Formulas which are provided on the final exam are at the end of this practice final.

1. Find the slope of the line containing the points $(-2, 4)$ and $(6, -3)$.
   A. 4 B. $-\frac{7}{8}$ C. $\frac{1}{4}$ D. $-\frac{8}{7}$ E. $-\frac{1}{2}$

2. Suppose 280 tons of corn were harvested in 5 days and 940 tons in 20 days. If the relationship between tons T and days d is linear, express T as a function of d.
   A. $T(d) = 5d + 280$ B. $T(d) = -44d + 500$ C. $T(d) = 44d + 60$ D. $T(d) = 60d + 44$
   E. $T(d) = 44d - 60$

3. When 30 orange trees are planted per acre each tree yields 150 oranges For each additional tree per acre, the yield decreases by 3 oranges per tree. Express the total yield of oranges per acre, Y, as a function of the number of trees planted per acre, x, if $x \geq 30$.
   A. $Y = 4500 + 60x - 3x^2$ B. $Y = \frac{1}{3}x + 80$ C. $Y = 150x - 3x^2$ D. $Y = 240x - 3x^2$
   E. $Y = 900 + 3x - 60x^2$

4. A manufacturer can sell dining-room tables for \$70 apiece. The manufacturer's total cost consists of a fixed overhead of \$8000 plus production costs of \$30 per table. How many tables must the manufacturer sell to break even?
   A. 80 B. 267 C. 200 D. 20 E. 136

5. If $f(x) = \sqrt{x+1}$ and $g(x) = x^2 + 7$ then $(f \circ g)(-1)) =$
   A. 0 B. 3 C. $\sqrt{7}$ D. 7 E. $\sqrt{8} + 1$

6. If $f(x) = \dfrac{2}{x}$ then $\dfrac{f(x+h) - f(x)}{h} =$
   A. $-\dfrac{2}{x^2}$ B. $\dfrac{2}{x+h} - \dfrac{2}{x}$ C. $\dfrac{2}{x(x+h)}$ D. $-\dfrac{2}{x(x+h)}$ E. $-\dfrac{2}{(x+h)^2}$

7. The domain of $f(x) = \dfrac{1}{\sqrt[3]{x-1}}$ is:
   A. $x < 1$ or $x > 1$ B. $x > 1$ C. $x > 0$ D. $x < 0$ or $x > 0$ E. $-1 < x < 1$

8. $\displaystyle\lim_{x \to 1} \frac{x^2 + 4x - 5}{x^2 - 1} =$
   A. ∞ B. 0 C. 3 D. -3 E. 5

9. $\displaystyle\lim_{x \to \infty} e^{-x} =$
   A. 0 B. 1 C. -1 D. ∞ E. e

10. Suppose
$$f(x) = \begin{cases} \dfrac{x^2 - 1}{x - 1} & \text{if } x < 0 \\ x^2 - 3x + 2 & \text{if } x \geq 0 \end{cases}$$
    Find all values where the function f is <u>discontinuous</u>.
    A. $x = -1$ B. $x = 0$ C. $x = 1$ D. $x = 2$ E. f is continuous for all values of x.

11. Find all values of x for which the function $f(x) = 2x^3 - 3x^2 - 12x + 12$ is increasing.
    A. $-1 < x < 2$ B. $x < -1$ C. $x > 2$ D. $x < -1$ and $x > 2$ E. $x < 2$ and $x > 2$

12. The derivative of $\dfrac{x^2+1}{x+5}$ is:

    A. $\dfrac{x^2+10x-1}{(x+5)^2}$ B. $2x$ C. $\dfrac{2x^2+10x}{(x^2+1)^2}$ D. $\dfrac{3x^2+10x+1}{(x+5)^2}$

    E. $\dfrac{-x^2-10x+1}{(x+5)^2}$

13. If $y=(3-x^2)^3$ then $y''=$
    A. $-6x(3-x^2)^2$ B. $24x^2(3-x^2)-6(3-x^2)^2$ C. $6(3-x^2)$
    D. $24x^2(3-x^2)$ E. $12x^2-6(3-x^2)$

14. The line tangent to the graph of $f(x)=x-\dfrac{1}{x}$ at $x=2$ has slope:

    A. $\frac{5}{4}$ B. $\frac{3}{4}$ C. $\frac{3}{2}$ D. 0 E. $\frac{1}{2}$

15. Find an equation for the tangent line to the curve $x^2y+xy^3=2$ at the point $(1,1)$.
    A. $2x+y=3$ B. $3x+4y=7$ C. $2x+3y=5$ D. $5x-2y=3$ E. $5x+3y=8$

