two distinct irrational solutions

50. $b^2 - 4ac = 5^2 - 4(10)(1)$

$= 25 - 40$

$= -15$

Two distinct complex solutions

52. $b^2 - 4ac = (-2)^2 - 4(3)(-5)$

$= 4 + 60$

$= 64$

Two distinct rational solutions

54. $b^2 - 4ac = 10^2 - 4(2)(6)$
$= 100 - 48$
$= 52$
Two distinct irrational solutions

56. $b^2 - 4ac = (-24)^2 - 4(9)(16)$
$= 576 - 576$
$= 0$
One (repeated) rational solution

58. $t^2 = 144$
$t = \pm\sqrt{144}$
$t = \pm 12$

60. $7u^2 + 49u = 0$
$7u(u + 7) = 0$
$7u = 0 \qquad u + 7 = 0$
$u = 0 \qquad u = -7$

62. $9(x + 4)^2 + 16 = 0$
$9(x + 4)^2 = -16$
$(x + 4)^2 = \dfrac{-16}{9}$
$x + 4 = \pm\sqrt{\frac{-16}{9}}$
$x = -4 \pm \frac{4}{3}i$

64. $4y(y + 7) - 5(y + 7) = 0$
$(y + 7)(4y - 5) = 0$
$y + 7 = 0 \qquad 4y - 5 = 0$
$y = -7 \qquad y = \frac{5}{4}$

66. $x^2 - 3x - 4 = 0$
$(x - 4)(x + 1) = 0$
$x - 4 = 0 \qquad x + 1 = 0$
$x = 4 \qquad x = -1$

68. $y^2 + 21y + 108 = 0$
$(y + 12)(y + 9) = 0$
$y + 12 = 0 \qquad y + 9 = 0$
$y = -12 \qquad y = -9$

70. $2x^2 - 15x + 225 = 0$
$x = \dfrac{-(-15) \pm \sqrt{(-15)^2 - 4(2)(225)}}{2(2)}$
$x = \dfrac{15 \pm \sqrt{225 - 1800}}{4}$
$x = \dfrac{15 \pm \sqrt{-1575}}{4}$
$x = \dfrac{15}{4} \pm \dfrac{15\sqrt{7}}{4}i$

72. $14x^2 + 11x - 40 = 0$
$x = \dfrac{-11 \pm \sqrt{11^2 - 4(14)(-40)}}{2(14)}$
$x = \dfrac{-11 \pm \sqrt{121 + 2240}}{28}$
$x = \dfrac{-11 \pm \sqrt{2361}}{28}$

74. $5x(x - 1) - 7 = 4x(x - 2)$
$5x^2 - 5x - 7 = 4x^2 - 8x$
$x^2 + 3x - 7 = 0$
$x = \dfrac{-3 \pm \sqrt{3^2 - 4(1)(-7)}}{2(1)}$
$x = \dfrac{-3 \pm \sqrt{9 + 28}}{2}$
$x = \dfrac{-3 \pm \sqrt{37}}{2}$

76. $x = -2 \qquad x = 3$
$x + 2 = 0 \qquad x - 3 = 0$
$(x + 2)(x - 3) = 0$
$x^2 - x - 6 = 0$

78. $x = 3 \qquad x = 9$
$x - 3 = 0 \qquad x - 9 = 0$
$(x - 3)(x - 9) = 0$
$x^2 - 12x + 27 = 0$

80. $x = -3 + \sqrt{5} \qquad x = -3 - \sqrt{5}$
$x - \left(-3 + \sqrt{5}\right) = 0 \qquad x - \left(-3 - \sqrt{5}\right) = 0$
$\left[x - \left(-3 + \sqrt{5}\right)\right]\left[x - \left(-3 - \sqrt{5}\right)\right] = 0$
$\left[(x + 3) - \sqrt{5}\right]\left[(x + 3) + \sqrt{5}\right] = 0$
$(x + 3)^2 - \left(\sqrt{5}\right)^2 = 0$
$x^2 + 6x + 9 - 5 = 0$
$x^2 + 6x + 4 = 0$

82. $x = 2i \qquad x = -2i$

$x - 2i = 0 \qquad x + 2i = 0$

$(x - 2i)(x + 2i) = 0$

$x^2 - 4i^2 = 0$

$x^2 + 4 = 0$

84. $x = -4 \qquad x = -4$

$x + 4 = 0 \qquad x + 4 = 0$

$(x + 4)(x + 4) = 0$

$x^2 + 8x + 16 = 0$

86. $y = x^2 + x + 1$

Keystrokes:

Y= X,T,θ x^2 + X,T,θ + 1 GRAPH

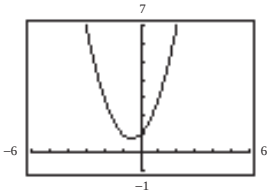
$0 = x^2 + x + 1$

$x = \dfrac{(-1) \pm \sqrt{1^2 - 4(1)(1)}}{2(1)}$

$x = \dfrac{-1 \pm \sqrt{1 - 4}}{2}$

$x = \dfrac{-1 \pm \sqrt{-3}}{2}$

$x = \dfrac{-1 \pm \sqrt{3}i}{2}$



No x-intercepts

88. $y = x^2 - 4x + 3$

Keystrokes:

Y= X,T,θ x^2 − 4 X,T,θ + 3 GRAPH

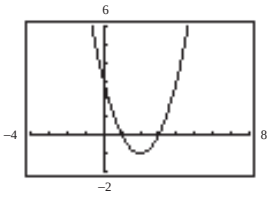
$0 = x^2 - 4x + 3$

$0 = (x - 3)(x - 1)$

$x - 3 = 0 \qquad x - 1 = 0$

$x = 3 \qquad x = 1$

(3, 0), (1, 0)



90. $y = 15x^2 + 3x - 105$

Keystrokes:

Y= 15 X,T,θ x^2 + 3 X,T,θ − 105 GRAPH

$x = \dfrac{-3 \pm \sqrt{3^2 - 4(15)(-105)}}{2(15)}$

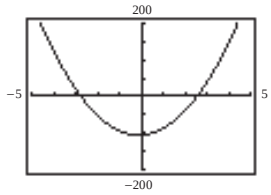
$x = \dfrac{-3 \pm \sqrt{9 + 6300}}{30}$

$x = \dfrac{-3 \pm \sqrt{6309}}{30}$

$x = \dfrac{-3 \pm 3\sqrt{701}}{30}$

$x = \dfrac{3\left(-1 \pm \sqrt{701}\right)}{30}$

$x = \dfrac{-1 \pm \sqrt{701}}{10}$



$x \approx 2.55, -2.75$

(2.55, 0), (−2.75, 0)

92. $y = 3.7x^2 - 10.2x + 3.2$

Keystrokes:

Y= 3.7 X,T,θ x^2 − 10.2 X,T,θ + 3.2 GRAPH

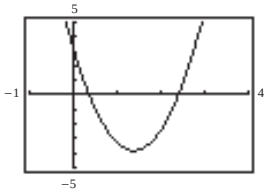
$x = \dfrac{10.2 \pm \sqrt{(-10.2)^2 - 4(3.7)(3.2)}}{2(3.7)}$

$x = \dfrac{10.2 \pm \sqrt{104.04 - 47.36}}{7.4}$

$x = \dfrac{10.2 \pm \sqrt{56.68}}{7.4}$

$x \approx 2.40, 0.36$

(2.40, 0), (0.36, 0)



94. $2x^2 - x - 1 = 0$

Keystrokes:

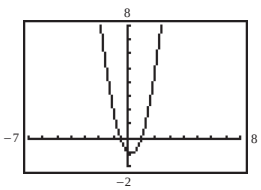
Y= 2 X,T,θ x^2 − X,T,θ − 1 GRAPH

$b^2 - 4ac = (-1)^2 - 4(2)(-1)$

$= 1 + 8$

$= 9$

Two real solutions



96. $\frac{1}{3}x^2 - 5x + 25 = 0$

Multiply by 3.

$x^2 - 15x + 75 = 0$

Keystrokes:

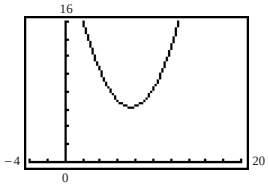
Y= X,T,θ x^2 − 15 X,T,θ + 75 GRAPH

$b^2 - 4ac = (-15)^2 - 4(1)(75)$

$= 225 - 300$

$= -75$

No real solutions



98. $f(x) = 2x^2 - 7x + 5$

$0 = 2x^2 - 7x + 5$

$0 = (2x - 5)(x - 1)$

$x = \frac{5}{2} \qquad x = 1$

100. $h(x) = 6x^2 + x + 10$

$-2 = 6x^2 + x + 10$

$0 = 6x^2 + x + 12$

$x = \dfrac{-1 \pm \sqrt{1^2 - 4(6)(12)}}{2(6)}$

$x = \dfrac{-1 \pm \sqrt{1 - 288}}{12}$

$x = \dfrac{-1 \pm \sqrt{-287}}{12}$

No real values

102. $\dfrac{x^2 - 9x}{6} = \dfrac{x - 1}{2}$

$2x^2 - 18x = 6x - 6$

$2x^2 - 24x + 6 = 0$

$x^2 - 12x + 3 = 0$

$x = \dfrac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(3)}}{2(1)}$

$x = \dfrac{12 \pm \sqrt{144 - 12}}{2} = \dfrac{12 \pm \sqrt{132}}{2}$

$x = \dfrac{12 \pm 2\sqrt{33}}{2}$

$x = 6 \pm \sqrt{33}$

104. $\sqrt{2x - 3} = x - 2$

$\left(\sqrt{2x - 3}\right)^2 = (x - 2)^2$

$2x - 3 = x^2 - 2(x)(2) + 2^2$

$2x - 3 = x^2 - 4x + 4$

$0 = x^2 - 6x + 7$

$x = \dfrac{6 \pm \sqrt{6^2 - 4(1)(7)}}{2(1)}$

$x = \dfrac{6 \pm \sqrt{36 - 28}}{2}$

$x = \dfrac{6 \pm \sqrt{8}}{2}$

$x = \dfrac{6 \pm 2\sqrt{2}}{2}$

$x = \dfrac{2\left(3 \pm \sqrt{2}\right)}{2}$

$x = 3 \pm \sqrt{2}$

$x = 3 - \sqrt{2}$ does not check.

Check: $\sqrt{2\left(3 + \sqrt{2}\right) - 3} \stackrel{?}{=} 3 + \sqrt{2} - 2$

$\sqrt{6 + 2\sqrt{2} - 3} \stackrel{?}{=} 1 + \sqrt{2}$

$\sqrt{3 + 2\sqrt{2}} \stackrel{?}{=} 1 + \sqrt{2}$

$2.4142 = 2.4142$

Check: $\sqrt{2\left(3 - \sqrt{2}\right) - 3} \stackrel{?}{=} 3 - \sqrt{2} - 2$

$\sqrt{6 - 2\sqrt{2} - 3} \stackrel{?}{=} 1 - \sqrt{2}$

$\sqrt{3 - 2\sqrt{2}} \stackrel{?}{=} 1 - \sqrt{2}$

$0.4142 \neq -0.4142$

106. $x^2 - 12x + c = 0$

(a) $b^2 - 4ac > 0$
$(-12)^2 - 4(1)c > 0$
$144 - 4c > 0$
$144 > 4c$
$36 > c$

(b) $b^2 - 4ac = 0$
$(-12)^2 - 4(1)c = 0$
$144 - 4c = 0$
$144 = 4c$
$36 = c$

(c) $b^2 - 4ac < 0$
$(-12)^2 - 4(1)c < 0$
$144 - 4c < 0$
$144 < 4c$
$36 < c$

108. $x^2 + 2x + c = 0$

(a) $b^2 - 4ac > 0$
$2^2 - 4(1)c > 0$
$4 - 4c > 0$
$4 > 4c$
$1 > c$

(b) $b^2 - 4ac = 0$
$2^2 - 4(1)c = 0$
$4 - 4c = 0$
$4 = 4c$
$1 = c$

(c) $b^2 - 4ac < 0$
$2^2 - 4(1)c < 0$
$4 - 4c < 0$
$4 < 4c$
$1 < c$

110. *Verbal Model:* $\boxed{\text{Area}} = \boxed{\text{Length}} \cdot \boxed{\text{Width}}$

Labels: Length $= x + 1.5$
Width $= x$

Equation: $18.36 = (x + 1.5) \cdot x$

$18.36 = x^2 + 1.5x$

$0 = x^2 + 1.5x - 18.36$

$$x = \frac{-1.5 \pm \sqrt{1.5^2 - 4(1)(-18.36)}}{2(1)}$$

$$x = \frac{-1.5 \pm \sqrt{2.25 + 73.44}}{2}$$

$$x = \frac{-1.5 + \sqrt{75.69}}{2} \approx 3.6$$

$x \approx 3.6$ inches

$x + 1.5 \approx 5.1$ inches

112. $h = -16t^2 + 20t + 40$

(a) $40 = -16t^2 + 20t + 40$
$0 = -16t^2 + 20t$
$0 = -4t(4t - 5)$
$-4t = 0 \qquad 4t - 5 = 0$
$t = 0 \qquad t = \frac{5}{4} = 1.25$ seconds

(b) $0 = -16t^2 + 20t + 40$
$0 = -4(4t^2 - 5t - 10)$

$$t = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(4)(-10)}}{2(4)}$$

$$t = \frac{5 \pm \sqrt{25 + 160}}{8}$$

$$t = \frac{5 \pm \sqrt{185}}{8}$$

$$t = \frac{5 + \sqrt{185}}{8} \approx 2.325 \text{ seconds}$$

reject $\frac{5 - \sqrt{185}}{8}$

114. (a) $61 = -16t^2 + 36t + 61$
$0 = -16t^2 + 36t$
$0 = -4t(4t - 9)$
$0 = -4t \qquad 4t - 9 = 0$
$0 = t \qquad t = \frac{9}{4} = 2.25$ seconds

