1) Which of the following angles is **not** coterminal with the angle $\frac{2\pi}{3}$?

Add or subtract multiples of 2π.

$$\frac{2\pi}{3}+2\pi=\frac{8\pi}{3} \qquad \frac{2\pi}{3}-2\pi=-\frac{4\pi}{3}$$
$$\frac{8\pi}{3}+2\pi=\frac{14\pi}{3} \qquad \frac{14\pi}{3}+2\pi=\frac{20\pi}{3}$$

Convert to degrees and add or subtract multiples of 360°.

$$\frac{2\pi}{3}\left(\frac{180^\circ}{\pi}\right)=120^\circ$$
$$120^\circ+360=480^\circ \qquad 480^\circ+360=840^\circ$$
$$120^\circ-360=-240^\circ$$

The answer that does not fit is $\frac{11\pi}{3}$.

2) Convert $112^\circ\,55'50''$ to radians, rounded to 4 decimal places.

$$112^\circ+55'\left(\frac{1^\circ}{60'}\right)+50''\left(\frac{1^\circ}{3600''}\right)$$

Convert to degrees decimal: $=112+0.91\overline{6}+0.013\overline{8}$

$=112.9305556^\circ$

Convert this to radians and round: $112.9305556^\circ\left(\frac{\pi}{180^\circ}\right)$

$=1.971010021$

Answer is 1.9710.

3) The distance between two points *A* and *B* on the earth is measured along a circle having center *C* of earth and radius equal to the distance from *C* to the surface. If the **diameter** of the earth is approximately 12,750 kilometers, approximate, to the nearest kilometer, the distance between *A* and *B*, if angle *ABC* is **45°.**

Radius is half of diameter or 6375 km. Angle θ **must be in radians:**

$$\theta=45^\circ\left(\frac{\pi}{180^\circ}\right)=\frac{\pi}{4}$$

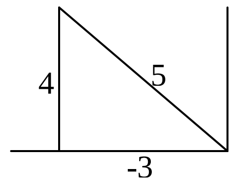
We need to find the distance along the surface of the earth, or an arc length.

$$s=r\theta$$
$$s=6375\left(\frac{\pi}{4}\right)\approx 5007$$

5007 km

4) Find the exact value of $\sec\alpha$, if α is in standard position and the terminal side of α is in QII and parallel to the line $4x+3y=12$.

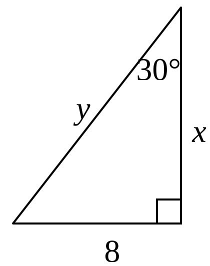
slope of line = $-\dfrac{4}{3}=\dfrac{\Delta y}{\Delta x}=\dfrac{opp}{adj}$ In Q II:



$$\sec\alpha=\frac{hyp}{adj}=\frac{5}{-3}$$

$$\boxed{\sec\alpha=-\frac{5}{3}}$$

5) Find the exact values of *x* and *y* for the right triangle at the right.

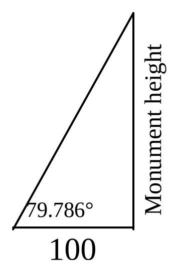


$$\tan 30^\circ=\frac{opp}{adj} \qquad \sin 30^\circ=\frac{opp}{hyp}$$

$$\frac{1}{\sqrt{3}}=\frac{8}{x} \qquad \frac{1}{2}=\frac{8}{y}$$

$$\boxed{x=8\sqrt{3} \qquad y=16}$$

6) At the distance of 100 feet from the base of the Washington Monument in Washington D.C. and on level ground, the angle from the ground to the top of the capstone of the monument is 79.786°. Approximate the height of the Washington Monument to the nearest foot.



$$\tan 79.786^\circ=\frac{monument\ height}{100}$$

$$100(5.549985)=height$$

$$554.9985=height$$

$$\boxed{555 \text{ feet}}$$

7) $(1+\tan x)(1-\tan x)$ is equivalent to which of the following?

$(1+\tan x)(1-\tan x)=1-\tan^2 x$ (Use FOIL)

$=1-(\sec^2 x-1)$ (Use a variation of Pythagorean Identity)

$=1-\sec^2 x+1$

$$\boxed{=2-\sec^2 x}$$

8) $\csc(u) + \cos(-u)\cot(-u)$ is equivalent to which of the following?

$$\csc(u) + \cos(-u)\cot(-u) = \csc u + (\cos u)(-\cot u) \quad \text{(Use negative formulas)}$$

$$= \frac{1}{\sin u} + \frac{\cos u}{1}\left(-\frac{\cos u}{\sin u}\right) \quad \text{(Use reciprocal and cotangent identiies)}$$

$$= \frac{1}{\sin u} + \frac{-\cos^2 u}{\sin u}$$

$$= \frac{1 - \cos^2 u}{\sin u}$$

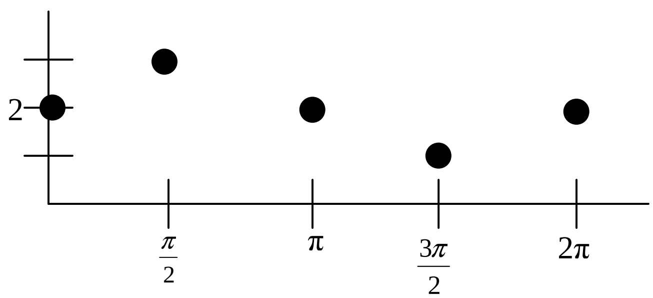
$$= \frac{\sin^2 u}{\sin u} \quad \text{(Pythagorean identity)}$$

$$\boxed{= \sin u}$$

9) Which of the following statements is (are) **true** concerning the graph of $y = \sin(x) + 2$?

