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MATH 2900
Fall 2008

Exam 3

Instructions: You have 50 minutes to complete your exam. The exam is closed book/notes and calculators are not allowed. You must show all work and reasoning on the paper provided for full credit. Please work in a clean, ordered and **honorable** fashion. Put a box around your final answer.

Good Luck.

1. (30pts) Find the derivatives of the following functions.

   (a) $y = (\sin x)e^x - \ln x$

   (b) $y = \sqrt{e^{2x} + e^{-x}}$

   (c) $y = \ln\left(\dfrac{x^2}{x+5}\right)$

2. (15pts) Consider the implicit function $x^2 + xy + y^2 = 12$.

(a) Find $\frac{dy}{dx}$.

(b) Find the equation of the tangent line at the point (-2,-2).

(c) Use a linear approximation to estimate y when $x = -2.05$.

3. (15pts) A 13 ft ladder is leaning against a house when its base starts to slide away. By the time the base is 12 ft from the house, the base is moving at the rate of 5 ft/sec. Start by drawing a picture and labeling all variables.

   (a) How fast is the top of the ladder sliding down the wall at that instant?

   (b) How fast is the area of the triangle formed by the ladder, wall, and ground changing at that instant?

4. (15pts) A rectangle rotated around one of its sides sweeps out a figure in the shape of a right circular cylinder. If the perimeter of the rectangle is 36 inches, what are the dimensions of the rectangle that maximize the volume of the cylinder? Note: the volume of a cylinder is given by $V = \pi r^2 h$ where r is the radius and h is the height.

5. (15pts) A dose of a drug is injected into the body of a patient. The drug amount in the body decreases at a rate of 10% per hour, that is

$$\frac{dA}{dt} = -0.1A,$$

where A is the amount in the body and t is the time, in hours.

(a) A dose of 3 cubic centimeters (cc) is administered. Find the function, $A(t)$, that satisfies the equation.

(b) How much of the initial dose of 3 cc will remain after 10 hours?

(c) After how long does half the original dose remain?

6. (5pts) Explain the major difference between an exponential growth model: $P_e(t) = P_0 e^{rt}$ and the logistic growth model $P_l(t) = \dfrac{K}{1 + (K/P_0 - 1)e^{-rt}}$, i.e. compare the limit as t goes to infinity for both models.

BONUS * 5pts *** BONUS**

If $y = 10^{x^2}$, then find $\dfrac{dy}{dx}$. Hint: take the natural log of both sides and use implicit differentiation.

Scratch Paper