1) Given ΔABC with $\angle C = 90°$, $c = 215.1$, and $\alpha = 49^{\circ}42'$; approximate b to the nearest tenth and $\angle B$ *or* β to the nearest minute. Hint: Draw a triangle.

A $b = 182.4,\ \angle B = 40^{\circ}18'$

B $b = 139.9,\ \angle B = 40^{\circ}58'$

C $b = 164.0,\ \angle B = 40^{\circ}18'$

D $b = 164.0,\ \angle B = 40^{\circ}58'$

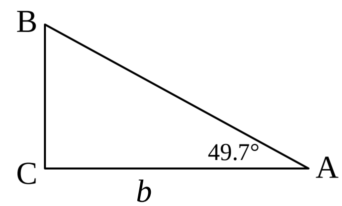
E $b = 139.1,\ \angle B = 40^{\circ}18'$

$\angle B = 90^{\circ} - 49^{\circ}42' = 40^{\circ}18'$

$\cos 49.7^{\circ} = \dfrac{b}{215.1}$

$b = 215.1\cos(49.7^{\circ})$

$b = 139.1$



2) A measure the height of a large billboard in Times Square in New York City, two sightings 500 feet along a street from below the billboard are taken. The angle of elevation to the bottom of the billboard is 55° and the angle of elevation to the top of the billboard is 61°30'. What is the height of the billboard to the nearest foot?

A 207 *ft.*

B 79 *ft.*

C 30 *ft.*

D 219 *ft.*

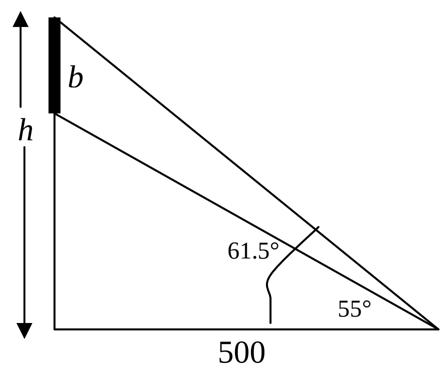
E None of the Above.

$\tan\left(55^{\circ}\right) = \dfrac{h-b}{500}$

$500\tan\left(55^{\circ}\right) = h - b$

$\tan\left(61.5^{\circ}\right) = \dfrac{h}{500}$

$500\tan(61.5^{\circ}) = h$



$500\tan(55) = 500\tan(61.5) - b$ (substitution)

$b = 500(\tan(61.5) - \tan(55))$

$b = 206.8$

3) Ship A leaves port at **2:00 PM** and sails in the direction S 23° E at a rate of 25 miles per hour. The second ship, B, leaves the port at **2:30 PM** and sails in the direction N 67° E at a rate of 30 miles per hour. What is the heading from ship A to ship B at 4:00 PM?

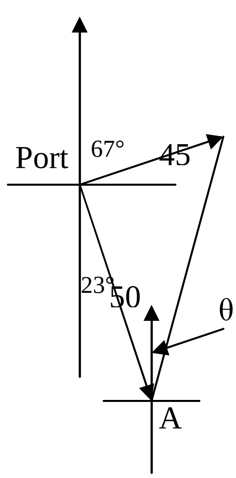
A $S\,19^{\circ}\,W$

B $N\,23^{\circ}\,E$

C $S\,48^{\circ}\,W$

D $N\,19^{\circ}\,E$

E $N\,42^{\circ}\,E$



heading = θ

$\angle A - 23^{\circ} = \theta$

$\tan A = \dfrac{45}{50} = 0.9$

$A = 42^{\circ}$

$\theta = 42^{\circ} - 23^{\circ} = 19^{\circ}$ $\quad N\,19^{\circ}\,E$

4) A man and his son are playing paintball. The man is on the roof of a building and wants to hit his son who is standing on the ground. The **angle of depression** from the man's paint gun to his son's feet is 65° and the height from the base of the building to the man's paint gun is 28 feet. How far will the paint ball travel through the air from the paint gun until it hits the feet of the son? Assume the paint ball travels in a straight line. Round to the nearest tenth of a foot.

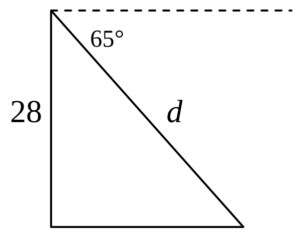
A 66.3 *ft*.

