1. Evaluate $\int \frac{1}{(3x+1)^3}dx$

A. $\frac{-12}{(3x+1)^4}+c$

B. $\frac{-1}{6(3x+1)^2}+c$

C. $\frac{1}{(3x+1)^2}+c$

D. $\frac{-1}{2(3x+1)^2}+c$

E. $\frac{-9}{(3x+1)^4}+c$

2. Evaluate $\int xe^{2x}dx$

A. $xe^{2x}-\frac{1}{4}e^{2x}+c$

B. $\frac{1}{4}x^2e^{2x}+c$

C. $2xe^{2x}+e^{2x}+c$

D. $\frac{1}{2}xe^{2x}-\frac{1}{4}e^{2x}+c$

E. $\frac{1}{2}xe^{2x}+\frac{1}{4}e^{2x}+c$

3. Find the volume of the solid of revolution generated by rotating the region under the graph of $y = x^3$ from $x = 1$ to $x = 2$ about the x-axis.

A. $\frac{7\pi}{3}$
B. $\frac{15}{4}\pi$
C. $\frac{63}{6}\pi$
D. $\frac{31}{5}\pi$
E. $\frac{127}{7}\pi$

4. Evaluate $\int_1^\infty \frac{dx}{\sqrt{x}}$

A. 1
B. $\frac{1}{2}$
C. 2
D. $\frac{3}{2}$
E. It is divergent.

5. An object moves so that its velocity after t minutes is given by the formula $v = 10e^{-0.1t}$. The distance it travels during the 10th minute is

A. $\int_0^{10} 10e^{-0.1t}\,dt$
B. $\int_0^{10} -10e^{-0.1t}\,dt$
C. $\int_9^{10} 10e^{-0.1t}\,dt$
D. $\int_9^{10} -10e^{-0.1t}\,dt$
E. $\int_9^{10} -e^{-0.1t}\,dt$

6. If $f(x, y) = \sin x \bullet \cos y$, then $f(\frac{\pi}{4}, \frac{\pi}{3}) =$

A. 0
B. $\frac{\sqrt{2}}{2}$
C. $\frac{\sqrt{6}}{4}$
D. $\frac{\sqrt{2}}{4}$
E. $\frac{\sqrt{3}}{6}$

7. If $f(x, y)=\sqrt{x^2+y^2}$, then $f_x(-3,4)=$

A. $\frac{1}{10}$
B. $\frac{3}{5}$
C. $\frac{4}{5}$
D. $\frac{-3}{5}$
E. $\frac{-4}{5}$

8. If $f(x, y)=\ln(xy)$, then $f_{xy}=$

A. 0
B. $\frac{1}{x}$
C. $\frac{1}{y}$
D. $\frac{1}{xy}$
E. $\frac{1}{x}+\frac{1}{y}$

9. If $f(x,y) = 2xy - x^3 - y^2$, please choose the correct statement about its critical points.

A. There are 1 relative max and 1 relative min.

B. There are 1 relative max and 1 saddle point.

C. There are 1 relative min and 1 saddle point

D. There is only 1 saddle point, no max/min.

E. There are 2 saddle points.

10. Compute $\int_0^2 \int_0^x (x^2 - y)dydx$

A. 4
B. $\frac{4}{3}$
C. $\frac{16}{3}$
D. $\frac{8}{3}$
E. $\frac{3}{2}$

11. $g(x)$ is the unique solution to the initial-value problem:

$$\begin{cases} g'(x) = \dfrac{\ln x}{x} \\ g(1) = 0 \end{cases}$$

Evaluate the value of $g(2)$

A. 2
B. $\ln(\ln 2)$
C. $(\ln 2)^2$
D. $\frac{1}{2}(\ln 2)^2$
E. $\ln 2$

12. Consider the initial-value problem:

$$\begin{cases} y' + y = e^{-x} \\ y(0) = 2 \end{cases}$$

Evaluate the value of $y(1)$

A. e
B. e^{-1}
C. $2e^{-1}$
D. $3e^{-1}$
E. $4e^{-1}$

13. Find all equilibrium values of the differential equation:
$y' = y^2 - 2y - 24$

A. $y = 1$
B. $y = 1, y = 2$
C. $y = -4, y = 2$
D. $y = -4, y = 6$
E. $y = -2, y = 12$

14. Solve the initial-value problem:

$$\begin{cases} y' = \dfrac{-x}{y} \\ y(0) = 1 \end{cases}$$

A. $y = x + 1$
B. $y = 1 - x$
C. $y = \sqrt{1 - x^2}$
D. $y = -\sqrt{1 - x^2}$
E. $y = \sqrt{1 + x^2}$

15. According to the Hullian model of learning, the probability P of mastering a certain concept after t learning trials may be modeled by the differential equation