I	The point $\left(\frac{\pi}{2}, 3\right)$ is on the graph.
II	The graph intersects the *y*-axis (*y*-intercept)at 2.
III	The range of the function is [1,3].

Sketch a sine graph shifted vertically 2 upward.



You can see that $\left(\frac{\pi}{2}, 3\right)$ is a point on the graph, so statement I is true. The *y*-intercept is obviously 2, so statement II is true. The graph would oscillate between 1 and 3, so the range is [1, 3], so statement III is true. I, II, and III

10) Find the reference angle θ_R if $\theta = -1.73$. Round to the nearest hundredth of a radian.

First make θ positive, by adding 2π. $-1.73 + 2\pi \approx 4.553185$

This is an angle in Q III (between 3.14 and 4.71) so reference angle is the difference between π and θ:

$$\theta_R = 4.553185 - \pi$$
$$\approx 1.41$$

$$\boxed{\theta_R = 1.41}$$

11) Approximate $\csc(2.35)$ to 4 decimal places. (Check mode on calculator.)

Make sure calculator is in radian mode.

$$\csc(2.35) = \frac{1}{\sin(2.35)} = \frac{1}{0.711473353} = 1.405534$$

csc(2.35)=1.4055

12) Approximate, to the nearest tenth of a degree, all angles θ in the interval $[0^\circ, 360^\circ)$ that satisfy the equation $\sec\theta = -1.3829$ (Check mode on calculator.)
You are given a trig value and asked to find any angles in the interval with that secant value.

Put calculator in degree mode. $\cos\theta = \frac{1}{\sec\theta} = \frac{1}{-1.3829} = -0.723118085$

Use the inverse cosine key function on calculator: $\cos^{-1}(-0.723118085) = 136.3125^\circ$

This result is one of the answers in the interval:

$\theta = 136.3^\circ$

However, this is **not** a reference angle because it is over 90°.

$$\theta_R = 180^\circ - 136.3125 = 43.6875^\circ$$

The secant is negative in quadrants II and III. To find the angle in QIII, add the reference angle to 180°.

$\theta = 180 + 43.6875^\circ$ $\theta = 223.7^\circ$

13) What is the amplitude, period, and phase shift of the graph with equation $y = -3\sin(2x+3)$?

Amplitude is **always positive**: $|-3| = 3$

Period: $\frac{2\pi}{|b|} = \frac{2\pi}{2} = \pi$ Phase Shift: $-\frac{c}{b} = -\frac{3}{2}$

Amplitude: 3, Period: π, Phase Shift: $-\frac{3}{2}$

14) Scientists sometimes use the formula $f(t) = a\sin(bt+c)+d$ to simulate temperature variations during the 24 hour day, with *t* in hours, temperature $f(t)$ in °C, and *t* = 0 corresponding to midnight. Assume that the temperature is decreasing at midnight.

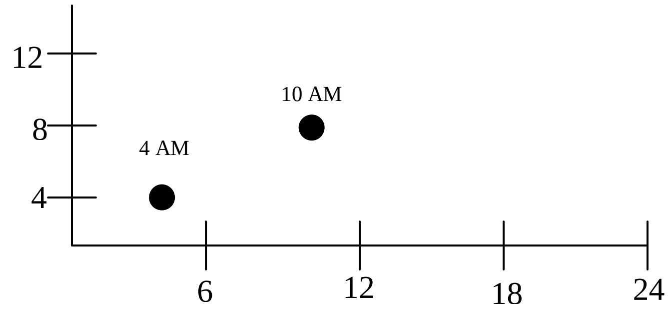
On a given day, the high temperature was 12°C and the low temperature of 4°C first occurred at 4 AM. Which equation represents $f(t)$?
Sketch a graph with a sine curve oscillating between 4 and 12 degrees and an average half-way between of 8 degrees.

d = 8 since graph is shifted from an average of 0 to 8.	*a* = 4 since the difference between the average and maximum or minimum is 4.

$$24 = \frac{2\pi}{b}$$

A 24 hour period leads to $24b = 2\pi$

$$\boxed{b = \frac{\pi}{12}}$$



The problem says the low first occurred at 4 AM, so 6 hours later (a quarter of a period) the average would occur at 10 AM. The sine curve is rising at this time, so the phase shift is 10.

$$10 = -\frac{c}{\frac{\pi}{12}} = -\frac{12c}{\pi}$$

$$10\pi = -12c$$

$$\boxed{-\frac{5\pi}{6} = c}$$

All values inserted in formula:
$$f(t) = 4\sin\left(\frac{\pi}{12}t - \frac{5\pi}{6}\right)$$

15) Which is the graph of $y = -2\sin\left(\frac{\pi}{2}x + \frac{\pi}{2}\right)$?

Amplitude would be 2. That eliminates no answers. All have amplitude 2.

Period: $\frac{2\pi}{\frac{\pi}{2}} = \frac{2\pi}{1}\left(\frac{2}{\pi}\right) = 4$ On examination, this eliminates choices D and E. They have a period (one cycle of sine curve) of 2.

Phase Shift: $-\frac{\frac{\pi}{2}}{\frac{\pi}{2}} = -1$ This eliminates choice C. It has no phase shift.

Lastly, the value of a is -2, so the graph is vertically reflected. Instead of the sine curve increasing at the phase shift, it should be **decreasing at the phase shift: choice A**