Ma 36200

3/31/09

Remarks on prob 17c) p 460

S a regular surface. Suppose it has two regular parametrizations [the second at the beginning of this problem is not regular at $(0,0) = (u,v)$].
So S is given as

$$\Phi(u,v), \quad \Psi(U,V)$$

where I use (U,V) as well as (u,v) for parameters for clarity

By today's class, we have

$$\Phi_u \times \Phi_v = \left(\frac{\partial(y,z)}{\partial(u,v)}, \frac{\partial(x,z)}{\partial(u,v)}, \frac{\partial(x,y)}{\partial(u,v)}\right),$$

$$\Psi_U \times \Psi_V = \left(\frac{\partial(y,z)}{\partial(U,V)}, \frac{\partial(x,z)}{\partial(U,V)}, \frac{\partial(x,y)}{\partial(U,V)}\right).$$

Now we need to check that these vectors are $\parallel$ and nonzero together. Let's just check the last component.
By the chain rule,

$$\frac{\partial x}{\partial U} = \frac{\partial x}{\partial u}\frac{\partial u}{\partial U} + \frac{\partial x}{\partial v}\frac{\partial v}{\partial V}, \text{ which we write as:}$$

$$x_U = x_u u_U + x_v v_V,$$

Similarly

$$x_V = x_u u_V + x_v v_V$$

$$y_U = y_u u_U + y_v v_U,$$

$$y_V = y_u u_V + y_v v_V.$$

So let's compute the last component of $\Psi_U \times \Psi_V$, and see what we get!

$$\frac{\partial(x,y)}{\partial(U,V)} = x_U\, y_V - x_V\, y_U$$

$$= (x_u u_U + x_v v_U)(y_u u_V + y_v v_V) - (x_u u_V + x_v v_V)(y_u u_U + y_v v_U)$$

collect terms; force signs

$$= (x_u y_v)\left[u_U v_V - u_V v_U\right]$$

$$-(x_v y_u)(-v_U u_V + u_U v_V)$$

$$+\, x_u y_u u_U u_V + x_v y_v v_U v_V - x_u u_V y_u u_U - x_v y_v v_U v_V$$

$$\overset{!}{=} (x_u y_v - x_v y_u)(u_U v_V - v_U u_V)$$

$$= \frac{\partial(x,y)}{\partial(u,v)} \cdot \left(\frac{\partial(u,v)}{\partial(U,V)}\right)$$

this factor will be the same in all!

$$\frac{\partial(x,z)}{\partial(U,V)} = \frac{\partial(x,z)}{\partial(u,v)}\,\frac{\partial(u,v)}{\partial(U,V)}$$

$$\frac{\partial(y,z)}{\partial(U,V)} = \frac{\partial(y,z)}{\partial(u,v)}\,\frac{\partial(u,v)}{\partial(U,V)}$$

(all matrix multiplications!)