MA 23100 Fall 2009

NAME:______________________________ Exam 3

PUID:______________________________

INSTRUCTIONS

- No books or notes are allowed.
- You may use a one-line scientific calculator. No other electronic device is allowed. Be sure to turn off your cellphone.
- Show all your work in the space provided. Little or no credit may be given for an answer with insufficient or inconsistent work, even if the answer happens to be correct.
- Write answers in the boxes provided. All answers are expected to be simplified ($\frac{2}{4} \to \frac{1}{2}$, $2x + x \to 3x$, $e^{\ln 2} \to 2$, etc).

| Question | Possible | Score |
|---|---|---|
| 1 | 30 | |
| 2 | 10 | |
| 3 | 12 | |
| 4 | 12 | |
| 5 | 12 | |
| 6 | 12 | |
| 7 | 12 | |
| Total | 100 | |

1.) (30 pts) Find $f'(x)$ for each of the following functions:

(a) $f(x) = e^{-x^3}$

$f'(x) =$

(b) $f(x) = \frac{4}{1+3e^{7x}}$

$f'(x) =$

(c) $f(x) = x^5 \ln(x^5)$

$f'(x) =$

(d) $f(x) = \ln\left(\frac{2x}{(x^2+1)^2}\right)$

$f'(x) =$

(e) $f(x) = 7xe^{-x^2}$

$f'(x) =$

2.) (10 pts) Let

$$f(x) = x^3 - 12x + 9$$

Find the absolute maximum and absolute minimum values of $f(x)$ over the interval $[-3, 1]$.

| |
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| maximum: |
| minimum: |

3.) (12 pts) Give the linearization for the function

$$f(x) = \sqrt[3]{x^3 + 19}$$

at the point $a = 2$.

| |
|---|
| $L(x) =$ |

4.) (12 pts) Suppose

$$x^2e^{2y} + y^3 = 8.$$

Find $\frac{dy}{dx}$.

$\frac{dy}{dx} =$

5.) (12pts) A certain radioactive isotope decays exponentially. That is

$$\frac{dA}{dt} = -kA$$

where $A$ is the amount of the isotope still present after $t$ years, and $k$ is a positive constant. It is known to have a half-life of 150 years. The isotope is created upon exposure of organic material to a special type of energy. An artifact is found to have at one time been exposed to that type of energy but has since lost 70% of the amount of the isotope originally created. How long ago was the artifact exposed to that energy?

Round your answer to at least 2 decimal places.

$t \approx$

6.) (12 pts) A spherical balloon is being inflated at a rate of $4\pi$ cubic inches of air each minute (this means that $\frac{dV}{dt} = 4\pi$). Assuming that the balloon maintains its spherical shape, how hast is the surface area of the ballon changing when the radius of the balloon is 5 inches?

The volume of a sphere with radius $r$ is given by $V = \frac{4}{3}\pi r^3$. The surface area of a sphere is given by $S = 4\pi r^2$.

Hint: One way to do this problem is to find how fast the radius is changing first, and then use that value to find how fast the surface area is changing.

| $\frac{dS}{dt} =$ |
|---|

7.) (12 pts) The owners of an apartment complex have found that they can fill all 200 of their apartments if they charge a monthly rent of \$800 for each unit. They have also found each time the monthly rent is raised by \$30, they lose 3 renters, so that when the rent is \$830, they fill only 197 units, when the rent is \$860, they only fill 194 units, etc.

Assuming this trend continues, how much monthly rent should the apartment complex charge so as to generate the greatest possible monthly revenue?

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| Rent: |