(b) $0 = -16t^2 + 36t + 61$

$$t = \frac{-36 \pm \sqrt{36^2 - 4(-16)(61)}}{2(-16)}$$

$$t = \frac{-36 \pm \sqrt{1296 + 3904}}{-32}$$

$$t = \frac{36 \pm \sqrt{5200}}{32}$$

$t \approx 3.38; -1.13$ reject

$t \approx 3.38$ seconds

116. (a) $42 = -16t^2 + 30t + 42$

$0 = -16t^2 + 30t$

$0 = -2t(8t - 15)$

$0 = -2t \qquad 8t - 15 = 0$

$0 = t \qquad t = \frac{15}{8}$ seconds

(b) $0 = -16t^2 + 30t + 42$

$0 = 8t^2 - 15t - 21$

$t = \frac{15 \pm \sqrt{(-15)^2 - 4(8)(-21)}}{2(8)}$

$t = \frac{15 \pm \sqrt{225 + 672}}{16}$

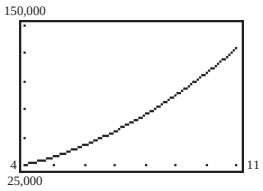
$t = \frac{15 \pm \sqrt{897}}{16}$

$t \approx 2.81; -0.93$ reject

$t \approx 2.81$ seconds

118. (a) *Keystrokes:*

Y= 1178.29 X,T,θ x² − 2816.5 X,T,θ + 17,457 GRAPH



(b) 1996

$s = 44$ million $= 44{,}000{,}000 = 44{,}000$ (in thousands)

$44{,}000 = 1178.29t^2 - 2816.5t + 17{,}457$

$0 = 1178.29t^2 - 2816.5t - 26{,}543$

$t = \frac{2816.5 \pm \sqrt{(-2816.5)^2 - 4(1178.29)(-26{,}543)}}{2(1178.29)}$

$t \approx 6;$ 1996

120. $ax^2 + bx + c = 0$

Solutions are x_1 and x_2.

$x_1 + x_2 = -\frac{b}{a}, \qquad x_1 \cdot x_2 = \frac{c}{a}$

122. Compute $-b$ plus or minus the square root of the quantity b squared minus $4ac$. This quantity divided by the quantity $2a$ is the Quadratic Formula.

124. The Quadratic Formula is derived by solving the general quadratic equation $ax^2 + bx + c = 0$ by the method of completing the square.

Section 8.4 Graphs of Quadratic Functions

2. $y = -(x + 1)^2$ (f)

4. $y = -x^2 + 3$ (c)

6. $y = 2 - (x - 2)^2$ (a)

8. $y = x^2 + 2x$

$y = (x^2 + 2x + 1) - 1$

$y = (x + 1)^2 - 1$

vertex $= (-1, -1)$

10. $y = x^2 + 6x - 5$

$y = (x^2 + 6x + 9) - 5 - 9$

$y = (x + 3)^2 - 14$

vertex $= (-3, -14)$

12. $y = x^2 - 4x + 5$

$y = (x^2 - 4x + 4) + 5 - 4$

$y = (x - 2)^2 + 1$

vertex $= (2, 1)$

14. $y = 4 - 8x - x^2$

$y = -(x^2 + 8x - 4)$

$y = -(x^2 + 8x + 16 - 16 - 4)$

$y = -(x^2 + 8x + 16) + 16 + 4$

$y = -(x + 4)^2 + 20$

vertex $= (-4, 20)$

16. $y = -x^2 - 10x + 10$

$= -(x^2 + 10x) + 10$

$= -(x^2 + 10x + 25 - 25) + 10$

$= -(x^2 + 10x + 25) + 25 + 10$

$= -(x + 5)^2 + 35$

vertex $= (-5, 35)$

18. $y = 3x^2 - 3x - 9$

$y = 3(x^2 - x - 3)$

$y = 3\left(x^2 - x + \frac{1}{4} - \frac{1}{4} - 3\right)$

$y = 3\left(x^2 - x + \frac{1}{4}\right) - \frac{3}{4} - 9$

$y = 3\left(x - \frac{1}{2}\right)^2 - \frac{3}{4} - \frac{36}{4}$

$y = 3\left(x - \frac{1}{2}\right)^2 - \frac{39}{4}$

vertex $= \left(\frac{1}{2}, -\frac{39}{4}\right)$

20. $f(x) = x^2 + 4x + 1$

$a = 1, \quad b = 4$

$x = \dfrac{-b}{2a} = \dfrac{-(4)}{2(1)} = -2$

$f\left(\dfrac{-b}{2a}\right) = (-2)^2 + 4(-2) + 1$

$= 4 - 8 + 1$

$= -3$

vertex $= (-2, -3)$

22. $h(x) = -x^2 + 14x - 14$

$a = -1, \quad b = 14$

$x = \dfrac{-b}{2a} = \dfrac{-14}{2(-1)} = 7$

$h\left(\dfrac{-b}{2a}\right) = -7^2 + 14(7) - 14$

$= -49 + 98 - 14 = 35$

vertex $= (7, 35)$

24. $y = 9x^2 - 12x$

$a = 9, \quad b = -12$

$x = \dfrac{-b}{2a} = \dfrac{-(-12)}{2(9)} = \dfrac{12}{18} = \dfrac{2}{3}$

$y = 9\left(\dfrac{2}{3}\right)^2 - 12\left(\dfrac{2}{3}\right)$

$= 9\left(\dfrac{4}{9}\right) - 8 = 4 - 8 = -4$

vertex $= \left(\dfrac{2}{3}, -4\right)$

26. $y = -3(x + 5)^2 - 3$

$-3 < 0$ opens downward.

vertex $= (-5, -3)$

28. $y = 2(x - 12)^2 + 3$

$2 > 0$ opens upward.

vertex $= (12, 3)$

30. $y = -(x + 1)^2$

$-1 < 0$ opens downward.

vertex $= (-1, 0)$

32. $y = x^2 - 6x$

$1 > 0$ opens upward.

$y = x^2 - 6x$

$y = x^2 - 6x + 9 - 9$

$y = (x - 3)^2 - 9$

vertex $= (3, -9)$

34. $y = -x^2 - 5$

$-1 < 0$ opens downward.

vertex $= (0, -5)$

36. $y = x^2 - 49$

$0 = x^2 - 49$

$0 = (x - 7)(x + 7)$

$x - 7 = 0 \qquad x + 7 = 0$

$x = 7 \qquad x = -7$

$(7, 0)$ and $(-7, 0)$

$y = x^2 - 49$

$y = 0^2 - 49$

$y = -49$

$(0, -49)$

38. $y = x^2 + 4x$

$0 = x^2 + 4x$

$0 = x(x + 4)$

$x = 0 \qquad x + 4 = 0$

$x = -4$

$(0, 0)$ and $(-4, 0)$

$y = x^2 + 4x$

$y = 0^2 + 4(0)$

$y = 0$

$(0, 0)$

40. $y = -x^2 + 4x - 5$

$0 = x^2 - 4x + 5$

$x = \dfrac{4 \pm \sqrt{16 - 20}}{2}$

$x = \dfrac{4 \pm \sqrt{-4}}{2}$

No x-intercepts

$y = -x^2 + 4x - 5$

$y = -0^2 + 4(0) - 5$

$y = -5$

$(0, -5)$

42. $y = 10 - x - 2x^2$

$0 = 10 - x - 2x^2$

$0 = (5 + 2x)(2 - x)$

$5 + 2x = 0 \qquad 2 - x = 0$

$2x = -5 \qquad 2 = x$

$x = -\frac{5}{2}$

$\left(-\frac{5}{2}, 0\right)$ and $(2, 0)$

$y = 10 - x - 2x^2$

$y = 10 - 0 - 2(0)^2$

$y = 10$

$(0, 10)$

44. $y = x^2 - 3x - 10$

$0 = x^2 - 3x - 10$

$0 = (x - 5)(x + 2)$

$x - 5 = 0 \qquad x + 2 = 0$

$x = 5 \qquad x = -2$

$(5, 0)$ and $(-2, 0)$

$y = x^2 - 3x - 10$

$y = 0^2 - 3(0) - 10$

$y = -10$

$(0, -10)$

46. $y = -4x^2 + 6x - 9$

$0 = 4x^2 - 6x + 9$

$x = \dfrac{6 \pm \sqrt{36 - 144}}{8}$

$x = \dfrac{6 \pm \sqrt{-108}}{8}$

No x-intercepts

$y = -4x^2 + 6x - 9$

$y = -4(0)^2 + 6(0) - 9$

$y = -9$

$(0, -9)$

48. $h(x) = x^2 - 9$

x-intercepts:

$0 = x^2 - 9$

$0 = (x - 3)(x + 3)$

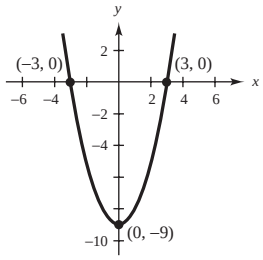
$x - 3 = 0 \qquad x + 3 = 0$

$x = 3 \qquad x = -3$

vertex:

$h(x) = x^2 - 9$

$h(x) = (x - 0)^2 - 9$



50. $f(x) = -x^2 + 9$

x-intercepts:

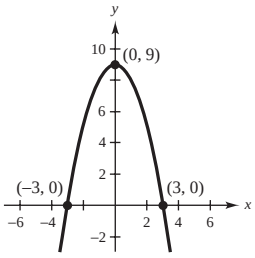
$0 = -x^2 + 9$

$x^2 = 9$

$x = \pm 3$

vertex:

$f(x) = -(x - 0)^2 + 9$



52. $g(x) = x^2 - 4x$

x-intercepts:

$0 = x^2 - 4x$

$0 = x(x - 4)$

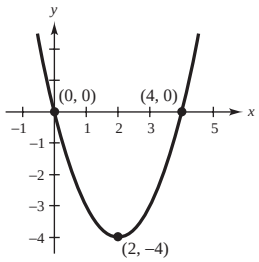
$x = 0 \qquad x - 4 = 0$

$x = 4$

vertex:

$y = (x^2 - 4x + 4) - 4$

$= (x - 2)^2 - 4$



54. $y = -x^2 + 4x$

x-intercepts:

$0 = -x^2 + 4x$

$0 = -x(x - 4)$

$-x = 0 \qquad x - 4 = 0$

$x = 0 \qquad x = 4$

vertex:

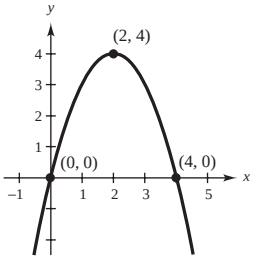
$y = -x^2 + 4x$

$= -(x^2 - 4x)$

$= -(x^2 - 4x + 4 - 4)$

$= -(x^2 - 4x + 4) + 4$

$= -(x - 2)^2 + 4$



56. $y = -(x + 4)^2$

x-intercepts:

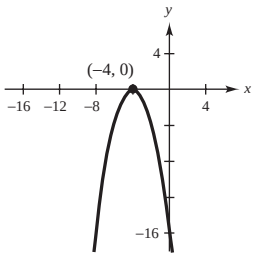
$0 = -(x + 4)^2$

$x + 4 = 0$

$x = -4$

vertex:

$f(x) = -(x + 4)^2$



58. $y = x^2 + 4x + 2$

x-intercepts:

$0 = x^2 + 4x + 2$

$x = \dfrac{-4 \pm \sqrt{4^2 - 4(1)(2)}}{2(1)}$

$x = \dfrac{-4 \pm \sqrt{16 - 8}}{2}$

$x = \dfrac{-4 \pm \sqrt{8}}{2}$

$x = \dfrac{-4 \pm 2\sqrt{2}}{2}$

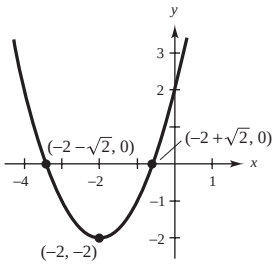
$x = -2 \pm \sqrt{2}$

vertex:

$y = x^2 + 4x + 2$

$= (x^2 + 4x + 4) + 2 - 4$

$= (x + 2)^2 - 2$



60. $y = -x^2 + 2x + 8$

x-intercepts:

$0 = -x^2 + 2x + 8$

$0 = -(x^2 - 2x - 8)$

$0 = -(x - 4)(x + 2)$

$-1 \neq 0 \quad x - 4 = 0 \quad x + 2 = 0$

$x = 4 \qquad x = -2$

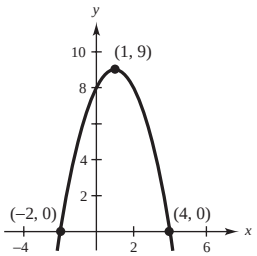
vertex:

$y = -(x^2 - 2x) + 8$

$= -(x^2 - 2x + 1 - 1) + 8$

$= -(x^2 - 2x + 1) + 1 + 8$

$= -(x - 1)^2 + 9$



62. $f(x) = x^2 + 4x + 7$

vertex:

$f(x) = (x^2 + 4x + 4) + 7 - 4$

$f(x) = (x + 2)^2 + 3$

x-intercepts:

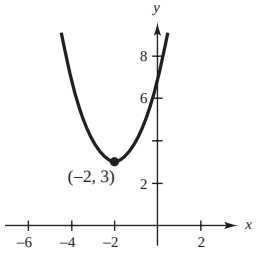
$0 = x^2 + 4x + 7$

$x = \dfrac{-4 \pm \sqrt{4^2 - 4(1)(7)}}{2(1)}$

$x = \dfrac{-4 \pm \sqrt{16 - 28}}{2}$

$x = \dfrac{-4 \pm \sqrt{-12}}{2}$

No x-intercepts



64. $y = 3x^2 - 6x + 4$

x-intercepts:

