Fall 2008

MA 265
Final Exam

Name ______________________________________

PUID# ______________________________________

Instructor ____________________________________

Section# ______________ **Class Time** ______________

Instructions

1. Make sure you have a complete test. There are **12** different test pages, including this cover page.
2. Your PUID# is your student identification number. DO NOT list your social security number.
3. <u>**Using a #2 pencil**</u>, fill in each of the following items on your <u>**answer sheet**</u>:
   (a) On the top left side, print your name (last name, first name), and fill in the little circles.
   (b) On the bottom left side, under SECTION, write in your 4 digit section number and fill in the little circles. If you do not know your division and section number, <u>ask your lecturer</u>.
   (c) On the bottom, under STUDENT IDENTIFICATION NUMBER, write in your Purdue student I.D. number and fill in the little circles.
   (d) On the bottom right, print your <u>instructor's name</u> and the <u>course number</u>.
4. Do any necessary work for each problem in the space provided or on the back of the pages of this test. No partial credit is given but your work may be considered if your grade is on the borderline. Circle your answers in this test.
5. Each problem is worth 10 points. The maximum possible score is 200 points.
6. <u>**Using a #2 pencil**</u>, put your answers to questions 1–20 on your answer sheet by filling in the circle of the letter of your response. Double check that you have filled in the circles you intended. If more than one circle is filled in for any question, your response will be considered incorrect.
7. NO books, notes or calculators are allowed on this exam.
8. Turn OFF all cellphones and iPods.
9. After you have finished the exam, hand in your answer sheet <u>and</u> this test to your lecturer.

1. Given the linear system $\left\{ \begin{array}{l} 3x - 2y = 7 \\ 6x - ty = 8 \end{array} \right.$

A. We have infinitely many values of t for which the system is consistent.

B. We have exactly one value of t for which the system is consistent.

C. We have no value of t for which the system is consistent.

D. We have exactly two values of t for which the system is consistent.

E. We have exactly three values of t for which the system is consistent.

2. How many 2×2 matrices A such that $A^2 = 0$ can we find ?

A. Only one, the zero matrix

B. Two, which are negative to each other

C. Exactly three

D. Infinity many

E. Exactly four

3. The first row in the *reduced* row echelon form of a 3×3 matrix A is $\begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$.
Which of the following statements are TRUE ?

(i) A is *not* row-equivalent to I_3

(ii) The third column of the *reduced* row echelon form of A is necessarily $\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$

A. Both **(i)** and **(ii)**.

B. Only **(i)**.

C. Only **(ii)**.

D. Neither are true.

E. **(i)** is true sometimes.

4. Consider the vector space P_2 of polynomials in t of degree ≤ 2. Let $v_1 = t^2 + 2t + 1$, $v_2 = t + 1$, and $v_3 = t^2 + t$. Which one of the following vectors is in the span of $\{v_1, v_2, v_3\}$?

A. $t^2 + 5t + 4$

B. $2t^2 + 3t + 2$

C. $-t^2 - 2t + 3$

D. $t^2 + 1$

E. $t^2 + t - 1$

5. When is the following system consistent?

$$\begin{aligned} 2x + 2y + 3z &= r \\ 3x - y + 5z &= s \\ x - 3y + 2z &= t \end{aligned}$$

A. If and only if $r + 2s + t = 0$.

B. If and only if $t = s - r$.

C. If and only if $r + s = -2t$.

D. If and only if $3r - s - 2t = 1$.

E. If and only if $r = s = t = 0$.

6. For what values of t is the determinant of the following matrix *zero*?

$$B = \begin{bmatrix} (t-1) & 0 & 1 \\ 0 & (t+2) & 0 \\ 3 & -1 & 2 \end{bmatrix}$$

A. Only $t = 5$

B. Only $t = -2$

C. $t = -2$ and $t = \frac{5}{2}$

D. All t except $t = -2$ and $t = 5$

E. $t = 1$ and $t = -2$

7. Let V be the set of all 3×3 matrices. Which of the following is **not** a subspace of V ?

(i) $W = \left\{ \begin{bmatrix} a & b & 0 \\ b & a & 0 \\ 0 & 0 & c \end{bmatrix} : a, b, c \in \mathbb{R} \text{ and } a + b = 1 \right\}$

(ii) $W = \left\{ \begin{bmatrix} a^2 & b^2 & a+b \\ b^2 & a^2 & 0 \\ a+b & 0 & c \end{bmatrix} : a, b, c \in \mathbb{R} \text{ and } (2a+b)^2 + b^2 = 0 \right\}$

(iii) $W = \left\{ \begin{bmatrix} a+1 & 0 & 0 \\ 0 & a+1 & 0 \\ 0 & 0 & b \end{bmatrix} : a, b \in \mathbb{R} \right\}$

(iv) $W = \left\{ \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} : a, b, c \in \mathbb{R} \right\}$

(v) $W = \left\{ \begin{bmatrix} a+c & b & 0 \\ b & c & 0 \\ 0 & 0 & a \end{bmatrix} : a, b, c \in \mathbb{R} \text{ and } 2a + b - c = 0 \right\}$

A. (i) and (iii)

B. (ii) and (iii)

C. (i)

D. (ii)

E. (iii)

8. Which of the following sets of vectors in $\mathbb{R}^3$ is linearly independent?

$$S_1 = \left\{ \begin{bmatrix} 1 \\ 0 \\ -5 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ -6 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix} \right\}$$

$$S_2 = \left\{ \begin{bmatrix} 1 \\ 0 \\ -5 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -10 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} \right\}$$

$$S_3 = \left\{ \begin{bmatrix} 1 \\ 0 \\ -5 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$

$$S_4 = \left\{ \begin{bmatrix} 1 \\ 0 \\ -5 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} \right\}$$

$$S_5 = \left\{ \begin{bmatrix} 1 \\ 0 \\ -5 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix} \right\}$$

