Lesson 37 Sections 7.3 & 7.4

Examine the following:

$\sqrt{4}\cdot\sqrt{9}=2\cdot 3=6$

$\sqrt{4\cdot 9}=\sqrt{36}=6$

Since both equal 6, the expressions are equal.

Conclusion: $\sqrt{4}\cdot\sqrt{9}=\sqrt{4\cdot 9}$

Likewise:

$\dfrac{\sqrt{16}}{\sqrt{4}}=\dfrac{4}{2}=2$

$\sqrt{\dfrac{16}{4}}=\sqrt{4}=2$

Since both equal 2, the expressions are equal.

Conclusion: $\dfrac{\sqrt{16}}{\sqrt{4}}=\sqrt{\dfrac{16}{4}}$

These observations lead to two very important rules: the Product and Quotient Rules for Radicals.

Product Rule for Radicals: $\sqrt[n]{a}\bullet\sqrt[n]{b}=\sqrt[n]{ab}$

Quotient Rule for Radicals: $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}=\sqrt[n]{\dfrac{a}{b}}$

Caution: These rules only apply when the indices (plural of index) are equal!

Use the rules above (if possible) to multiply, divide, or otherwise simplify.

1. $(\sqrt[3]{13})(\sqrt[3]{6})=$

2. $\sqrt{\dfrac{2}{3}}\cdot\sqrt{\dfrac{t}{4}}=$

3. $\sqrt{2x-3}\cdot\sqrt{2x+3}=$

4. $\sqrt{\dfrac{25}{x^2}} =$

5. $\sqrt[3]{\dfrac{2a^6}{27}} =$

6. $\dfrac{\sqrt{80}}{\sqrt{5}} =$

7. $\left(\sqrt{3}\right)\left(\sqrt[3]{x}\right) =$

<u>The product rule can also be used to simplify a radical by using factoring.</u>

Look at the following example.

$$\sqrt{20} = \sqrt{4 \cdot 5} = \sqrt{4} \cdot \sqrt{5} = 2\sqrt{5}$$

To simplify a radical with index *n* (using factoring or the product rule), use the following steps.

1. Express the radicand as a product in which one factor is the largest perfect *n*th power possible.
2. Take the *n*th root of each factor.
3. Simplification is complete when no radicand has a factor that is a perfect *n*th power.

Simplify the following radicals.

4. $\sqrt{300} =$

5. $\sqrt{8x^3} =$

6. $\sqrt[3]{54} =$

7. $\sqrt[3]{27p^5} =$

8. $\sqrt{72x^3y^6} =$

9. $\sqrt[4]{32a^9} =$

10. $\sqrt[5]{-243a^7b^3} =$

Many directions and procedures in algebra, trigonometry, and calculus require that radical answers be given with no radical sign in a denominator. To clear a radical sign in a denominator is sometimes easy, such as in the case of $\sqrt[3]{\frac{3}{8}}$. Simply use the quotient rule: $\sqrt[3]{\frac{3}{8}} = \frac{\sqrt[3]{3}}{\sqrt[3]{8}} = \frac{\sqrt[3]{3}}{2}$ However, sometimes a process call **rationalizing the denominator** must be used. Examine the next example.

$$\sqrt{\frac{2}{7}} = \sqrt{\frac{2 \cdot 7}{7 \cdot 7}} = \frac{\sqrt{14}}{7}$$

Rationalize each denominator.

11. $\frac{\sqrt{6}}{\sqrt{5}} =$

12. $\sqrt{\frac{4}{11}} =$

13. $\frac{5}{\sqrt{x}} =$

Sometimes it is necessary to **simplify any radical (in numerator and/or denominator) <u>before</u> rationalizing!!!**

14. $\frac{2}{\sqrt{8}} =$

15. $\sqrt{\frac{64}{27}} =$

16. $\frac{\sqrt{12}}{\sqrt{x^3}} =$

In this class, we will only do rationalizing with square roots. In future math courses, you may study rationalizing with cube roots, etc.