$0 = 3x^2 - 6x + 4$

$x = \dfrac{6 \pm \sqrt{(-6)^2 - 4(3)(4)}}{2(3)}$

$x = \dfrac{6 \pm \sqrt{36 - 48}}{6}$

$x = \dfrac{6 \pm \sqrt{-12}}{6}$

No x-intercepts

vertex:

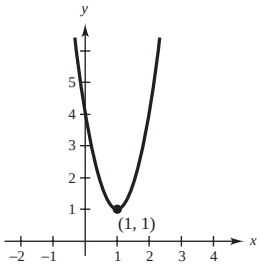
$y = 3x^2 - 6x + 4$

$= 3(x^2 - 2x) + 4$

$= 3(x^2 - 2x + 1 - 1) + 4$

$= 3(x^2 - 2x + 1) - 3 + 4$

$= 3(x - 1)^2 + 1$



66. $y = -\frac{1}{2}(x^2 - 6x + 7)$

x-intercepts:

$0 = -\frac{1}{2}(x^2 - 6x + 7)$

$x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(7)}}{2(1)}$

$x = \frac{6 \pm \sqrt{36 - 28}}{2}$

$x = \frac{6 \pm \sqrt{8}}{2}$

$x = \frac{6 \pm 2\sqrt{2}}{2}$

$x = 3 \pm \sqrt{2}$

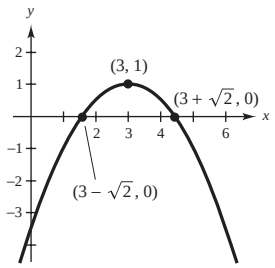
vertex:

$y = -\frac{1}{2}(x^2 - 6x + 7)$

$= -\frac{1}{2}(x^2 - 6x + 9 - 9 + 7)$

$= -\frac{1}{2}(x^2 - 6x + 9) + \frac{9}{2} - \frac{7}{2}$

$= -\frac{1}{2}(x - 3)^2 + 1$



68. $y = \frac{1}{5}(2x^2 - 4x + 7)$

x-intercepts:

$0 = \frac{1}{5}(2x^2 - 4x + 7)$

$x = \frac{4 \pm \sqrt{16 - 4(2)(7)}}{2(2)}$

$x = \frac{4 \pm \sqrt{16 - 56}}{4}$

$x = \frac{4 \pm \sqrt{-40}}{4}$

no x-intercepts

vertex:

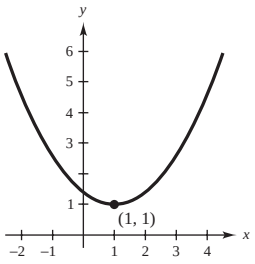
$y = \frac{1}{5}(2x^2 - 4x + 7)$

$= \frac{2}{5}(x^2 - 2x) + \frac{7}{5}$

$= \frac{2}{5}(x^2 - 2x + 1 - 1) + \frac{7}{5}$

$= \frac{2}{5}(x^2 - 2x + 1) - \frac{2}{5} + \frac{7}{5}$

$= \frac{2}{5}(x - 1)^2 + 1$



70. $f(x) = \frac{1}{3}x^2 - 2$

x-intercepts:

$0 = \frac{1}{3}x^2 - 2$

$2 = \frac{x^2}{3}$

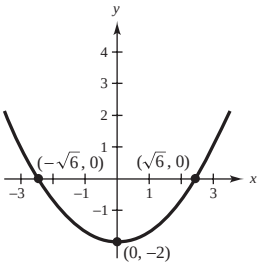
$6 = x^2$

$\pm\sqrt{6} = x$

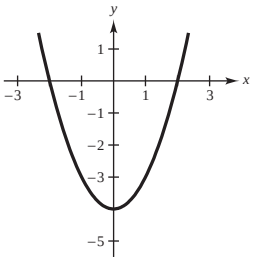
vertex:

$f(x) = \frac{1}{3}(x^2) - 2$

$f(x) = \frac{1}{3}(x - 0)^2 - 2$

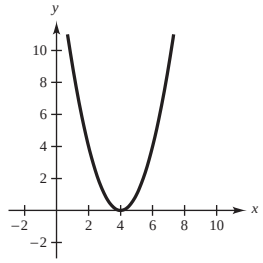


72. $h(x) = x^2 - 4$



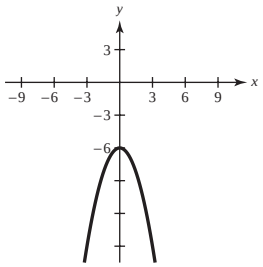
Vertical shift 4 units down

74. $h(x) = (x - 4)^2$



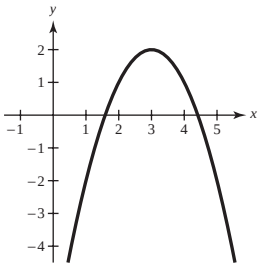
Horizontal shift 4 units right

76. $h(x) = -x^2 - 6$



Vertical shift 6 units down
Reflection in the x-axis

78. $h(x) = -(x - 3)^2 + 2$



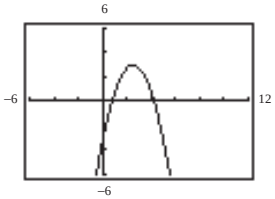
Horizontal shift 3 units right
Vertical shift 2 units down

80. $y = -\frac{1}{4}(4x^2 - 20x + 13)$

Keystrokes:

Y= ((−) 1 ÷ 4) (4 X,T,θ x² − 20 X,T,θ + 13) GRAPH

vertex $= \left(\frac{5}{2}, 3\right)$



Check:

$y = -x^2 + 5x - \frac{13}{4}$

$a = -1, \quad b = 5$

$x = \frac{-b}{2a} = \frac{-5}{2(-1)} = \frac{5}{2}$

$y = -\left(\frac{5}{2}\right)^2 + 5\left(\frac{5}{2}\right) - \frac{13}{4}$

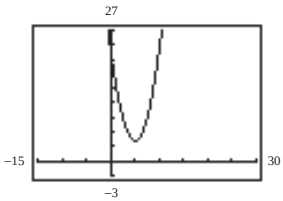
$= -\frac{25}{4} + \frac{25}{2} - \frac{13}{4} = \frac{-25 + 50 - 13}{4} = \frac{12}{4} = 3$

82. $y = 0.75x^2 - 7.50x + 23.00$

Keystrokes:

Y= .75 X,T,θ x² − 7.5 X,T,θ + 23 GRAPH

vertex $= (5, 4.25)$



Check:

$y = 0.75x^2 - 7.50x + 23$

$a = 0.75, \quad b = -7.5$

$x = \frac{-b}{2a} = \frac{-(-7.5)}{2(0.75)} = 5$

$y = 0.75(5)^2 - 7.5(5) + 23$

$= 18.75 - 37.5 + 23$

$= 4.25$

84. vertex $= (2, 0)$; point $= (0, 4)$

$4 = a(0 - 2)^2 + 0$ $\qquad$ $y = 1(x - 2)^2 + 0$

$4 = a(4)$ $\qquad$ $y = (x - 2)^2$

$1 = a$ $\qquad$ $y = x^2 - 4x + 4$

86. vertex $= (2, 6)$; point $= (0, 4)$

$y = a(x - 2)^2 + 6$ $\qquad$ $y = -\frac{1}{2}(x - 2)^2 + 6$

$4 = a(0 - 2)^2 + 6$ $\qquad$ $y = -\frac{1}{2}(x^2 - 4x + 4) + 6$

$4 = a(4) + 6$ $\qquad$ $y = -\frac{1}{2}x^2 + 2x - 2 + 6$

$-2 = a(4)$ $\qquad$ $y = -\frac{1}{2}x^2 + 2x + 4$

$-\frac{2}{4} = a$

88. vertex $= (-3, -3)$; $a = 1$

$y = a(x - h)^2 + k$

$y = 1(x - (-3))^2 + (-3)$

$y = (x + 3)^2 - 3$

$y = x^2 + 2x(3) + 3^2 - 3$

$y = x^2 + 6x + 9 - 3$

$y = x^2 + 6x + 6$

90. vertex $= (-2, -4)$; point $= (0, 0)$

$y = a(x - h)^2 + k$

$y = a(x - (-2))^2 + (-4)$

$y = a(x + 2)^2 - 4$

$0 = a(0 + 2)^2 - 4$

$0 = 4a - 4$

$4 = 4a$

$a = 1$

$y = (x + 2)^2 - 4$

$y = x^2 + 4x + 4 - 4$

$y = x^2 + 4x$

92. vertex $= (-1, -1)$; point $= (0, 4)$

$y = a(x - h)^2 + k$

$y = a(x - (-1))^2 + (-1)$

$y = a(x + 1)^2 - 1$

$4 = a(0 + 1)^2 - 1$

$4 = a - 1$

$5 = a$

$y = 5(x + 1)^2 - 1$

$y = 5(x^2 + 2x + 1) - 1$

$y = 5x^2 + 10x + 5 - 1$

$y = 5x^2 + 10x + 4$

94. vertex $= (5, 2)$; point $= (10, 3)$

$y = a(x - h)^2 + k$ $\qquad$ $y = \frac{1}{25}(x - 5)^2 + 2$

$y = a(x - 5)^2 + 2$ $\qquad$ $y = \frac{1}{25}(x^2 - 10x + 25) + 2$

$3 = a(10 - 5)^2 + 2$ $\qquad$ $y = \frac{1}{25}x^2 - \frac{2}{5}x + 1 + 2$

$3 = a(25) + 2$ $\qquad$ $y = \frac{1}{25}x^2 - \frac{2}{5}x + 3$

$1 = a(25)$

$\frac{1}{25} = a$

96. $y = -\frac{1}{16}x^2 + 2x + 5$

(a) $y = -\frac{1}{16}(0)^2 + 2(0) + 5$

$= 5$ feet

(b) $y = -\frac{1}{16}(x^2 - 32x + 256) + 5 + 16$

$= -\frac{1}{16}(x - 16)^2 + 21$

Maximum height $= 21$ feet

(c) $0 = -\frac{1}{16}x^2 + 2x + 5$

$0 = x^2 - 32x - 80$

$x = \frac{-(-32) \pm \sqrt{(-32)^2 - 4(1)(-80)}}{2(1)}$

$x = \frac{32 \pm \sqrt{1024 + 320}}{2}$

$= \frac{32 \pm \sqrt{1344}}{2} = \frac{32 \pm 8\sqrt{21}}{2}$

$= 16 + 4\sqrt{21} \approx 34.3$ feet

98. $y = -\frac{1}{90}x^2 + \frac{1}{5}x + 9$

(a) $y = -\frac{1}{90}(0)^2 + \frac{1}{5}(0) + 9$

$y = 9$ feet

(b) $y = -\frac{1}{90}(x^2 - 18x + 81) + 9 + \frac{9}{10}$

$y = -\frac{1}{90}(x - 9)^2 + 9\frac{9}{10}$

Maximum height $= 9\frac{9}{10}$ feet

(c) $0 = -\frac{1}{90}x^2 + \frac{1}{5}x + 9$

$0 = x^2 - 18x - 810$

$x = \frac{-(-18) \pm \sqrt{(-18)^2 - 4(1)(-810)}}{2(1)}$

$x = \frac{18 \pm \sqrt{324 + 3240}}{2}$

$x = \frac{18 \pm \sqrt{3564}}{2}$

$x \approx 38.8; -20.8$ reject

$x \approx 38.8$ feet

100. $y = -\frac{1}{70}x^2 + 2x + 2$

(a) $y = -\frac{1}{70}(0)^2 + 2(0) + 2$

$y = 2$ feet

(b) $y = -\frac{1}{70}(x^2 - 140x + 4900) + 2 + 70$

$y = -\frac{1}{70}(x - 70)^2 + 72$

Maximum height $= 72$ feet

(c) $0 = -\frac{1}{70}x^2 + 2x + 2$

$0 = x^2 - 140x - 140$

$x = \frac{-(-140) \pm \sqrt{(-140)^2 - 4(1)(-140)}}{2(1)}$

$x = \frac{140 \pm \sqrt{19{,}600 + 560}}{2}$

$x = \frac{140 \pm \sqrt{20{,}160}}{2}$

$x \approx 140.99; -0.99$ reject

$x \approx 140.99$ feet

102. $y = \frac{-4}{3}x^2 + \frac{10}{3}x + 10$

$y = \frac{-4}{3}\left(x^2 - \frac{5}{2}x + \frac{25}{16}\right) + 10 + \frac{25}{12}$

$y = \frac{-4}{3}\left(x - \frac{5}{4}\right)^2 + \frac{145}{12}$

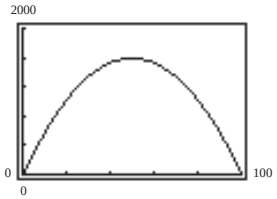
Maximum height $= \frac{145}{12} \approx 12.08$ feet

104. $A = \frac{2}{\pi}(100x - x^2)$

Keystrokes:

Y= (2 ÷ π) (100 X,T,θ − X,T,θ x²) GRAPH

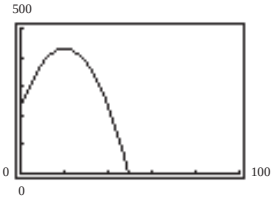
$x \approx 50$ when A is maximum.



