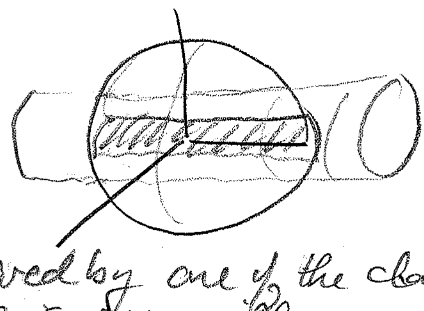
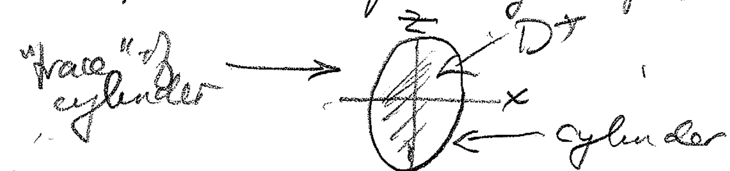
Ma 362

p 364 #6 Inside both the ball $x^2+y^2+z^2=4$ and $2x^2+z^2=1$.

Sketch!



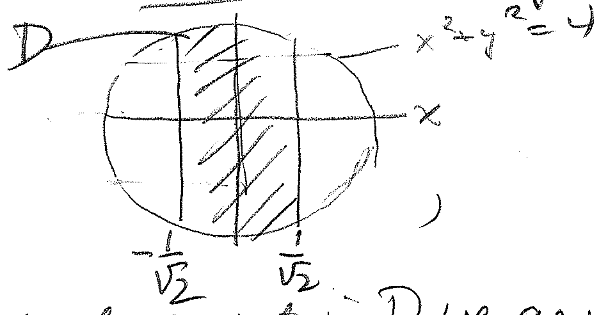
① As observed by one of the class members, the "shadow" in the x-z plane is especially simple



for each point $(x,0,z)$ in D^*, we move up and down with y until we hit the sphere. So we have

$$\underbrace{\iint}_{D^*} \int_{-\sqrt{4-x^2-z^2}}^{\sqrt{4-x^2-z^2}} dy\,dx\,dz = 2\int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}} \int_{-\sqrt{1-2x^2}}^{\sqrt{1-2x^2}} \sqrt{4-x^2-z^2}\; dz\,dx$$

② I tried the harder - more foolish - way, but let's see that we can at least set it up. From the sketch at the top, we see that the shadow in the xy-plane is



and for each point in D we go up and down.

The complication is that when we go up or down, we stop sometimes when we reach the sphere (near the top and bottom of D, we hit the sphere first, but near the middle we hit the cylinder first.

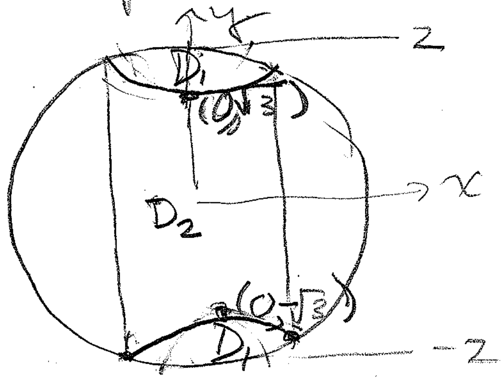
Let D_1 be the part of D where we hit the sphere first, and D_2 where we hit the cylinder first.

To decide which applies, we ask when the two equations for z agree:

$$4-x^2-y^2=1-2x^2$$

$$(*) \qquad y^2-x^2=3 \quad \text{(hyperbola)}$$

This gives the picture



The boundaries of D_1 and D_2 meet where $x^2=\frac{1}{2}$ and $x^2+y^2=4$. For when $x^2=\frac{1}{2}$, $(*)$ shows that $y^2=3.5$ and so $x^2+y^2=4$.

A glace at the picture shows it is easiest to integrate first with respect to y. So we get

D_2:

$$\int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}} \int_{-\sqrt{3-x^2}}^{\sqrt{3-x^2}} 2\sqrt{4-x^2-y^2}\, dy\, dx$$

D_1

$$\int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}} \int_{-\sqrt{4-x^2}}^{-\sqrt{3-x^2}} 2\sqrt{1-2x^2}\, dy\, dx + \int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}} \int_{\sqrt{3-x^2}}^{\sqrt{4-x^2}} 2\sqrt{4-x^2-y^2}\, dy\, dx$$