A. S_1

B. S_2

C. S_3

D. S_4

E. S_5

9. Which one of the following sets is a basis for $\mathbb{R}^4$?

A. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 2 \\ 1 \end{bmatrix} \right\}$

C. $\left\{ \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 2 \\ 1 \end{bmatrix} \right\}$

D. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 10 \\ 20 \\ 30 \\ 40 \end{bmatrix} \right\}$

E. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 3 \end{bmatrix} \right\}$

10. The dimension of the solution space of $\left\{ \begin{array}{rrrrl} x_1 & +2x_2 & -x_3 & -2x_4 & = 0 \\ x_1 & +3x_2 & -2x_3 & +x_4 & = 0 \\ 3x_1 & +8x_2 & -5x_3 & & = 0 \end{array} \right.$ is equal to

A. 0

B. 1

C. 2

D. 3

E. 4

11. Let A be a 4×7 matrix, and let $B = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 0 & 2 & 2 & 2 \\ 0 & 0 & 0 & 1 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ be the *reduced row echelon form* of A. Which of the following statements is TRUE ?

(i) The *rank* of A is 6.

(ii) The dimension of $Null(A)$ is 4.

(iii) The rank of A cannot be determined because we only know B.

(iv) The columns of B span the *column space* of A.

(v) None of the above.

A. Only (i)

B. Only (ii)

C. Only (iii)

D. Only (iv)

E. Only (v)

12. Consider P_3 with the inner product defined by $(p, q) = \int_0^1 p(t)q(t)\, dt$.
Which of the following polynomials is orthogonal to t ?

A. $1 + t^3$

B. t^2

C. $4t^2 - 2$

D. $4t^2 + t^3$

E. None of the above.

13. Let W be a subspace of $\mathbb{R}_3$ with a basis $\{[0\ 1\ 1], [-1\ 1\ 0]\}$, and let $v = [2\ 0\ -4]$. Find the vector w in W *closest* to v.

A. $w = [1\quad 3\ -2]$
B. $w = [0\ -2\ -2]$
C. $w = [0\quad 1\ -1]$
D. $w = [1\quad 0\ -2]$
E. $w = [1\ -3\ -2]$

14. Which of the following transformations is **<u>not</u>** a linear transformation ?

A. $L : \mathbb{R}_2 \to \mathbb{R}_3$ defined by $L([u_1\ u_2]) = [\{u_1 + u_1\}\ u_2\ \{u_1 + u_2\}]$.
B. $L : \mathbb{R}_3 \to \mathbb{R}_2$ defined by $L([u_1\ u_2\ u_3]) = [u_3\ \{u_1 + u_2\}]$.
C. $L : P_3 \to P_2$ defined by $L(f(t)) = f'(t)$, where P_m denotes the space of polynomials with degree $\leq m$.
D. $L : M_{nn} \to \mathbb{R}$ defined by $L(A) = \det(A)$, where M_{nn} denotes the space of $n \times n$ matrices.
E. $L : M_{22} \to \mathbb{R}$ defined by $L(A) = A^T$, where M_{22} denotes the space of 2×2 matrices.

15. Let $V = \mathbb{R}^n$ and L be the linear transformation on V that sends the column vector

$$\begin{bmatrix} x_1 \\ . \\ . \\ x_{n-1} \\ x_n \end{bmatrix} \quad \text{to} \quad \begin{bmatrix} x_1 \\ . \\ . \\ x_{n-1} \\ 0 \end{bmatrix}$$

Which one of the following statements is TRUE ?

A. 1 is the only eigenvalue of L.

B. 0 is the only eigenvalue of L.

C. L has no real eigenvalue.

D. Both 0 and 1 are eigenvalues of L.

E. L is not a linear transformation.

16. Which one of the following statements is TRUE ?

A. The only square matrices which are diagonalizable are those with distinct eigenvalues.

B. For a square matrix to be diagonalizable it must be non-singular.

C. If a square matrix has distinct eigenvalues then it is diagonalizable.

D. Only scalar multiples of the identity matrix are diagonalizable.

E. Every square matrix is diagonalizable.

17. Let $W = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$. How many different matrices are there in the following list ?

$$\{W,\ W^5,\ -W,\ W^T,\ \overline{W},\ -W^{-1}\}$$

A. 2

B. 3

C. 4

D. 5

E. 6

18. The eigenvalues of the matrix $\begin{bmatrix} 3 & 0 & -1 \\ 0 & 1 & 0 \\ 4 & 0 & 3 \end{bmatrix}$ are

A. $1,\ 3+\sqrt{22},\ 3-\sqrt{22}$

B. $1,\ 3,\ 3$

C. $i,\ -i,\ 0$

D. $1,\ 3+i\sqrt{22},\ 3-i\sqrt{22}$

E. $1,\ 3+2i,\ 3-2i$

19. Let us consider the following three equations in two variables,

$$\begin{bmatrix} 1 & 0 \\ 2 & 3 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix}$$

Then the least square solution for x_1 is

A. $x_1 = 0$

B. $x_1 = 1$

C. $x_1 = 2$

D. $x_1 = 4$

E. $x_1 = 5$

20. Let $x_1(t)$, $x_2(t)$ be solutions of the system of linear differential equations

$$\begin{bmatrix} x_1'(t) \\ x_2'(t) \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

with the initial conditions $x_1(0) = e^2$ and $x_2(0) = e$. Then $x_1(1) + x_2(1)$ equals

A. $e^2 + e + 2$

B. $2e^3 + 3e^2$

C. $e^4 + e^6$

D. $2e^4$

E. $e^2 + e^3 + e^4$