106. $P = 230 + 20s - \frac{1}{2}s^2$

Keystrokes:

Y= 230 + 20 X,T,θ − .5 X,T,θ x² GRAPH



$P = 230 + 20s - \frac{1}{2}s^2$

$P = -\frac{1}{2}s^2 + 20s + 230$

$P = -\frac{1}{2}(s^2 - 40s + 400) + 230 + 200$

$P = -\frac{1}{2}(s - 20)^2 + 430$

Amount of advertising that yields a maximum profit $= \$2000$

Maximum profit $= \$430{,}000$

108. $100 = a(500 - 0)^2 + 0$

$100 = a(250{,}000)$

$\frac{100}{250{,}000} = a$

$\frac{1}{2500} = a$

$y = \frac{1}{2500}(x - 0)^2 + 0$

$y = \frac{1}{2500}x^2$

110. To find the vertex of the graph of a quadratic function, use the method of completing the square to write the function in standard form $f(x) = a(x - h)^2 + k$. The vertex is located at point (h, k).

112. The graph of a quadratic function $f(x) = ax^2 + bx + c$ opens upward if $a > 0$ and opens downward if $a < 0$.

114. It is not possible for the graph of a quadratic function to have two y-intercepts. All functions must have only one y-value for each x-value.

Section 8.5 Applications of Quadratic Equations

2. *Verbal Model:* [Selling price per computer] = [Cost per computer] + [Profit per computer]

Equation: $\frac{27{,}000}{x} = \frac{27{,}000}{x + 3} + 750$

$27{,}000(x + 3) = 27{,}000x + 750x(x + 3)$

$27{,}000x + 81{,}000 = 27{,}000x + 750x^2 + 2250x$

$0 = 750x^2 + 2250x - 81{,}000$

$0 = x^2 + 3x - 108$

$0 = (x + 12)(x - 9)$

$x = -12 \quad x = 9$ computers

Selling price $= \frac{27{,}000}{9} = \$3000$

4. *Verbal Model:* [Selling price per sweatshirt] = [Cost per sweatshirt] + [Profit per sweatshirt]

Equation: $\frac{850}{x} = \frac{850}{x + 16} + 8$

$850(x + 16) = 850x + 8x(x + 16)$

$850x + 13600 = 850x + 8x^2 + 128x$

$0 = 8x^2 + 128x - 13600$

$0 = x^2 + 16x - 1700$

$0 = (x + 50)(x - 34)$

$x = -50 \quad x = 34$ sweatshirts

reject Selling price $= \frac{850}{34} = \$25$

6. *Verbal Model:* $2\boxed{\text{Length}} + 2\boxed{\text{Width}} = \boxed{\text{Perimeter}}$

Equation: $2(3.5w) + 2w = 60$

$7w + 2w = 60$

$9w = 60$

$w = \frac{20}{3}$

$l = 3.5w = \frac{70}{3}$

Verbal Model: $\boxed{\text{Length}} \cdot \boxed{\text{Width}} = \boxed{\text{Area}}$

Equation: $\frac{70}{3} \cdot \frac{20}{3} = A$

$\frac{1400}{9} = 155\frac{5}{9}\ \text{m}^2 = A$

8. *Verbal Model:* $\boxed{\text{Length}} \cdot \boxed{\text{Width}} = \boxed{\text{Area}}$

Equation: $1.5w \cdot w = 216$

$1.5w^2 = 216$

$w^2 = 144$

$w = 12$

$l = 1.5w = 12$

Verbal Model: $2\boxed{\text{Length}} + 2\boxed{\text{Width}} = \boxed{\text{Perimeter}}$

Equation: $2(18) + 2(12) = P$

$60 \text{ centimeters} = P$

10. *Verbal Model:* $\boxed{\text{Length}} \cdot \boxed{\text{Width}} = \boxed{\text{Area}}$

Equation: $l \cdot \frac{3}{4}l = 2700$

$\frac{3}{4}l^2 = 2700$

$l^2 = 3600$

$l = 60$

$w = \frac{3}{4}l = 45$

Verbal Model: $2\boxed{\text{Length}} + 2\boxed{\text{Width}} = \boxed{\text{Perimeter}}$

Equation: $2(60) + 2(45) = P$

$210 \text{ inches} = P$

12. *Verbal Model:* $2\boxed{\text{Length}} + 2\boxed{\text{Width}} = \boxed{\text{Perimeter}}$

Equation: $2l + 2(l - 6) = 108$

$2l + 2l - 12 = 108$

$4l = 120$

$l = 30$

$w = l - 6 = 24$

Verbal Model: $\boxed{\text{Length}} \cdot \boxed{\text{Width}} = \boxed{\text{Area}}$

Equation: $30 \cdot 24 = A$

$720 \text{ ft}^2 = A$

14. *Verbal Model:* $\boxed{\text{Area}} = \boxed{\text{Length}} \cdot \boxed{\text{Width}}$

Equation: $500 = (w + 5) \cdot w$

$500 = w^2 + 5w$

$0 = w^2 + 5w$

$0 = (w + 25)(w - 20)$

$w = -25 \qquad w = 20$

reject $\qquad l = w + 5 = 25$

Verbal Model: $2\boxed{\text{Length}} + 2\boxed{\text{Width}} = \boxed{\text{Perimeter}}$

Equation: $2(25) + 2(20) = P$

$90 \text{ feet} = P$

16. *Verbal Model:* $\boxed{\text{Area}} = \frac{1}{2} \cdot \boxed{\text{Height}} \cdot \boxed{\text{Base}}$

Labels: Height $= x - 8$

Base $= x$

Equation: $192 = \frac{1}{2}(x - 8)x$

$384 = x^2 - 8x$

$0 = x^2 - 8x - 384$

$0 = (x - 24)(x + 16)$

$x = 24 \text{ inches} \quad \text{reject } x = -16$

$x - 8 = 16 \text{ inches}$

18. *Verbal Model:* $\boxed{\text{Length}} \cdot \boxed{\text{Width}} = \boxed{\text{Area}}$

Equation: $(x + 20) \cdot x = 25{,}500$

$x^2 + 20x = 25{,}500$

$x^2 + 20x - 25{,}500 = 0$

$(x - 150)(x + 170) = 0$

$x - 150 = 0 \qquad x + 170 = 0$

$x = 150 \qquad x = -170$

The lot is $150 + 20 = 170$ feet $\times$ 150 feet.

20. (a) *Verbal Model:* $\boxed{\text{Area}} = \boxed{\text{Length}} \cdot \boxed{\text{Width}}$

Equation: $630 = x \cdot (50 - x)$

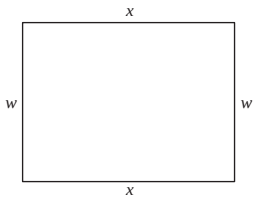
$630 = 50x - x^2$

$x^2 - 50x + 630 = 0$

$x = \dfrac{-(-50) \pm \sqrt{(-50)^2 - 4(1)(630)}}{2(1)}$

$x = \dfrac{50 \pm \sqrt{2500 - 2520}}{2} = \dfrac{50 \pm \sqrt{-20}}{2}$

Not real therefore cannot enclose a rectangular region.



$2x + 2w = 100$

$x + w = 50$

$w = 50 - x$

(b) *Verbal Model:* $\boxed{\text{Area}} = \pi \cdot \boxed{\text{Radius}}^2$

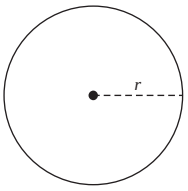
Equation: $A = \pi\left(\dfrac{50}{\pi}\right)^2$

$A = \pi \cdot \dfrac{2500}{\pi^2}$

$A = \dfrac{2500}{\pi}$

$A \approx 796$ square feet

Yes, can enclose a circular area.



$C = 2\pi r$

$100 = 2\pi r$

$\dfrac{100}{2\pi} = r$

$\dfrac{50}{\pi} = r$

22. *Verbal Model:* $\boxed{\text{Length}} \cdot \boxed{\text{Width}} = \boxed{\text{Area}}$

Labels: Length $= 8 - 2x$

Width $= 6 - 2x$

Equation: $(8 - 2x)(6 - 2x) = \frac{1}{2}(48)$

$48 - 28x + 4x^2 = 24$

$4x^2 - 28x + 24 = 0$

$x^2 - 7x + 6 = 0$

$(x - 6)(x - 1) = 0$

$x = 6$ $\quad$ $x = 1$

reject $\quad$ $2x = 2$ inches

Each side should be reduced by 2 inches.

24. $A = P(1 + r)^2$

$3499.20 = 3000(1 + r)^2$

$1.1664 = (1 + r)^2$

$1.08 = 1 + r$

$0.08 = r$ or 8%

26. $A = P(1 + r)^2$

$280.90 = 250.00(1 + r)^2$

$\dfrac{280.90}{250.00} = (1 + r)^2$

$1.1236 = (1 + r)^2$

$1.06 = 1 + r$

$0.06 = r$

$6\% = r$

28. $A = P(1 + r)^2$

$8421.41 = 8000.00(1 + r)^2$

$1.052676 = (1 + r)^2$

$1.0260 \approx 1 + r$

$0.0260 \approx r$ or 2.6%

30. *Verbal Model:* $\boxed{\text{Cost per member}} \cdot \boxed{\text{Number of members}} = \boxed{\$240}$

Labels: Number of members $= x$

Number going to game $= x + 8$

Equation: $\left(\frac{240}{x} - 1\right) \cdot (x + 8) = 240$

$$\left(\frac{240 - x}{x}\right)(x + 8) = 240$$

$$(240 - x)(x + 8) = 240x$$

$$240x + 1920 - x^2 - 8x = 240x$$

$$-x^2 - 8x + 1920 = 0$$

$$x^2 + 8x - 1920 = 0$$

$$(x + 48)(x - 40) = 0$$

$$x = -48 \quad x = 40$$

$$x + 8 = 48$$

32. *Verbal Model:* $\boxed{\text{Investment per person; current group}} - \boxed{\text{Investment per person; new group}} = \boxed{6000}$

Labels: Number in current group $= x$

Number in new group $= x + 3$

Equation: $\frac{80{,}000}{x} - \frac{80{,}000}{x + 3} = 6000$

$$x(x + 3)\left(\frac{80{,}000}{x} - \frac{80{,}000}{x + 3}\right) = (6000)x(x + 3)$$

$$80{,}000(x + 3) - 80{,}000x = 6000(x^2 + 3x)$$

$$80{,}000x + 240{,}000 - 80{,}000x = 6000x^2 + 18{,}000x$$

$$0 = 6000x^2 + 18{,}000x - 240{,}000$$

$$0 = x^2 + 3x - 40$$

$$0 = (x + 8)(x - 5)$$

$x + 8 = 0$ $\qquad$ $x - 5 = 0$

~~$x = -8$~~ $\qquad$ $x = 5$ investors

34. *Common formula:* $a^2 + b^2 + c^2$

Equation: $x^2 + (100 - x)^2 = 80^2$

$$x^2 + 10{,}000 - 200x + x^2 = 6400$$

$$2x^2 - 200x + 3600 = 0$$

$$x^2 - 100x + 1800 = 0$$

$$x = \frac{-(-100) \pm \sqrt{(-100)^2 - 4(1)(1800)}}{2(1)}$$

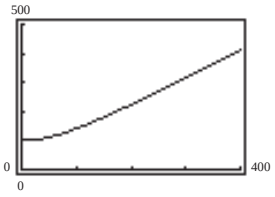
$$x = \frac{100 \pm \sqrt{10{,}000 - 7200}}{2} = \frac{100 \pm \sqrt{2800}}{2} \approx 76.5 \text{ yards}, 23.5 \text{ yards}$$

36. (a) $d = \sqrt{100^2 + h^2}$

$d = \sqrt{10{,}000 + h^2}$

(b) *Keystrokes:*

Y= $\sqrt{\ }$ (10,000 + X,T,θ x^2) GRAPH



(c) When $d = 200$ feet h is approximately 173.2 feet.

(d)

h	0	100	200	300
d	100	141.4	223.6	316.2

$d = \sqrt{10{,}000 + 0^2}$
$= \sqrt{10{,}000}$
$= 100$

$d = \sqrt{10{,}000 + 100^2}$
$= \sqrt{20{,}000}$
≈ 141.4

$d = \sqrt{10{,}000 + 200^2}$
$= \sqrt{10{,}000 + 40{,}000}$
$= \sqrt{50{,}000}$
≈ 223.6

$d = \sqrt{10{,}000 + 300^2}$
$= \sqrt{10{,}000 + 90{,}000}$
$= \sqrt{100{,}000}$
≈ 316.2

38. *Verbal Model:* [Work done by Person 1] + [Work done by Person 2] = [One complete job]

Equation: $\frac{1}{x}(6) + \frac{1}{x+2}(6) = 1$

$\frac{6}{x} + \frac{6}{x+2} = 1$

$x(x+2)\left[\frac{6}{x} + \frac{6}{x+2}\right] = x(x+2)$

$6(x+2) + 6x = x^2 + 2x$

$6x + 12 + 6x = x^2 + 2x$

$12x + 12 = x^2 + 2x$

$0 = x^2 - 10x - 12$

$x = \frac{10 \pm \sqrt{(-10)^2 - 4(1)(-12)}}{2(1)}$

$x = \frac{10 \pm \sqrt{100 + 48}}{2}$

$x = \frac{10 \pm \sqrt{148}}{2}$

$x = \frac{10 \pm 2\sqrt{37}}{2} = 5 \pm \sqrt{37} \approx 11.1, -1.1$

Thus, it would take person one ≈ 11.1 hours to complete the task alone and it would take person two $11.1 + 2 \approx 13.1$ hours.