16. After t years the population of a certain town is $P(t)=50+5t$ thousand people. A population P has an associated CO_2 level, $C(P)=(\sqrt{P^2+1})/2$. After 2 years, the rate at which CO_2 level is changing with respect to t will be:
    A. $5/(2\sqrt{5})$ B. $150/\sqrt{3601}$ C. $30\sqrt{3601}$ D. $30/\sqrt{3601}$ E. $50/\sqrt{3601}$

17. If $yx^2+y^3=x-y$. Then $y'=$
    A. $1-2xy-3y^2$ B. $1-2xy-x^2-3y^2$ C. $(1-2xy)/(3y^2+1)$ D. $(1-2xy)/(x^2+3y^2+1)$
    E. $(2xy)/(x^2+3y^2+1)$

18. If the concentration $C(t)$ of a certain drug remaining in the bloodstream t minutes after it is injected is given by $C(t)=t/(5t^2+125)$, then the concentration is a maximum when $t=$
    A. 25 B. 15 C. 5 D. 10 E. 20

19. If $f(x)=2x^4-6x^2$ then which one of the following is true?
    A. f has a relative max. at $x=\pm\sqrt{3/2}$ and a relative min. at $x=0$.
    B. f has a relative max. at $x=0$ and a relative min. at $x=\pm\sqrt{3/2}$.
    C. f has a relative max. at $x=-\sqrt{3/2}$ and a relative min. at $x=\sqrt{3/2}$.
    D. f has a relative max. at $x=\sqrt{3/2}$ and a relative min. at $x=-\sqrt{3/2}$.
    E. f has no relative max. points, but has relative min. at $x=\pm\sqrt{3/2}$.

20. The derivative of a function f is $f'(x)=x^2-\dfrac{8}{x}$. Then at $x=2$, f has:
    A. an inflection point B. a relative maximum C. a vertical tangent
    D. a discontinuity E. a relative minimum

21. If $f(x)=\frac{1}{3}x^3-9x+2$, then on the closed interval $0\le x\le 4$,
    A. f has an absolute max. at $x=3$ and an absolute min. at $x=0$.
    B. f has an absolute max. at $x=4$ and an absolute min. at $x=3$.
    C. f has an absolute max. at $x=0$ and an absolute min. at $x=4$.
    D. f has an absolute max. at $x=0$ and an absolute min. at $x=3$.
    E. f has an absolute max. at $x=4$ and an absolute min. at $x=0$.

22. The total cost in dollars to manufacture x units is given by the function $C = 3x^2 + x + 48$. For what value of x is the average cost a minimum?

    A. 4 B. 0.17 C. 12 D. 6.93 E. 16

23. A display case is in the shape of a rectangular box with a square base. Suppose the volume is 21 cubic ft and it costs \$1 per square ft. to build the glass top and \$0.50 per sq. ft. to build the sides and base. If x is the length of one side of the base, what value should x have to minimize the cost? Round your answer to two decimal places.

    A. 3.04 ft. B. 2.41 ft. C. 3.74 ft. D. 2.24 ft. E. 3.36 ft.

24. What is the area of the largest rectangle with sides parallel to the axes which can be inscribed in the first quadrant under the parabola $y = 4 - x^2$? Round your answer to 2 decimal places.
    A. 1.15 B. 1.33 C. 3.08 D. 4.00 E. 2.67

25. The radius of a circular oil spill is increasing at the rate of 3 ft/min. How fast is the area increasing when the radius is 4 ft?
    A. $24\pi \text{ft}^2/\text{min}$ B. $48\pi \text{ft}^2/\text{min}$ C. $8\pi \text{ft}^2/\text{min}$ D. $16\pi \text{ft}^2/\text{min}$ E. $32\pi \text{ft}^2/\text{min}$

26. Use differentials to approximate $\sqrt{3.96}$. Round your answer to 3 decimal places.

    A. 1.989 B. 1.990 C. 1.980 D. 1.975 E. 1.995

27. Water is flowing into a tank which is in the shape of a right circular cylinder standing on its circular base. If the water is flowing in at a rate of 80 cu. ft. per min. and the radius of the base of the tank is 4 ft., how fast is the water rising when the water is 10 ft. deep?

    A. $\frac{\pi}{5}$ ft/min B. 5π ft/min C. $\frac{50}{\pi}$ ft/min D. $\frac{5}{\pi}$ ft/min E. 50π ft/min

28. A manufacturer has been selling lamps at \$6 apiece and, at that price, consumers have been buying 3,000 lamps per month. The manufacturer wishes to raise the price and estimates that for each \$1 increase in the price, 1000 fewer lamps will be sold each month. The manufacturer can produce the lamps at a cost of \$4 per lamp. At what price should the manufacturer sell each lamp to generate the greatest possible profit?