B 25.4 *ft*.

C 60.0 *ft*.

D 30.9 *ft*.

E None of the Above.

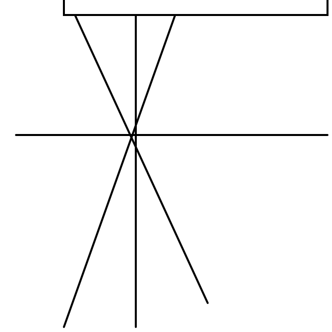


$$\cos(25^\circ) = \frac{28}{d}$$

$$d = \frac{28}{\cos(25^\circ)}$$

$$d = 30.9\ ft.$$

α will be in all 4 quadrants with a reference angle of $\frac{\pi}{3}$



5) Find all solutions of the equation below using *n* as an arbitrary integers.

$$4\cos^2\alpha - 1 = 0$$

A $\alpha = \frac{\pi}{3} + \pi n,\ \frac{2\pi}{3} + \pi n$

B $\alpha = \frac{\pi}{6} + \pi n,\ \frac{5\pi}{6} + \pi n$

C $\alpha = \frac{\pi}{3} + 2\pi n,\ \frac{2\pi}{3} + 2\pi n$

D $\alpha = \frac{\pi}{6} + 2\pi n,\ \frac{5\pi}{6} + 2\pi n$

E None of the Above.

$$4\cos^2\alpha = 1$$

$$\cos^2\alpha = \frac{1}{4}$$

$$\cos\alpha = \pm\frac{1}{2}$$

$$\alpha = \frac{\pi}{3} + \pi n \text{ and } \alpha = \frac{2\pi}{3} + \pi n$$

6) Find all solutions **in the interval $[0, 2\pi)$** for the equation below. How many solutions are there?

$$\tan\left(2x - \frac{\pi}{2}\right) = -\frac{1}{\sqrt{3}}$$

Reference angle is $\frac{\pi}{6}$ in Q II and Q IV(tan is negative). Terminal sides are in opposite directions. Use $\frac{5\pi}{6} + \pi n$ equal to angle.

A 2

B 3

C 4

D 5

E 6

$$2x - \frac{\pi}{2} = \frac{5\pi}{6} + \pi n$$

$$2x = \frac{4\pi}{3} + \pi n$$

$$x = \frac{2\pi}{3} + \frac{\pi}{2} n$$

n	*x*
-1	π/6
0	2π/3
1	7π/6
2	5π/3

4 solutions

7) Which statement is **false**? Use your knowledge of the sum/difference formulas, double-angle formulas, cofunction formulas, and/or inverse trig functions.

A $\cot(23^\circ 40') = \tan(66^\circ 20')$

B $\sin\left(\frac{\pi}{3} - \theta\right) = \sin\left(\frac{\pi}{3}\right)\cos\theta - \cos\left(\frac{\pi}{3}\right)\sin\theta$

C $\tan\left(55^\circ\right) = \dfrac{\tan\left(25^\circ\right) - \tan\left(30^\circ\right)}{1 + \tan(25^\circ)\tan(30^\circ)}$

D $\sin\left(\frac{\pi}{3}\right) = 2\sin\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}\right)$

E $\arcsin\left(\sin\frac{5\pi}{4}\right) = -\frac{\pi}{4}$

The false one is the one with tangent or 55°. The numerator should have a + and the denominator a – to be the sum of two angles tangent formula.

8) Find the exact value of $\sin(120^\circ - 225^\circ)$.

A $\dfrac{-\sqrt{6}+\sqrt{2}}{4}$

B $\dfrac{-\sqrt{6}-\sqrt{2}}{4}$

C $\dfrac{-\sqrt{3}-3}{4}$

D $\dfrac{\sqrt{6}-\sqrt{2}}{4}$

E $\dfrac{\sqrt{6}+\sqrt{2}}{4}$

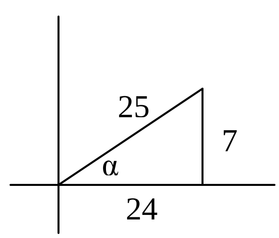
$= \sin(120)\cos(225) - \cos(120)\sin(225)$

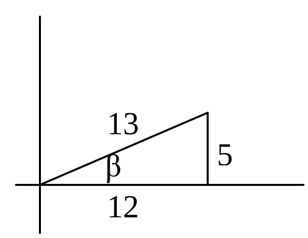
Remember: The sin is positive in Q II and negative in Q III; the cos in negative in both these quadrants.

$$= \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) - \left(-\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)$$

$$= -\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{-\sqrt{6}-\sqrt{2}}{4}$$

9) If α and β are **acute** angles such that $\sin\alpha = \frac{7}{25}$ and $\cos\beta = \frac{12}{13}$, find the exact value of $\cos(\alpha - \beta)$.