40. *Verbal Model:* [Rate Company A] + [Rate Company B] = [Rate together]

Labels: Time Company A $= x + 3$

Time Company B $= x$

Equation: $\frac{1}{x+3} + \frac{1}{x} = \frac{1}{4}$

$4x(x+3)\left(\frac{1}{x+3} + \frac{1}{x}\right) = \left(\frac{1}{4}\right)4x(x+3)$

$4x + 4(x+3) = x(x+3)$

$4x + 4x + 12 = x^2 + 3x$

$0 = x^2 - 5x - 12$

$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-12)}}{2(1)}$

$x = \frac{5 \pm \sqrt{25 + 48}}{2}$

$x = \frac{5 \pm \sqrt{73}}{2}$

$x \approx 6.8$ days ~~$x \approx -1.8$~~

$x + 3 \approx 9.8$ days

42. $h = h_0 - 16t^2$

$0 = 729 - 16t^2$

$16t^2 = 729$

$t^2 = 45.5625$

$t = 6.75$ or $6\frac{3}{4}$ seconds

44. $h = h_0 - 16t^2$

$0 = 984 - 16t^2$

$16t^2 = 984$

$t^2 = 61.5$

$t \approx 7.8$ seconds

46. $4 = 5 + 25t - 16t^2$

$16t^2 - 25t - 1 = 0$

$$t = \frac{25 \pm \sqrt{(-25)^2 - 4(16)(-1)}}{2(16)}$$

$$t = \frac{25 \pm \sqrt{625 + 64}}{32}$$

$$t = \frac{25 \pm \sqrt{689}}{32}$$

$$t \approx \frac{25 \pm 26.25}{32}$$

$t \approx 1.6, -0.04$

1.6 seconds will pass before you hit the ball.

48. $h = -16t^2 + 21t + 5$

(a) $11 = -16t^2 + 21t + 5$

$0 = -16t^2 + 21t - 6$

$$t = \frac{-21 \pm \sqrt{21^2 - 4(-16)(-6)}}{2(-16)}$$

$$t = \frac{-21 \pm \sqrt{441 - 384}}{-32}$$

$$t = \frac{-21 \pm \sqrt{57}}{-32} \approx 0.42 \text{ second and } 0.89 \text{ second}$$

(b) $0 = -16t^2 + 21t + 5$

$$t = \frac{-21 \pm \sqrt{21^2 - 4(-16)(5)}}{2(-16)}$$

$$t = \frac{-21 \pm \sqrt{441 + 320}}{-32}$$

$$t = \frac{-21 \pm \sqrt{761}}{-32}$$

$t \approx -0.21$ reject $t \approx 1.52$ seconds

(c) $t = \dfrac{-21}{2(-16)} = \dfrac{21}{32}$

$$h = -16\left(\frac{21}{32}\right)^2 + 21\left(\frac{21}{32}\right) + 5 \approx 11.89 \text{ feet}$$

Maximum height is 11.89 feet.

50. *Verbal Model:* [Integer] · [Integer] = [Product]

Equation: $n \cdot (n + 1) = 1806$

$n^2 + n - 1806 = 0$

$(n + 43)(n - 42) = 0$

$n + 43 = 0$ $\quad$ $n - 42 = 0$

reject $\begin{cases} n = -43 & n = 42 \\ n + 1 = -42 & n + 1 = 43 \end{cases}$

52. *Verbal Model:* [Even integer] · [Even integer] = [Product]

Equation: $2n(2n + 2) = 2808$

$4n^2 + 4n = 2808$

$4n^2 + 4n - 2808 = 0$

$4(n^2 + n - 702) = 0$

$4(n + 27)(n - 26) = 0$

$n + 27 = 0$ $\quad$ $n - 26 = 0$

$\cancel{n = -27}$ $\quad$ $n = 26$

reject $\quad$ $2n = 52$

$2n + 2 = 54$

54. *Verbal Model:* [Odd integer] · [Odd integer] = [Product]

Equation: $(2n + 1)(2n + 3) = 1443$

$4n^2 + 8n + 3 = 1443$

$4n^2 + 8n - 1440 = 0$

$n^2 + 2n - 360 = 0$

$(n + 20)(n - 18) = 0$

$n + 20 = 0$ $\qquad$ $n - 18 = 0$

$n = -20$ $\qquad$ $n = 18$

reject $\qquad$ $2n + 1 = 37$

$2n + 3 = 39$

The integers are 37 and 39.

56. *Verbal Model:* [Time for part 1] + [Time for part 2] = 5

Equation: $\dfrac{100}{r} + \dfrac{135}{r - 5} = 5$

$r(r - 5)\left[\dfrac{100}{r} + \dfrac{135}{r - 5}\right] = 5r(r - 5)$

$100(r - 5) + 135r = 5r^2 - 25r$

$100r - 500 + 135r = 5r^2 - 25r$

$0 = 5r^2 - 260r + 500$

$0 = 5(r^2 - 52r + 100)$

$0 = 5(r - 50)(r - 2)$

$r - 50 = 0$ $\qquad$ $r - 2 = 0$

$r = 50$ $\qquad$ $r = 2$ reject

Thus, the average speed for the first part of the trip was 50 miles per hour.

58. $d_1 = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$13 = \sqrt{(x_1 - 1)^2 + (14 - 2)^2}$

$169 = (x_1 - 1)^2 + 144$

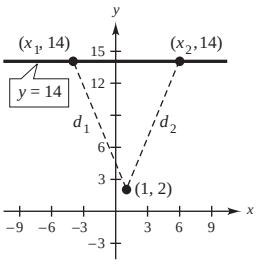
$25 = (x_1 - 1)^2$

$\pm\sqrt{25} = (x_1 - 1)$

$1 \pm 5 = x_1$

$x_1 = 6$ $\qquad$ $x_1 = -4$

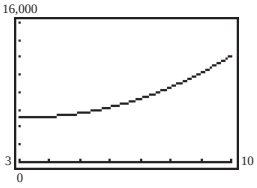
$(6, 14)$ $\qquad$ $(-4, 14)$



60. $S = 156.45t^2 - 1035.5t + 6875$

(a) *Keystrokes:*

Y= 156.45 X,T,θ x² − 1035.5 X,T,θ + 6875 GRAPH



(b) Sales were approximately \$6.3 billion (6300 million) in the year 1996.

$6300 = 156.45t^2 - 1035.5t + 6875$

$0 = 156.45t^2 - 1035.5t + 575$

Use the Quadratic Formula to solve for t.

62. The four strategies that can be used to solve a quadratic equation are factoring, the Square Root Property, completing the square and the Quadratic Formula.

64. $\dfrac{20 \text{ feet}}{\cancel{\text{minute}}} \cdot \dfrac{1 \cancel{\text{minute}}}{60 \cancel{\text{seconds}}} \cdot (45 \cancel{\text{seconds}}) = 15 \text{ feet}$

Section 8.6 Quadratic and Rational Inequalities

2. $5x(x-3)=0$

$5x=0 \qquad x-3=0$

$x=0 \qquad x=3$

Critical numbers $=0, 3$

4. $9y^2-16=0$

$(3y-4)(3y+4)=0$

$3y-4=0 \qquad 3y+4=0$

$3y=4 \qquad 3y=-4$

$y=\frac{4}{3} \qquad y=-\frac{4}{3}$

Critical numbers $=\frac{4}{3}, -\frac{4}{3}$

6. $y(y-4)-3(y-4)=0$

$(y-4)(y-3)=0$

$y-4=0 \qquad y-3=0$

$y=4 \qquad y=3$

Critical numbers: 3, 4

8. $3x^2-2x-8=0$

$(3x+4)(x-2)=0$

$3x+4=0 \qquad x-2=0$

$3x=-4 \qquad x=2$

$x=-\frac{4}{3}$

Critical numbers $=-\frac{4}{3}, 2$

10. $4x^2-4x-3=0$

$(2x+1)(2x-3)=0$

$2x+1=0 \qquad 2x-3=0$

$2x=-1 \qquad 2x=3$

$x=-\frac{1}{2} \qquad x=\frac{3}{2}$

Critical numbers: $-\frac{1}{2}, \frac{3}{2}$

12. $3-x$

Positive: $(-\infty, 3)$

Negative: $(3, \infty)$

14. $\frac{2}{3}x-8$

Negative: $(-\infty, 12)$

Positive: $(12, \infty)$

16. $7x(3-x)$

Negative: $(-\infty, 0)$

Positive: $(0, 3)$

Negative: $(3, \infty)$

18. $x^2-9=(x-3)(x+3)$

Positive: $(-\infty, -3)$

Negative: $(-3, 3)$

Positive: $(3, \infty)$

20. $2x^2-4x-3$

$$x=\frac{4\pm\sqrt{(-4)^2-4(2)(-3)}}{2(2)}$$

$$x=\frac{4\pm\sqrt{16+24}}{4}$$

$$x=\frac{4\pm\sqrt{40}}{4}$$

$$x=\frac{4\pm 2\sqrt{10}}{4}$$

$$x=\frac{2\left(2\pm\sqrt{10}\right)}{4}$$

$$x=\frac{2\pm\sqrt{10}}{2}$$

Positive: $\left(-\infty, \frac{2-\sqrt{10}}{2}\right)$

Negative: $\left(\frac{2-\sqrt{10}}{2}, \frac{2+\sqrt{10}}{2}\right)$

Positive: $\left(\frac{2+\sqrt{10}}{2}, \infty\right)$

22. $2x(x-6)>0$

Critical numbers: $x=0, 6$

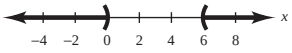
Test intervals:

Positive: $(-\infty, 0)$

Negative: $(0, 6)$

Positive: $(6, \infty)$

Solution: $(-\infty, 0)\cup(6, \infty)$



24. $2x(6-x)>0$

Critical numbers: 0, 6

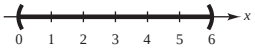
Test intervals:

Negative: $(-\infty, 0)$

Positive: $(0, 6)$

Negative: $(6, \infty)$

Solution: $(0, 6)$



26. $z^2\le 9$

$z^2-9\le 0$

$(z-3)(z+3)\le 0$

Critical numbers: $z=-3, 3$

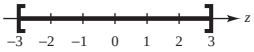
Test intervals:

Positive: $(-\infty, -3)$

Negative: $(-3, 3)$

Positive: $(3, \infty)$

Solution: $[-3, 3]$



28. $x^2 + 8x + 7 < 0$

$(x + 7)(x + 1) < 0$

Critical numbers: $x = -7, -1$

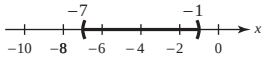
Test intervals:

Positive: $(-\infty, -7)$

Negative: $(-7, -1)$

Positive: $(-1, \infty)$

Solution: $(-7, -1)$



30. $x^2 - 5x \geq 0$

$x(x - 5) \geq 0$

Critical numbers: $x = 0, 5$

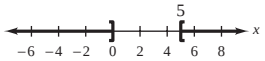
Test intervals:

Positive: $(-\infty, 0)$

Negative: $(0, 5)$

Positive: $(5, \infty)$

Solution: $(-\infty, 0] \cup [5, \infty)$



32. $t^2 - 4t > 12$

$t^2 - 4t - 12 > 0$

$(t - 6)(t + 2) > 0$

Critical numbers: $x = 6, -2$

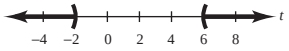
Test intervals:

Positive: $(-\infty, -2)$

Negative: $(-2, 6)$

Positive: $(6, \infty)$

Solution: $(-\infty, -2) \cup (6, \infty)$



34. $t^2 - 15t + 50 < 0$

$(t - 10)(t - 5) < 0$

Critical numbers: 5, 10

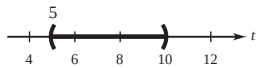
Test intervals:

Positive: $(-\infty, 5)$

Negative: $(5, 10)$

Positive: $(10, \infty)$

Solution: $(5, 10)$



36. $x^2 + 6x + 10 > 0$

$$x = \frac{-6 \pm \sqrt{36 - 4(10)}}{2}$$

$$x = \frac{-6 \pm \sqrt{-4}}{2}$$

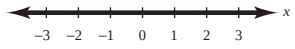
$$x = \frac{-6 \pm 2i}{2}$$

$x = -3 \pm i$

No critical numbers

$x^2 + 6x + 10$ is greater than 0 for all values of x.

Solution: $(-\infty, \infty)$



38. $y^2 - 5y + 6 > 0$

$(y - 3)(y - 2) > 0$

Critical numbers: $y = 2, 3$

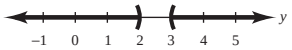
Test intervals:

Positive: $(-\infty, 2)$

Negative: $(2, 3)$

Positive: $(3, \infty)$

Solution: $(-\infty, 2) \cup (3, \infty)$



40. $-x^2 + 8x - 11 \leq 0$

$0 \leq x^2 - 8x + 11$

$$x = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(11)}}{2(1)}$$

$$x = \frac{8 \pm \sqrt{64 - 44}}{2}$$

$$x = \frac{8 \pm \sqrt{20}}{2}$$

$$x = \frac{8 \pm 2\sqrt{5}}{2}$$

$$x = \frac{2\left(4 \pm \sqrt{5}\right)}{2}$$

$x = 4 \pm \sqrt{5}$

Critical numbers: $x = 4 \pm \sqrt{5}$

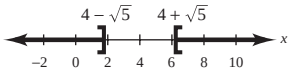
Test intervals:

Positive: $\left(-\infty, 4 - \sqrt{5}\right)$

Negative: $\left(4 - \sqrt{5}, 4 + \sqrt{5}\right)$

Positive: $\left(4 + \sqrt{5}, \infty\right)$

Solution: $\left(-\infty, 4 - \sqrt{5}\right] \cup \left[4 + \sqrt{5}, \infty\right)$



42. $x^2 + 8x + 16 < 0$

$(x + 4)^2 < 0$

$(x + 4)^2$ is not less than zero for any value of x.