    A. \$6.25 B. \$6.50 C. \$7.00 D. \$7.50 E. \$7.75

29. A population grows exponentially. In 1960, it was 50,000 and in 1965, it was 100,000. What was the population in 1970?

    A. 200,000 B. 150,000 C. 250,000 D. 300,000 E. 225,000

30. If $18^x = \sqrt{3}$, then in which of the following intervals does x lie?

    A. $0 < x < 1$ B. $-1 < x < 0$ C. $1 < x < 2$ D. $-2 < x < -1$ E. $2 < x < 3$

31. If $y = \ln\sqrt{1-x^2}$ then $\frac{dy}{dx} =$

    A. $\frac{1}{\sqrt{1-x^2}}$ B. $-\frac{2x}{\sqrt{1-x^2}}$ C. $-\frac{x}{1-x^2}$ D. $\frac{1}{2(1-x^2)}$ E. $\frac{1}{2\sqrt{1-x^2}}$

32. The amount of a certain radioactive substance remaining after t years is given by a function of the form $Q(t) = Q_0e^{-0.003t}$. The half-life of the substance is:

    A. 53 years B. 0.00435 years C. 333 years D. 231 years E. 167 years

33. If $y = e^{x^2}$ then $\frac{dy}{dx} =$

    A. e^{x^2} B. $x^2e^{x^2-1}$ C. $2xe^{x^2-1}$ D. $2xe^{x^2}$ E. e^{2x}

34. What lump sum of money should be deposited in a money market certificate paying 8.25% interest compounded monthly to amount to 5000 in 10 years? Round your answer to the nearest dollar.

    A. \$2514 B. \$4669 C. \$2740 D. \$2262 E. \$2197

35. How quickly will money double if it is invested at a rate of 8 percent compounded continuously? Round your answer to two decimal places.

    A. 0.87 years B. 25 years C. 5.55 years D. 8.66 years E. 6.33 years

36. Suppose the total cost in dollars of producing q units is $C(q) = 2e^{-q} + 3q^2 - 2$. Calculate the marginal cost when 5 units have been produced and calculate the actual cost of producing the 6th unit. Round your answer to the nearest cent.

    A. marginal cost $=$ \$29.99, actual cost $=$ \$32.99 B. marginal cost $=$ actual cost $=$ \$29.99
    C. marginal cost $=$ \$29.99, actual cost $=$ \$36.00 D. marginal cost $=$ \$30.01,
    actual cost $=$ \$32.99 E. marginal cost $=$ actual cost $=$ \$30.01

37. A cylindrical can with no top has been made from 27π square inches of metal. Express the volume, V, of the can as a function of its radius, r.

    A. $V = 27\pi r^2$ B. $V = \frac{\pi}{2}r(27 - r^2)$ C. $V = \pi r^2(27 - r^2 - 2r)$
    D. $V = 27\pi^2 r^2$ E. $V = \frac{4}{3}\pi r^2(27 - r^2)$

38. For what value of a does the function $f(x) = x^2 + ax$ have a relative minimum at $x = 1$?

    A. -2 B. 0 C. 2 D. -1 E. 1

39. The total cost of manufacturing q units of a certain commodity is $C(q) = 3q^2 + 5q + 75$. At what level of production is the average cost per unit equal to the marginal cost?

    A. $q = 2$ B. $q = 3$ C. $q = 4$ D. $q = 5$ E. $q = 6$

FORMULAS

Volume & Surface Area

Right Circular Cylinder

$V = \pi r^2 h$

$$SA = \begin{cases} 2\pi r^2 + 2\pi rh \\ \pi r^2 + 2\pi rh \end{cases}$$

Sphere

$V = \frac{4}{3}\pi r^3$

$SA = 4\pi r^2$

Interest Formulas

$B(t) = P(1 + \frac{r}{k})^{kt}$

$B(t) = Pe^{rt}$

Exponential Growth & Decay

$Q(t) = Q_0 e^{kt}$

$Q(t) = Q_0 e^{-kt}$

Answers

1. B; 2. C; 3. D; 4. C; 5. B; 6. D; 7. A; 8. C; 9. A; 10. B; 11. D; 12. A; 13. B; 14. A; 15. B; 16. B; 17. D; 18. C; 19. B; 20. E; 21. D; 22. A; 23. B; 24. C; 25. A; 26. B; 27. D; 28. B; 29. A; 30. A; 31. C; 32. D; 33. D; 34. E; 35. D; 36. A; 37. B; 38. A; 39. D.