Computer Project # 1
Nonlinear Springs

Goal: Investigate the behavior of nonlinear springs.

Tools needed: **ode45**, **plot**.

Description: For certain (nonlinear) spring-mass systems, the spring force is not given by Hooke's Law but instead satisfies

$$F_{\text{Spring}} = k\,u + \epsilon\,u^3$$

where $k > 0$ is the spring constant and ϵ is small but may be positive or negative and represents the "strength" of the spring ($\epsilon = 0$ gives Hooke's Law). The spring is called a hard spring if $\epsilon > 0$ and a soft spring if $\epsilon < 0$.

Questions: Suppose a nonlinear spring-mass system satisfies the initial value problem

$$\begin{cases} u'' + u + \epsilon\,u^3 = 0 \\ u(0) = 0,\ \ u'(0) = 1 \end{cases}.$$

Use **ode45** and **plot** to answer the following:

(1) Let $\epsilon = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0$ and plot the solutions of the above initial value problem for $0 \le t \le 20$. Estimate the amplitude of the spring. Experiment with various $\epsilon > 0$. What appears to happen to the amplitude as $\epsilon \longrightarrow \infty$? Let μ^+ denote the first time the mass reaches equilibrium after $t = 0$. Estimate μ^+ when $\epsilon = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0$. What appears to happen to μ^+ as $\epsilon \longrightarrow \infty$?

(2) Let $\epsilon = -0.1, -0.2, -0.3, -0.4$ and plot the solutions of the above initial value problem for $0 \le t \le 20$. Estimate the amplitude of the spring. Experiment with various $\epsilon < 0$. What appears to happen to the amplitude as $\epsilon \longrightarrow -\infty$? Let μ^- denote the first time the mass reaches equilibrium after $t = 0$. Estimate μ^- when $\epsilon = -0.1, -0.2, -0.3, -0.4$. What appears to happen to μ^- as $\epsilon \longrightarrow -\infty$?

(3) Given that a certain nonlinear *hard* spring satisfies the initial value problem

$$\begin{cases} u'' + \frac{1}{5}u' + (u + \frac{1}{5}u^3) = \cos\omega t \\ u(0) = 0,\ \ u'(0) = 0 \end{cases},$$

plot the solution $u(t)$ over the interval $0 \le t \le 60$ for $\omega = 0.5, 0.7, 1.0, 1.3, 2.0$. Continue to experiment with various values of ω, where $0.5 \le \omega \le 2.0$, to find a value ω^* for which $|u(t)|$ is largest over the interval $40 \le t \le 60$.