Solution: none

44. $y^2 + 16y + 64 \le 0$

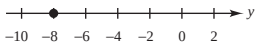
$(y + 8)^2 \le 0$

Critical number: $y = -8$

Test intervals:

Positive: $(-\infty, \infty)$

Solution: -8



46. $2t^2 - 3t - 20 \ge 0$

$(2t + 5)(t - 4) \ge 0$

Critical numbers: $t = -\frac{5}{2}, 4$

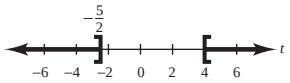
Test intervals:

Positive: $\left(-\infty, -\frac{5}{2}\right]$

Negative: $\left[-\frac{5}{2}, 4\right]$

Positive: $[4, \infty)$

Solution: $\left(-\infty, -\frac{5}{2}\right] \cup [4, \infty)$



48. $4x^2 - 4x - 63 < 0$

$(2x + 7)(2x - 9) < 0$

Critical numbers: $x = -\frac{7}{2}, \frac{9}{2}$

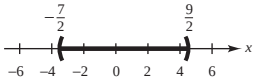
Test intervals:

Positive: $\left(-\infty, -\frac{7}{2}\right)$

Negative: $\left(-\frac{7}{2}, \frac{9}{2}\right)$

Positive: $\left(\frac{9}{2}, \infty\right)$

Solution: $\left(-\frac{7}{2}, \frac{9}{2}\right)$



50. $-3x^2 - 4x + 4 \le 0$

$0 \le 3x^2 + 4x - 4$

$0 \le (3x - 2)(x + 2)$

Critical numbers: $x = -2, \frac{2}{3}$

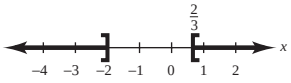
Test intervals:

Positive: $(-\infty, -2)$

Negative: $\left(-2, \frac{2}{3}\right)$

Positive: $\left(\frac{2}{3}, \infty\right)$

Solution: $(-\infty, -2] \cup \left[\frac{2}{3}, \infty\right)$

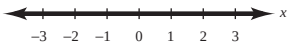


52. $9x^2 - 24x + 16 \ge 0$

$(3x - 4)^2 \ge 0$

$(3x - 4)^2 \ge 0$ for all real numbers.

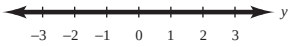
Solution: $(-\infty, \infty)$



54. $(y + 3)^2 \ge 0$

$(y + 3)^2 \ge 0$ for all real numbers.

Solution: $(-\infty, \infty)$



56. $(y + 3)^2 - 6 \ge 0$

$y^2 + 6y + 9 - 6 \ge 0$

$y^2 + 6y + 3 \ge 0$

$$y = \frac{-6 \pm \sqrt{6^2 - 4(1)(3)}}{2(1)}$$

$$y = \frac{-6 \pm \sqrt{36 - 12}}{2}$$

$$y = \frac{-6 \pm \sqrt{24}}{2}$$

$$y = \frac{-6 \pm 2\sqrt{6}}{2}$$

$$y = \frac{2\left(-3 \pm \sqrt{6}\right)}{2}$$

$$y = -3 \pm \sqrt{6}$$

Critical numbers: $x = -3 \pm \sqrt{6}$

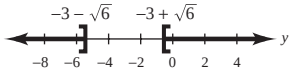
Test intervals:

Positive: $\left(-\infty, -3 - \sqrt{6}\right)$

Negative: $\left(-3 - \sqrt{6}, -3 + \sqrt{6}\right)$

Positive: $\left(-3 + \sqrt{6}, \infty\right)$

Solution: $\left(-\infty, -3 - \sqrt{6}\right] \cup \left[-3 + \sqrt{6}, \infty\right)$



58. $25 \geq (x-3)^2$

$25 \geq x^2 - 6x + 9$

$0 \geq x^2 - 6x - 16$

$0 \geq (x-8)(x+2)$

Critical numbers: $-2, 8$

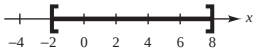
Test intervals:

Positive: $(-\infty, -2)$

Negative: $(-2, 8)$

Positive: $(8, \infty)$

Solution: $[-2, 8]$



60. $x(x-1)(x+4) \leq 0$

Critical numbers: $-4, 0, 1$

Test intervals:

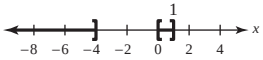
Negative: $(-\infty, -4)$

Positive: $(-4, 0)$

Negative: $(0, 1)$

Positive: $(1, \infty)$

Solution: $(-\infty, -4] \cup [0, 1]$



62. *Keystrokes:*

Y= 2 X,T,θ x² + 5 X,T,θ TEST 3 0 GRAPH

$2x^2 + 5x > 0$

$x(2x+5) > 0$

Critical numbers: $x = -\frac{5}{2}, 0$

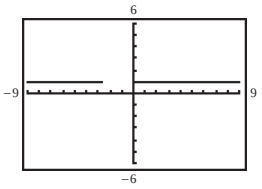
Test intervals:

Positive: $\left(-\infty, -\frac{5}{2}\right)$

Negative: $\left(-\frac{5}{2}, 0\right)$

Positive: $(0, \infty)$

Solution: $\left(-\infty, -\frac{5}{2}\right) \cup (0, \infty)$



64. *Keystrokes:*

Y= (1 ÷ 3) X,T,θ x² − 3 X,T,θ TEST 5 0 GRAPH

$\frac{1}{3}x^2 - 3x < 0$

$\frac{1}{3}x(x-9) < 0$

Critical numbers: $x = 0, 9$

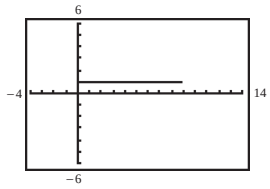
Test intervals:

Positive: $(-\infty, 0)$

Negative: $(0, 9)$

Positive: $(9, \infty)$

Solution: $(0, 9)$



66. *Keystrokes:*

Y= X,T,θ x² − 6 X,T,θ + 9 TEST 5 16 GRAPH

$x^2 - 6x + 9 < 16$

$x^2 - 6x - 7 < 0$

$(x-7)(x+1) < 0$

Critical numbers: $x = -1, 7$

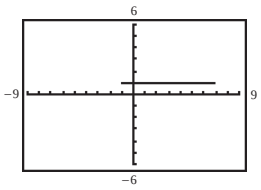
Test intervals:

Positive: $(-\infty, -1)$

Negative: $(-1, 7)$

Positive: $(7, \infty)$

Solution: $(-1, 7)$



68. *Keystrokes:*

Y= 8 X,T,θ − X,T,θ x² TEST 3 12 GRAPH

$8x - x^2 > 12$

$-x^2 + 8x - 12 > 0$

$x^2 - 8x + 12 < 0$

$(x-6)(x-2) < 0$

Critical numbers: $x = 2, 6$

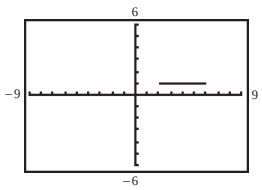
Test intervals:

Positive: $(-\infty, 2)$

Negative: $(2, 6)$

Positive: $(6, \infty)$

Solution: $(2, 6)$



70. $x + 2 = 0$

$x = -2$

Critical number: $x = -2$

72. $x - 2 = 0$ or $x - 10 = 0$

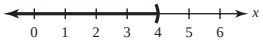
$x = 2$ $x = 10$

Critical numbers: $x = 2, 10$

74. $\dfrac{3}{4-x} > 0$

Critical number: $x = 4$

Test intervals:



Positive: $(-\infty, 4)$

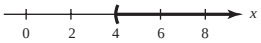
Negative: $(4, \infty)$

Solution: $(-\infty, 4)$

76. $\dfrac{-3}{4-x} > 0$

Critical number: $x = 4$

Test intervals:



Positive: $(-\infty, 4)$

Negative: $(4, \infty)$

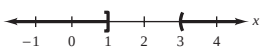
Solution: $(4, \infty)$

78. $\dfrac{2}{x-3} \geq -1$

$\dfrac{2}{x-3} + 1 \geq 0$

$\dfrac{2+x-3}{x-3} \geq 0$

$\dfrac{x-1}{x-3} \geq 0$



Critical numbers: $x = 1, 3$

Test intervals:

Positive: $(-\infty, 1]$

Negative: $[1, 3)$

Positive: $(3, \infty)$

Solution: $(-\infty, 1] \cup (3, \infty)$

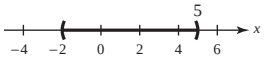
80. $\dfrac{x-5}{x+2} < 0$

Critical numbers: $x = -2, 5$

Test intervals:

Positive: $(-\infty, -2)$

Negative: $(-2, 5)$



Positive: $(5, \infty)$

Solution: $(-2, 5)$

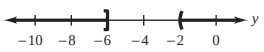
82. $\dfrac{y+6}{y+2} \geq 0$

Critical numbers: $y = -6, -2$

Test intervals:

Positive: $(-\infty, -6]$

Negative: $[-6, -2)$



Positive: $(-2, \infty)$

Solution: $(-\infty, -6] \cup (-2, \infty)$

84. $\dfrac{3x+4}{2x-1} < 0$

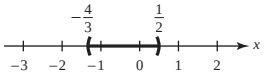
Critical numbers: $x = -\dfrac{4}{3}, \dfrac{1}{2}$

Test intervals:

Positive: $\left(-\infty, -\dfrac{4}{3}\right)$

Negative: $\left(-\dfrac{4}{3}, \dfrac{1}{2}\right)$

Positive: $\left(\dfrac{1}{2}, \infty\right)$



Solution: $\left(-\dfrac{4}{3}, \dfrac{1}{2}\right)$

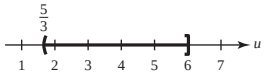
86. $\dfrac{u-6}{3u-5} \leq 0$

Critical numbers: $u = 6, \frac{5}{3}$

Test intervals:

Positive: $\left(-\infty, \frac{5}{3}\right)$

Negative: $\left(\frac{5}{3}, 6\right]$



Positive: $[6, \infty)$

Solution: $\left(\frac{5}{3}, 6\right]$

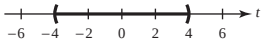
88. $\dfrac{2(4-t)}{4+t} > 0$

Critical numbers: $t = -4, 4$

Test intervals:

Negative: $(-\infty, 4)$

Positive: $(-4, 4)$



Negative: $(4, \infty)$

Solution: $(-4, 4)$

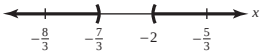
90. $\dfrac{1}{x+2} > -3$

$\dfrac{1}{x+2} + 3 > 0$

$\dfrac{1+3(x+2)}{x+2} > 0$

$\dfrac{1+3x+6}{x+2} > 0$

$\dfrac{3x+7}{x+2} > 0$



Critical numbers: $x = -\dfrac{7}{3}, -2$

Test intervals:

Positive: $\left(-\infty, -\dfrac{7}{3}\right)$

Negative: $\left(-\dfrac{7}{3}, -2\right)$

Positive: $(-2, \infty)$

Solution: $\left(-\infty, -\dfrac{7}{3}\right) \cup (-2, \infty)$

92. $\dfrac{6x}{x-4} < 5$

$\dfrac{6x}{x-4} - 5 < 0$

$\dfrac{6x-5(x-4)}{x-4} < 0$

$\dfrac{6x-5x+20}{x-4} < 0$

$\dfrac{x+20}{x-4} < 0$

Critical numbers: $x = -20, 4$

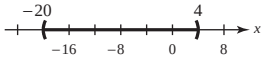
Test intervals:

Positive: $(-\infty, -20)$

Negative: $(-20, 4)$

Positive: $(4, \infty)$

Solution: $(-20, 4)$



94. $\dfrac{x+4}{x-5} \geq 10$

$\dfrac{x+4}{x-5} - 10 \geq 0$

$\dfrac{x+4-10(x-5)}{x-5} \geq 0$

$\dfrac{x+4-10x+50}{x-5} \geq 0$

$\dfrac{-9x+54}{x-5} \geq 0$

$\dfrac{-9(x-6)}{x-5} \geq 0$

Critical numbers: $x = 5, 6$

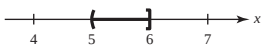
Test intervals:

Negative: $(-\infty, 5)$

Positive: $(5, 6]$

Negative: $[6, \infty)$

Solution: $(5, 6]$



96. $\dfrac{1}{x} - 4 < 0$

Keystrokes:

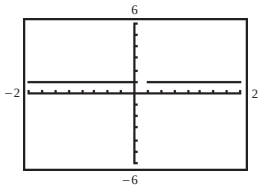
Y= 1 ÷ X,T,θ − 4 TEST 5 0 GRAPH

$\dfrac{1}{x} - 4 < 0$

$\dfrac{1}{x} - \dfrac{4x}{x} < 0$

$\dfrac{1-4x}{x} < 0$

$\dfrac{4x-1}{x} > 0$



Critical numbers: $x = 0, \dfrac{1}{4}$

Test intervals:

Positive: $(-\infty, 0)$

Negative: $\left(0, \dfrac{1}{4}\right)$

Positive: $\left(\dfrac{1}{4}, \infty\right)$

Solution: $(-\infty, 0) \cup \left(\dfrac{1}{4}, \infty\right)$

98. $\dfrac{x+12}{x+2} - 3 \geq 0$

Keystrokes:

Y= (X,T,θ + 12) ÷ (X,T,θ + 2) − 3 TEST 4 0 GRAPH

$\dfrac{x+12}{x+2} - 3 \geq 0$

$\dfrac{x+12}{x+2} - \dfrac{3(x+2)}{x+2} \geq 0$

$\dfrac{x+12-3x-6}{x+2} \geq 0$

$\dfrac{-2x+6}{x+2} \geq 0$

$\dfrac{2x-6}{x+2} \leq 0$

Critical numbers: $x = -2, 3$

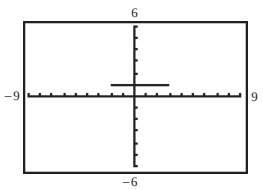
Test intervals:

Positive: $(-\infty, -2)$

Negative: $(-2, 3]$

Positive: $[3, \infty)$

Solution: $(-2, 3]$



100. $\dfrac{3x-4}{x-4} < -5$

Keystrokes:

Y= (3 X,T,θ − 4) ÷ (X,T,θ − 4) TEST 5 (−) 5 GRAPH

$$\frac{3x-4}{x-4} < -5$$

$$\frac{3x-4}{x-4} + \frac{5(x-4)}{x-4} < 0$$

$$\frac{3x-4+5x-20}{x-4} < 0$$

$$\frac{8x-24}{x-4} < 0$$

$$\frac{8(x-3)}{x-4} < 0$$

Critical numbers: $x = 3, 4$

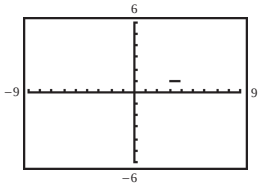
Test intervals:

Positive: $(-\infty, 3)$

Negative: $(3, 4)$

Positive: $(4, \infty)$

Solution: $(3, 4)$



102. $4 - \dfrac{1}{x^2} > 1$

Keystrokes:

Y= 4 − 1 ÷ X,T,θ x^2 TEST 3 1 GRAPH

$$4 - \frac{1}{x^2} > 1$$

$$\frac{4x^2}{x^2} - \frac{1}{x^2} - \frac{1x^2}{x^2} > 0$$

$$\frac{3x^2-1}{x^2} > 0$$

Critical numbers: $x = -0.58, 0, 0.58$

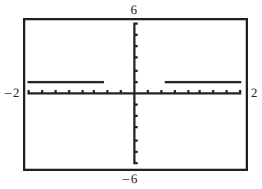
Test intervals:

Positive: $(-\infty, -0.58)$

Negative: $(-0.58, 0), (0, 0.58)$

Positive: $(0.58, \infty)$

Solution: $(-\infty, -0.58) \cup (0.58, \infty)$



104. $y = \dfrac{2(x-2)}{x+1}$ (a) $y \le 0$ (b) $y \ge 8$

Keystrokes:

Y= (2 (X,T,θ − 2)) ÷ (X,T,θ + 1) GRAPH

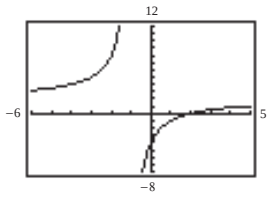
Solution:

(a) $(-1, 2]$

(Look at x-axis and vertical asymptote $x = -1$.)

(b) $[-2, -1)$

(Graph $y = 8$ and find intersection.)



106. $y = \dfrac{5x}{x^2+4}$ (a) $y \ge 1$ (b) $y \ge 0$

Keystrokes:

Y= 5 X,T,θ ÷ (X,T,θ x^2 + 4) GRAPH

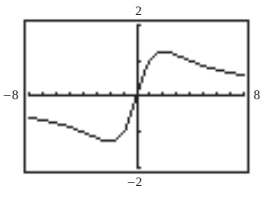
Solution:

(a) $[1, 4]$

(Graph $y = 1$ and find intersection.)

(b) $[0, \infty)$

(Look at x-axis.)



108. $h = -16t^2 + 88t$

$$\text{height} > 50$$
$$-16t^2 + 88t > 50$$
$$-16t^2 + 88t - 50 > 0$$
$$8t^2 - 44t + 25 < 0$$

Critical numbers: $t = \dfrac{11 + \sqrt{71}}{4}$ $t = \dfrac{11 - \sqrt{71}}{4}$

Test intervals:

Positive: $\left(-\infty, \dfrac{11 - \sqrt{71}}{4}\right)$

Negative: $\left(\dfrac{11 - \sqrt{71}}{4}, \dfrac{11 + \sqrt{71}}{4}\right)$

Positive: $\left(\dfrac{11 + \sqrt{71}}{4}, \infty\right)$

Solution: $\left(\dfrac{11 - \sqrt{71}}{4}, \dfrac{11 + \sqrt{71}}{4}\right)$

$(0.64, 4.86)$

110.
$$500(1 + r)^2 > 550$$
$$(1 + r)^2 > \frac{550}{500}$$
$$(1 + r)^2 > \frac{11}{10}$$
$$1 + 2r + r^2 > \frac{11}{10}$$
$$10 + 20r + 10r^2 - 11 > 0$$
$$10r^2 + 20r - 1 > 0$$

Critical numbers: $r = \dfrac{-10 + \sqrt{110}}{10}, \dfrac{-10 - \sqrt{110}}{10}$

r cannot be negative.

Test intervals:

Positive: $\left(\dfrac{-10 + \sqrt{110}}{10}, \infty\right)$

Solution: $\left(\dfrac{-10 + \sqrt{110}}{10}, \infty\right)$

$(0.0488, \infty)$, $r > 4.88\%$

112.
$$\text{Area} \geq 500$$
$$l(50 - l) \geq 500$$
$$50l - l^2 \geq 500$$
$$0 \geq l^2 - 50l + 500$$
$$l = \frac{50 \pm \sqrt{(-50)^2 - 4(1)(500)}}{2(1)}$$
$$l = \frac{50 \pm \sqrt{2500 - 2000}}{2}$$
$$l = \frac{50 \pm \sqrt{500}}{2}$$
$$l = \frac{50 \pm 10\sqrt{5}}{2}$$
$$l = \frac{2\left(25 \pm 5\sqrt{5}\right)}{2}$$
$$l = 25 \pm 5\sqrt{5}$$

Critical numbers: $l = 25 \pm 5\sqrt{5}$

Test intervals:

Positive: $\left[0, 25 - 5\sqrt{5}\right]$

Negative: $\left[25 - 5\sqrt{5}, 25 + 5\sqrt{5}\right]$

Positive: $\left[25 + 5\sqrt{5}, 50\right]$

Solution: $\left[25 - 5\sqrt{5}, 25 + 5\sqrt{5}\right]$

114. *Verbal Model:* $\boxed{\text{Profit}} = \boxed{\text{Revenue}} - \boxed{\text{Cost}}$

$$P(x) = x(125 - 0.0005x) - (3.5x + 185{,}000)$$
$$= 125x - 0.0005x^2 - 3.5x - 185{,}000$$
$$= -0.0005x^2 + 121.5x - 185{,}000$$

Inequality:
$$-0.0005x^2 + 121.5x - 185{,}000 \geq 6{,}000{,}000$$
$$-0.0005x^2 + 121.5x - 6{,}185{,}000 \geq 0$$
$$x = \frac{-121.5 \pm \sqrt{(121.5)^2 - 4(-0.0005)(-6{,}185{,}000)}}{2(-0.0005)}$$
$$x = \frac{121.5 \pm \sqrt{1476.25 - 12{,}370}}{0.001} = \frac{121.5 \pm \sqrt{2392.25}}{0.001}$$
$$x \approx 72{,}590;\ 170{,}410$$

Critical numbers: $x \approx 72{,}590,\ 170{,}410$

Test intervals:

Positive: $(-\infty, 72{,}590]$

Negative: $[72{,}590, 170{,}410]$

Positive: $[170{,}410, \infty]$

Solution: $[72{,}590, 170{,}410]$

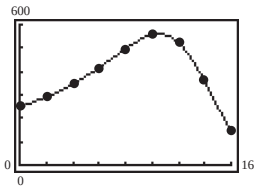
116. (a) *Keystrokes:*

Y= (248.5 − 13.72 X,T,θ) ÷ (1 − .13

X,T,θ + .005 X,T,θ x^2) GRAPH

(b) Let $y_2 = 400$ and find the intersection of the graphs.

Solution: $[5.6, 13.5]$, $5.6 \le t \le 13.5$



118. The direction of the inequality is reversed, when both sides are multiplied by a negative real number.

120. An algebraic expression can change signs only at the x-values that make the expression zero or undefined. The zeros and undefined values make up the critical numbers of the expression, and they are used to determine the test intervals in solving quadratic and rational inequalities.

122. $x^2 + 1 < 0$ is one example of a quadratic inequality that has no real solution. Any inequality of the form $x^2 + c < 0$, c any positive constant or $-x^2 - c > 0$, c any positive constant will not have a real solution.

Review Exercises for Chapter 8

2. $u^2 - 18u = 0$

$u(u - 18) = 0$

$u = 0 \qquad u - 18 = 0$

$u = 18$

4. $2z^2 - 72 = 0$

$2(z^2 - 36) = 0$

$2(z - 6)(z + 6) = 0$

$z - 6 = 0 \qquad z + 6 = 0$

$z = 6 \qquad z = -6$

6. $x^2 + \frac{8}{3}x + \frac{16}{9} = 0$

$9x^2 + 24x + 16 = 0$

$(3x + 4)(3x + 4) = 0$

$3x + 4 = 0 \qquad 3x + 4 = 0$

$x = -\frac{4}{3} \qquad x = -\frac{4}{3}$

8. $15x^2 - 30x - 45 = 0$

$15(x^2 - 2x - 3) = 0$

$15(x - 3)(x + 1) = 0$

$x - 3 = 0 \qquad x + 1 = 0$

$x = 3 \qquad x = -1$

10. $10x - 8 = 3x^2 - 9x + 12$

$0 = 3x^2 - 19x + 20$

$0 = (3x - 4)(x - 5)$

$3x - 4 = 0 \qquad x - 5 = 0$

$x = \frac{4}{3} \qquad x = 5$

12. $2x^2 = 98$

$x^2 = 49$

$x = \pm\sqrt{49}$

$x = \pm 7$

14. $y^2 - 8 = 0$

$y^2 = 8$

$y = \pm\sqrt{8}$

$y = \pm 2\sqrt{2}$

16. $(x + 3)^2 = 900$

$x + 3 = \pm 30$

$x = -3 \pm 30$

$x = 27, -33$

18. $u^2 = -25$

$u = \pm\sqrt{-25}$

$u = \pm 5i$

20. $x^2 + 48 = 0$

$x^2 = -48$

$x = \pm\sqrt{-48}$

$x = \pm 4\sqrt{3}i$

22. $(x - 2)^2 + 24 = 0$

$(x - 2)^2 = -24$

$x - 2 = \pm\sqrt{-24}$

$x = 2 \pm 2\sqrt{6}i$

24. $x^4 - 10x^2 + 9 = 0$

$(x^2 - 9)(x^2 - 1) = 0$

$x^2 - 9 = 0 \qquad x^2 - 1 = 0$

$x^2 = 9 \qquad x^2 = 1$

$x = \pm\sqrt{9} \qquad x = \pm\sqrt{1}$

$x = \pm 3 \qquad x = \pm 1$

26. $x + 2\sqrt{x} - 3 = 0$

$(\sqrt{x} + 3)(\sqrt{x} - 1) = 0$

$\sqrt{x} + 3 = 0$ $\qquad$ $\sqrt{x} - 1 = 0$

$\sqrt{x} = -3$ $\qquad$ $\sqrt{x} = 1$

$(\sqrt{x})^2 = (-3)^2$ $\qquad$ $(\sqrt{x})^2 = 1^2$

$x = 9$ $\qquad$ $x = 1$

Not a solution

Check: $9 + 2\sqrt{9} - 3 \stackrel{?}{=} 0$

$9 + 6 - 3 \stackrel{?}{=} 0$

$12 \neq 0$

Check: $1 + 2\sqrt{1} - 3 \stackrel{?}{=} 0$

$1 + 2 - 3 \stackrel{?}{=} 0$

$0 = 0$

28. $(\sqrt{x} - 2)^2 + 2(\sqrt{x} - 2) - 3 = 0$

$((\sqrt{x} - 2) + 3)((\sqrt{x} - 2) - 1) = 0$

$\sqrt{x} - 2 + 3 = 0$ $\qquad$ $\sqrt{x} - 2 - 1 = 0$

$\sqrt{x} + 1 = 0$ $\qquad$ $\sqrt{x} - 3 = 0$

$\sqrt{x} = -1$ $\qquad$ $\sqrt{x} = 3$

$(\sqrt{x})^2 = (-1)^2$ $\qquad$ $(\sqrt{x})^2 = 3^2$

$x = 1$ $\qquad$ $x = 9$

Not a solution

Check: $(\sqrt{1} - 2)^2 + 2(\sqrt{1} - 2) - 3 \stackrel{?}{=} 0$

$(1 - 2)^2 + 2(1 - 2) - 3 \stackrel{?}{=} 0$

$(-1)^2 + 2(-1) - 3 \stackrel{?}{=} 0$

$1 - 2 - 3 \stackrel{?}{=} 0$

$-4 \neq 0$

Check: $(\sqrt{9} - 2)^2 + 2(\sqrt{9} - 2) - 3 \stackrel{?}{=} 0$

$(3 - 2)^2 + 2(3 - 2) - 3 \stackrel{?}{=} 0$

$1^2 + 2(1) - 3 \stackrel{?}{=} 0$

$1 + 2 - 3 \stackrel{?}{=} 0$

$0 = 0$

30. $x^{2/5} + 4x^{1/5} + 3 = 0$

$(x^{1/5} + 3)(x^{1/5} + 1) = 0$

$x^{1/5} + 3 = 0$ $\qquad$ $x^{1/5} + 1 = 0$

$x^{1/5} = -3$ $\qquad$ $x^{1/5} = -1$

$\sqrt[5]{x} = -3$ $\qquad$ $\sqrt[5]{x} = -1$

$(\sqrt[5]{x})^5 = (-3)^5$ $\qquad$ $(\sqrt[5]{x})^5 = (-1)^5$

$x = -243$ $\qquad$ $x = -1$

32. $y^2 - 80y + 1600$

$\left[1600 = \left(-\frac{80}{2}\right)^2\right]$

34. $x^2 + 21x + \frac{441}{4}$

$\left[\frac{441}{4} = \left(\frac{21}{2}\right)^2\right]$

36. $x^2 - \frac{3}{4}x + \frac{9}{64}$

$\left[\frac{9}{64} = \left(\frac{-3/4}{2}\right)^2\right]$

$\left[\frac{9}{64} = \left(-\frac{3}{8}\right)^2\right]$

38. $x^2 + 12x + 6 = 0$

$x^2 + 12x + 36 = -6 + 36$

$(x + 6)^2 = 30$

$x + 6 = \pm\sqrt{30}$

$x = -6 \pm \sqrt{30}$

$x \approx -0.52; -11.48$

40. $u^2 - 5u + 6 = 0$

$\left(u^2 - 5u + \frac{25}{4}\right) = -6 + \frac{25}{4}$

$\left(u - \frac{5}{2}\right)^2 = \frac{1}{4}$

$u - \frac{5}{2} = \pm\sqrt{\frac{1}{4}}$

$u = \frac{5}{2} \pm \frac{1}{2} = \frac{6}{2}, \frac{4}{2} = 3, 2$

42. $t^2 + \frac{1}{2}t - 1 = 0$

$$t^2 + \frac{1}{2}t + \frac{1}{16} = 1 + \frac{1}{16}$$
$$\left(t + \frac{1}{4}\right)^2 = \frac{16 + 1}{16}$$
$$\left(t + \frac{1}{4}\right)^2 = \frac{17}{16}$$
$$t + \frac{1}{4} = \pm\sqrt{\frac{17}{16}}$$
$$t + \frac{1}{4} = \pm\frac{\sqrt{17}}{4}$$
$$t = -\frac{1}{4} \pm \frac{\sqrt{17}}{4}$$
$$t = \frac{-1 \pm \sqrt{17}}{4}$$
$$t \approx 0.78; -1.28$$