A $\dfrac{253}{325}$

B $\dfrac{204}{325}$

C $-\dfrac{36}{325}$

D $\dfrac{323}{325}$

E None of the Above.

$= \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$$= \left(\frac{24}{25}\right)\left(\frac{12}{13}\right) + \left(\frac{7}{25}\right)\left(\frac{5}{13}\right)$$

$$= \frac{288}{325} + \frac{35}{325} = \frac{323}{325}$$

10) Find the exact solutions of the equation below **in the interval $[0, 2\pi)$.**

$$\sin t - \sin(2t) = 0$$

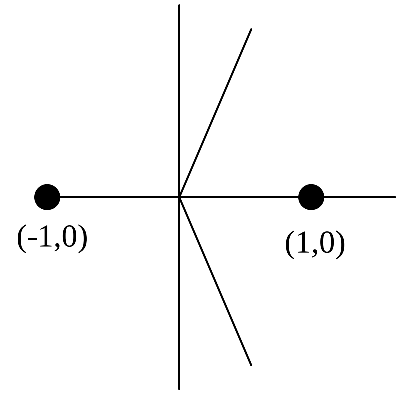
$\sin t - 2\sin t\cos t = 0$

$\sin t(1 - 2\cos t) = 0$

$\sin t = 0 \qquad 1 - 2\cos t = 0$

$\cos t = \frac{1}{2}$

$t = 0, \pi \qquad t = \frac{\pi}{3}, \frac{5\pi}{3}$



A $t = 0, \frac{\pi}{3}, \pi, \frac{4\pi}{3}$

B $t = \frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{3}$

C $t = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

D $t = \frac{\pi}{3}, \frac{5\pi}{3}$

E None of the Above.

11) If a projectile is fired from ground level with an initial velocity of v feet per second and at an angle of θ degrees with the horizontal, the range R of the projectile is given by $R = \frac{v^2}{16}\sin\theta\cos\theta$. If v = 90 feet per second, approximate the angle(s) that result(s) in a range of 200 feet. Round to the nearest tenth of a degree.

A $\theta = 52.2^\circ$

B $\theta = 26.1^\circ, \ 63.9^\circ$

C $\theta = 11.6^\circ, 78.4^\circ$

D $\theta = 23.3^\circ$

E None of the Above.

$200 = \frac{90^2}{16}\sin\theta\cos\theta$

$200 = \frac{4050}{16}(2)\sin\theta\cos\theta$

$0.790123457 = \sin(2\theta)$

$2\theta = 52.197^\circ \qquad 2\theta = 127.803^\circ$

$\theta = 26.1^\circ \qquad \theta = 63.9^\circ$

12) The expression $\frac{\sin^2(2x)}{3\cos^2 x}$ is equivalent to which of the following?

A $\frac{2\sin^2 x}{3}$

B $\frac{\sin(4x)}{3\cos^2 x}$

C $\frac{2\sin x}{3\cos x}$

D $\frac{4\sin x}{3}$

E $\frac{4\sin^2 x}{3}$

$$\frac{\sin^2(2x)}{3\cos^2 x} = \frac{(2\sin x\cos x)^2}{3\cos^2 x}$$

$$= \frac{4\sin^2 x\cos^2 x}{3\cos^2 x}$$

$$= \frac{4\sin^2 x}{3}$$

13) Find the exact value of the following: $\sin^{-1}\left[\cos\left(\frac{7\pi}{6}\right)\right]$.

A $\frac{4\pi}{3}$

B $\frac{\pi}{3}$

C $\frac{5\pi}{3}$

D $-\frac{\pi}{6}$

E $-\frac{\pi}{3}$

$$\sin^{-1}\left[\cos\left(\frac{7\pi}{6}\right)\right]=\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

$$=-\frac{\pi}{3}$$ (Inverse sin returns a negative value as a negative angle in Q IV.)