$$\frac{dP}{dt} = 1 - P; \quad P(0) = 0$$

After 5 trials, what is the probability of mastering the concept?

A. 63.21%
B. 86.47%
C. 95.02%
D. 98.65%
E. 99.32%

16. Find the slop of the direction field at $(-2,1)$ of $y' = \frac{x}{y}$

A. 1
B. -1
C. 2
D. -2
E. 0

17. Given $M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $N = \begin{bmatrix} -1 & 0 & 2 \\ 1 & 1 & 3 \end{bmatrix}$, compute $MN =$

A. $\begin{bmatrix} 1 & 2 & 8 \\ 1 & 3 & 12 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 2 & 8 \\ 1 & 4 & 9 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 4 & 8 \\ 1 & 2 & 18 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 2 & 8 \\ 1 & 4 & 18 \end{bmatrix}$

E. $\begin{bmatrix} 1 & 3 & 8 \\ 1 & 2 & 9 \end{bmatrix}$

18. The Leslie Matrix for a population of birds is $G=\begin{bmatrix}0.8 & 2\\ 0.5 & 0.4\end{bmatrix}$. The population vector in year 1 is $p=[\begin{matrix}hatchlings\\ adults\end{matrix}]=\begin{bmatrix}100\\ 60\end{bmatrix}$. Estimate the population vector for year 2.

A. $\begin{bmatrix}200\\ 120\end{bmatrix}$

B. $\begin{bmatrix}200\\ 74\end{bmatrix}$

C. $\begin{bmatrix}80\\ 120\end{bmatrix}$

D. $\begin{bmatrix}50\\ 120\end{bmatrix}$

E. $\begin{bmatrix}120\\ 74\end{bmatrix}$

19. Find the inverse of matrix $M=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$

A. $\begin{bmatrix}2 & 1\\ -3 & 2\end{bmatrix}$

B. $\begin{bmatrix}-2 & -3\\ -1 & -2\end{bmatrix}$

C. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

E. $\begin{bmatrix} -2 & 3 \\ 1 & -2 \end{bmatrix}$

20. Find the inverse of matrix $M = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

A. $\begin{bmatrix} 1 & -1 & -2 \\ 0 & -1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & -1 & 2 \end{bmatrix}$

C. $\begin{bmatrix} -1 & 1 & 2 \\ 0 & -1 & 0 \\ 1 & 1 & -1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 3 \\ 0 & 2 & -5 \end{bmatrix}$

E. $\begin{bmatrix} -1 & 1 & 2 \\ 0 & 1 & 0 \\ 1 & -1 & -1 \end{bmatrix}$

21. What is the determinant of matrix $M = \begin{bmatrix} 3 & 1 & 6 \\ -5 & 0 & -2 \\ 4 & 1 & 0 \end{bmatrix}$

A. -16
B. -32
C. -48
D. 28
E. 44

22. Find all eigenvalues of matrix $M = \begin{bmatrix} 3 & 6 \\ 1 & 4 \end{bmatrix}$

A. $r = 3, r = 4$
B. $r = 1, r = 6$
C. $r = 1, r = 3$
D. $r = 4, r = 6$
E. $r = 0, r = 5$

23. Which vector is an eigenvector of matrix $\begin{bmatrix} 3 & 6 \\ 1 & 4 \end{bmatrix}$?

A. $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

B. $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$

C. $\begin{bmatrix} 2 \\ -3 \end{bmatrix}$

D. $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$

E. $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$

24. Consider the particular solution to the difference equation:

$x_{n+1} = 2x_n + 8x_{n-1}$; $\quad x_0 = 1, \quad x_1 = -2$

Then find out x_{100}

A. 4^{100}

B. -4^{100}

C. 2^{100}

D. -2^{100}

E. 4^{101}