44. $x^2 - x - 72 = 0$

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-72)}}{2(1)}$$
$$x = \frac{1 \pm \sqrt{1 + 288}}{2}$$
$$x = \frac{1 \pm \sqrt{289}}{2}$$
$$x = \frac{1 \pm 17}{2}$$
$$x = -8, 9$$

46. $2x^2 - 3x - 20 = 0$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(2)(-20)}}{2(2)}$$
$$x = \frac{3 \pm \sqrt{9 + 160}}{4}$$
$$x = \frac{3 \pm \sqrt{169}}{4}$$
$$x = \frac{3 \pm 13}{4}$$
$$x = -\frac{5}{2}, 4$$

48. $3x^2 + 12x + 4 = 0$

$$x = \frac{-12 \pm \sqrt{12^2 - 4(3)(4)}}{2(3)}$$
$$x = \frac{-12 \pm \sqrt{144 - 48}}{6}$$
$$x = \frac{-12 \pm \sqrt{96}}{6}$$
$$x = \frac{-12 \pm 4\sqrt{6}}{6}$$
$$x = \frac{-6 \pm 2\sqrt{6}}{3}$$

50. $y^2 - 26y + 169 = 0$

$$b^2 - 4ac = (-26)^2 - 4(1)(169)$$
$$= 676 - 676$$
$$= 0$$

One repeated rational solution

52. $r^2 - 5r - 45 = 0$

$$b^2 - 4ac = (-5)^2 - 4(1)(-45)$$
$$= 25 + 180$$
$$= 205$$

Two distinct irrational solutions

54. $7x^2 + 3x - 18 = 0$

$$b^2 - 4ac = 3^2 - 4(7)(-18)$$
$$= 9 + 504$$
$$= 513$$

Two distinct irrational solutions

56. $9y^2 + 1 = 0$

$$b^2 - 4ac = 0^2 - 4(9)(1)$$
$$= -36$$

Two distinct complex solutions

58. $x = -1$ $x = 10$

$$x + 1 = 0 \qquad x - 10 = 0$$
$$(x + 1)(x - 10) = 0$$
$$x^2 - 9x - 10 = 0$$

60. $x = 2 + \sqrt{2}$ $x = 2 - \sqrt{2}$

$$x - \left(2 + \sqrt{2}\right) = 0 \qquad x - \left(2 - \sqrt{2}\right) = 0$$
$$(x - 2) - \sqrt{2} = 0 \qquad (x - 2) + \sqrt{2} = 0$$
$$\left[(x - 2) - \sqrt{2}\right]\left[(x - 2) + \sqrt{2}\right] = 0$$
$$(x - 2)^2 - \left(\sqrt{2}\right)^2 = 0$$
$$x^2 - 4x + 4 - 2 = 0$$
$$x^2 - 4x + 2 = 0$$

62. $x = 3 + 4i$ $x = 3 - 4i$

$$x - (3 + 4i) = 0 \qquad x - (3 - 4i) = 0$$
$$(x - 3) - 4i = 0 \qquad (x - 3) + 4i = 0$$
$$[(x - 3) - 4i][(x - 3) + 4i] = 0$$
$$(x - 3)^2 - (4i)^2 = 0$$
$$x^2 - 6x + 9 + 16 = 0$$
$$x^2 - 6x + 25 = 0$$

64. $g(x) = x^2 + 12x - 9$

$= (x^2 + 12x + 36) - 9 - 36$

$= (x + 6)^2 - 45$

Vertex: $(-6, -45)$

66. $y = 3x^2 + 2x - 6$

$= 3\left(x^2 + \frac{2}{3}x\right) - 6$

$= 3\left(x^2 + \frac{2}{3}x + \frac{1}{9}\right) - 6 - \frac{1}{3}$

$= 3\left(x + \frac{1}{3}\right)^2 - \frac{19}{3}$

Vertex: $\left(-\frac{1}{3}, -\frac{19}{3}\right)$

68. $y = -x^2 + 3x$

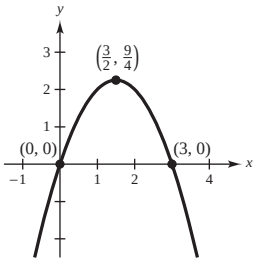
x-intercepts:

$0 = -x^2 + 3x$

$0 = -x(x - 3)$

$-x = 0 \qquad x - 3 = 0$

$x = 0 \qquad x = 3$



Vertex:

$y = -x^2 + 3x$

$y = -\left(x^2 - 3x + \frac{9}{4}\right) + \frac{9}{4}$

$= -\left(x - \frac{3}{2}\right)^2 + \frac{9}{4}$

$\left(\frac{3}{2}, \frac{9}{4}\right)$

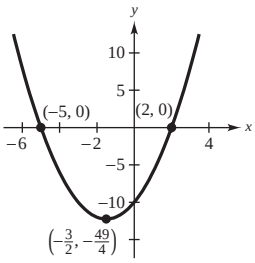
70. $y = x^2 + 3x - 10$

x-intercepts:

$0 = x^2 + 3x - 10$

$0 = (x + 5)(x - 2)$

$x = -5 \qquad x = 2$



Vertex:

$y = \left(x^2 + 3x + \frac{9}{4}\right) - 10 - \frac{9}{4}$

$= \left(x + \frac{3}{2}\right) - \frac{49}{4}$

$\left(-\frac{3}{2}, -\frac{49}{4}\right)$

72. Vertex: $(-4, 0)$

y-intercept: $(0, -6)$

$y = a(x - h)^2 + k$

$y = a(x + 4)^2 + 0$

$-6 = a(0 + 4)^2$

$-\frac{6}{16} = a$

$y = -\frac{3}{8}(x + 4)^2$

74. Vertex: $(-2, 5)$

Point: $(0, 1)$

$y = a(x - h)^2 + k$

$y = a(x - (-2))^2 + 5$

$y = a(x + 2)^2 + 5$

$1 = a(0 + 2)^2 + 5$

$1 = 4a + 5$

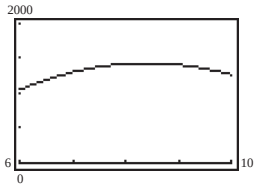
$-4 = 4a$

$-1 = a$

$y = -(x + 2)^2 + 5$

76. (a) *Keystrokes:*

Y= (−) 66.36 X,T,θ x² + 1116.2 X,T,θ − 3259 GRAPH



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76. —CONTINUED—

(b) Maximum number of bankruptcies $\approx$ 1,435,000 occuring in the year 1998

$N(8) = -66.36(8)^2 + 1116.2(8) - 3259$

$\approx 1423.56 \times 1000$

$\approx 1{,}423{,}560$

Maximum occurs when $t = 8$.

78. *Verbal Model:* $\boxed{\text{Selling price per computer}} = \boxed{\text{Cost per computer}} + \boxed{\text{Profit per computer}}$

Labels: Number computers sold $= x$

Number computers purchased $= x + 5$

Equation: $$\frac{27{,}000}{x} = \frac{27{,}000}{x+5} + 900$$

$$x(x+5)\left(\frac{2700}{x}\right) = \left(\frac{27{,}000}{x+5} + 900\right)x(x+5)$$

$$27{,}000(x+5) = 27{,}000x + 900\,x(x+5)$$

$$27{,}000x + 135{,}000 = 27{,}000x + 900x^2 + 4500x$$

$$0 = 900x^2 + 4500x - 135{,}000$$

$$0 = x^2 + 5x - 150$$

$$0 = (x+15)(x-10)$$

$x = -15$ reject $\qquad x = 10$ computers

Selling price of each computer $= \dfrac{27{,}000}{10} = \$2700$

80. $A = P(1+r)^2$

$38{,}955.88 = 35{,}000(1+r)^2$

$1.113025143 = (1+r)^2$

$1.055 \approx 1 + r$

$0.055 \approx r \approx 5.5\%$

82. *Formula:* $c^2 = a^2 + b^2$

Labels: $c = 39$ $\qquad a + b = 51$

$a = x$ $\qquad x + b = 51$

$b = 51 - x$ $\qquad b = 51 - x$

Equation: $39^2 = x^2 + (51-x)^2$

$1521 = x^2 + 2601 - 102x + x^2$

$0 = 2x^2 - 102x + 1080$

$0 = x^2 - 51x + 540$

$0 = (x-15)(x-36)$

$x = 15$ $\qquad x = 36$

$51 - x = 36$ $\quad 51 - x = 15$

15 feet and 36 feet

84. (a) $256 = -16t^2 + 64t + 192$

$0 = -16t^2 + 64t - 64$

$0 = t^2 - 4t + 4$

$0 = (t-2)^2$

$t = 2$ seconds

(b) $0 = -16t^2 + 64t + 192$

$0 = -16(t^2 - 4t - 12)$

$0 = -16(t+2)(t-6)$

$t + 2 = 0$ $\qquad t - 6 = 0$

discard $t = -2$ $\qquad t = 6$ seconds

86. $5x(x - 13)$

$5x = 0 \qquad x - 13 = 0$

$x = 0 \qquad x = 13$

Critical numbers: $x = 0, 13$

88. $2x^2 + 11x + 5$

$(2x + 1)(x + 5)$

$2x + 1 = 0 \qquad x + 5 = 0$

$x = -\frac{1}{2} \qquad x = -5$

Critical numbers: $x = -\frac{1}{2}, -5$

90. $-2x(x - 10) \le 0$

Critical numbers: $x = 0, 10$

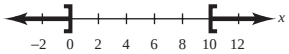
Test intervals:

Negative: $(-\infty, 0]$

Positive: $[0, 10]$

Negative: $[10, \infty)$

Solution: $(-\infty, 0] \cup [10, \infty)$



92. $(x - 5)^2 - 36 > 0$

$[(x - 5) - 6][(x - 5) + 6] > 0$

$(x - 11)(x + 1) > 0$

Critical numbers: $x = -1, 11$

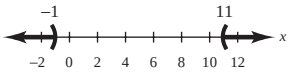
Test intervals:

Positive: $(-\infty, -1)$

Negative: $(-1, 11)$

Positive: $(11, \infty)$

Solution: $(-\infty, -1) \cup (11, \infty)$



94. $3x^2 - 2x - 8 > 0$

$(3x + 4)(x - 2) > 0$

Critical numbers: $x = -\frac{4}{3}, 2$

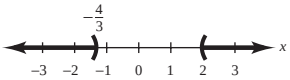
Test intervals:

Positive: $\left(-\infty, -\frac{4}{3}\right)$

Negative: $\left(-\frac{4}{3}, 2\right)$

Positive: $(2, \infty)$

Solution: $\left(-\infty, -\frac{4}{3}\right) \cup (2, \infty)$



96. $\dfrac{3x + 2}{x - 3} > 0$

Critical numbers: $x = -\frac{2}{3}, 3$

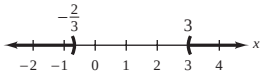
Test intervals:

Positive: $\left(-\infty, -\frac{2}{3}\right)$

Negative: $\left(-\frac{2}{3}, 3\right)$

Positive: $(3, \infty)$

Solution: $\left(-\infty, -\frac{2}{3}\right) \cup (3, \infty)$



98. $\dfrac{2x - 9}{x - 1} \le 0$

Critical numbers: $x = 1, \frac{9}{2}$

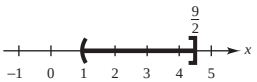
Test intervals:

Positive: $(-\infty, 1)$

Negative: $\left(1, \frac{9}{2}\right]$

Positive: $\left[\frac{9}{2}, \infty\right)$

Solution: $\left(1, \frac{9}{2}\right]$



100. $\overline{C} = \dfrac{C}{x} = \dfrac{100{,}000 + 0.9x}{x} = \dfrac{100{,}000}{x} + 0.9$

$\dfrac{100{,}000}{x} + 0.9 < 2$

$\dfrac{100{,}000}{x} - 1.1 < 0$

$\dfrac{100{,}000 - 1.1x}{x} < 0$

Critical numbers: $x = 0, 90{,}909$

Test intervals:

x must be positive.

Positive: $(0, 90{,}909)$

Negative: $(90{,}909, \infty)$

Solution: $(90{,}909, \infty)$; $x > 90{,}909$