1/20/2009 **MA 510** Solutions

I am anxious to see how well you understand the material, and so it is not necessary to complete all work, so long as when reading the paper I feel confident that all you have omitted is routine algebra, etc. Please write neatly so I can follow your work, that is part of your job. You should always give reasons when you make assertiions. I prefer pencil.

(20) 1(a). Find a plane through $(1,4,3)$ which is perpendicular to $(5,7,2)$. How many such planes are there?

a plane throu p_0 with $\underline{v}$ as normal vector is

$(p-p_0)\cdot v = 0$. Here we get

$(x-1, y-4, z-3)\cdot(5,7,2) = 0$

$5x + 7y + 2z = 39$

There is only one such plane

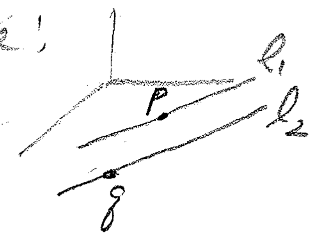
(b) Where does this plane intersect with the $x-y$–plane? Is it a point, a line or...? Give the equation of this intersection.

Set $z=0$ in 1a, and we get the line in the x-y plane

$5x + 7y = 39$

(20) 2(a). Why are the lines $\ell_1 : (1,4,2) + t(2,-1,3)$ and $\ell_2 : (2,4,1) + u(4,-2,6)$ parallel? Do they intersect?

A line thru p_0 ∥ to v has equation $p = p_0 + tv$, - vectors, so the lines have parallel directions. Picture!



Thus, they never meet or coincide, and to decide which we have to "get our hands dirty." We can ask: is $(2,4,1)$ on ℓ_1. So we would need (since $(1,4,2) \in \ell_1$)

$$\begin{aligned} 1 &= 2 + 4u \\ 4 &= 4 - 2u \\ 2 &= 1 + 6u \end{aligned} \quad \text{for the } \underline{\text{same}}\ u,$$

But the first equation gives $u = -\frac{1}{4}$, and that value of u doesn't work for either of the two other equations.

(b). Find the equation of the plane which contains these two lines.

We can see from the picture that ℓ_1 and ℓ_2 span a plane, but for a plane we need a ⊥ vector $\underline{v} = (A,B,C)$. All we know at first hand is that $(A,B,C)\cdot(2,-1,3) = 0$, since $(2,-1,3)$ is ∥ to the plane. We need another vector ∥ to the plane, and I'd take (illustration) $p - q$: $(1,4,2) - (2,4,1) = (-1,0,1)$, so the direction normal to our plane is

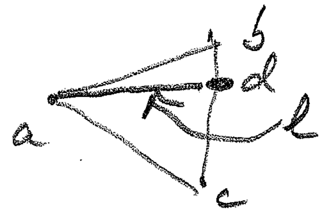
$$\begin{vmatrix} i & j & k \\ 2 & -1 & 3 \\ -1 & 0 & 1 \end{vmatrix} = (-1,-5,-1);$$

plane is

$$-(x-1) - 5(y-4) - 1(z-2) = 0$$

(20) 3. Let a, b, c be the vertices of a triangle (in two or three dimensions). Find the equation of the line joining a to the point $(1/3)$ of the way from b to c.

Good exercise with vectors.



ℓ has end points a, d.
of course $a = a$. And for d, we get (vector addition)

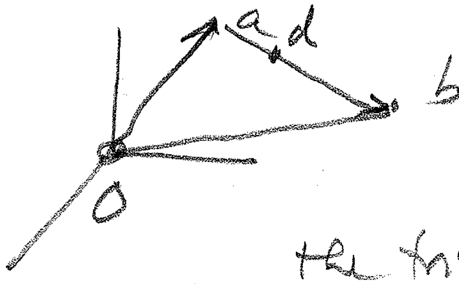
$d = b + \frac{1}{3}(c - b)$ (check this out!)

Then the equation for ℓ is

$$a + t(d-a) = a + t\left(b - a + \frac{1}{3}(c-b)\right)$$

$$0 \le t \le 1$$

(20) 4. Use vector notation to describe the triangle (in 3 dimensions) whose vertices are the vectors $0, a, b$ (0 is the origin). (A sketch might help you.)



Many ways to go. Here is one. Let d be any point on $[a, b]$. Then all points on $[0, d]$ lie in the triangle, and every point in the triangle can be recovered this way. Now

$$d = a + t(b-a), \quad 0 \le t \le 1,$$

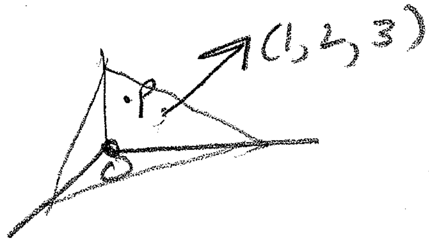
and the line segment $[0, d]$ is

$$sd, \quad 0 \le s \le 1$$

Combining, we get for the triangle

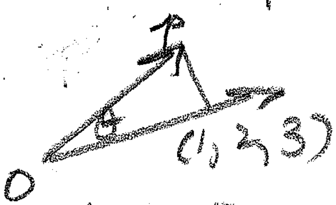
$$sd = s(a + t(b-a))$$
$$= a(s - st) + stb, \quad 0 \le s, t \le 1$$

(20) 5. How far is the plane $x + 2y + 3z = 6$ from the origin? (A figure might help.)



There is a formula in the book but this is more direct:

Let $p = (0,0,2)$ be on the plane



We know $(1,2,3)$ is $\perp$ to the plane

So the dist is

$$|p|\cdot\cos\theta = \cancel{|p|}\,\frac{p\cdot(1,2,3)}{\cancel{|p|}\sqrt{14}}$$

$$= \frac{6}{\sqrt{14}}$$