14) Use the quadratic formula (and an inverse trigonometric function) to approximate the solutions of the following equation in the interval $[0,2\pi)$ to three decimal places. Check mode on calculator.

$$3\sin^2 x+4\sin x-1=0$$

A $x=12.430$

B $x=0.217,\ 2.925$

C $x=3.359,\ 6.066$

D No solution.

E None of the Above.

$$\sin x=\frac{-4\pm\sqrt{16-4(3)(-1)}}{2(3)}=\frac{-4\pm\sqrt{28}}{6}$$

$\sin x=0.21525$ $\quad\quad$ $\sin x=-1.548$

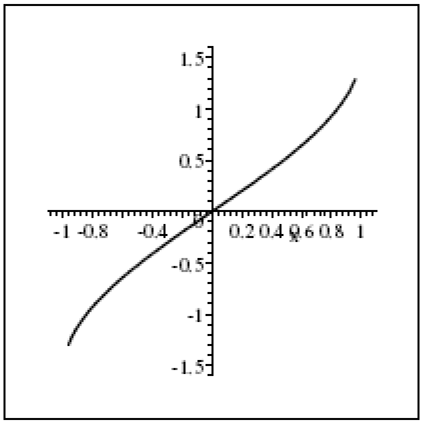
$x_R=0.216948$ $\quad\quad$ (This is impossible)

$x=0.217$

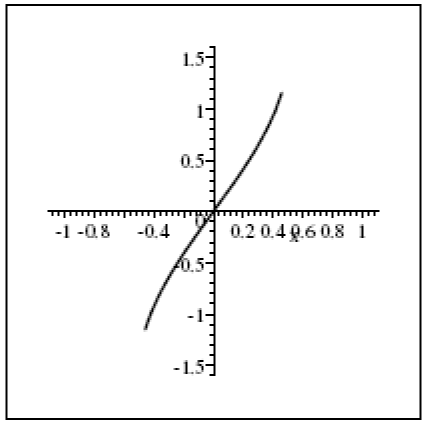
$x=\pi-x_R=2.925$

15) Which is the graph of $y = \cos^{-1}\left(\frac{1}{2}x\right)$?

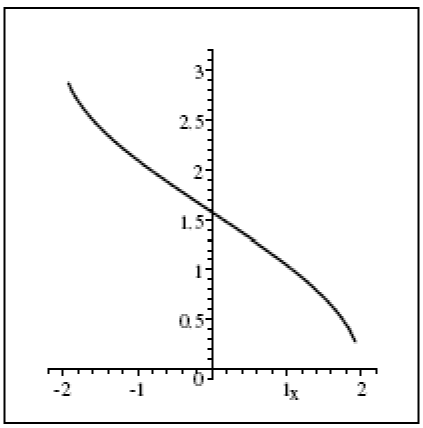
The domain of the function has been 'doubled'. The domain is [-2, 2]. The range of an inverse cosine function is [0, π]. The graph that fits on this form of the exam is C.



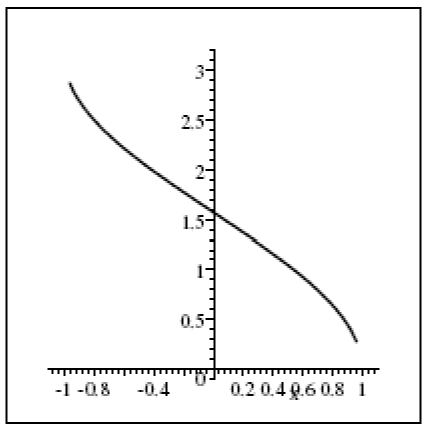
A



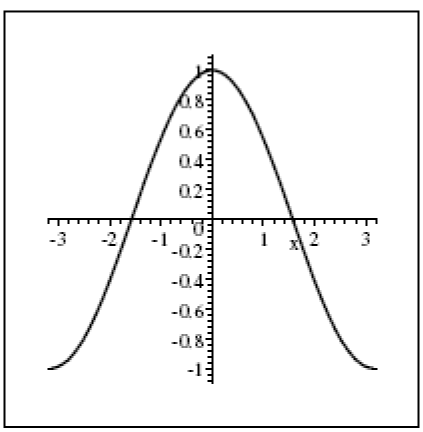
B



C



D



E