Computer Project 1. Nonlinear Springs

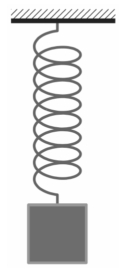
Goal: Investigate the behavior of nonlinear springs.

Tools needed: `ode45`, `plot`

Description: For certain (nonlinear) spring-mass systems, the spring force is not given by Hooke's Law but instead satisfies

$$F_{\text{spring}} = ku + \epsilon u^3,$$

where $k > 0$ is the spring constant and ϵ is small but may be positive or negative and represents the "strength" of the spring ($\epsilon = 0$ gives Hooke's Law). The spring is called a *hard spring* if $\epsilon > 0$ and a *soft spring* if $\epsilon < 0$.



Questions: Suppose a nonlinear spring-mass system satisfies the initial value problem

$$\begin{cases} u'' + u + \epsilon u^3 = 0 \\ u(0) = 0, \quad u'(0) = 1 \end{cases}$$

Use `ode45` and `plot` to answer the following:

1. Let $\epsilon = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0$ and plot the solutions of the above initial value problem for $0 \leq t \leq 20$. Estimate the amplitude of the spring. Experiment with various $\epsilon > 0$. What appears to happen to the amplitude as $\epsilon \to \infty$? Let μ^+ denote the first time the mass reaches equilibrium after $t = 0$. Estimate μ^+ when $\epsilon = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0$. What appears to happen to μ^+ as $\epsilon \to \infty$?

2. Let $\epsilon = -0.1, -0.2, -0.3, -0.4$ and plot the solutions of the above initial value problem for $0 \leq t \leq 20$. Estimate the amplitude of the spring. Experiment with various $\epsilon < 0$. What appears to happen to the amplitude as $\epsilon \to -\infty$? Let μ^- denote the first time the mass reaches equilibrium after $t = 0$. Estimate μ^- when $\epsilon = -0.1, -0.2, -0.3, -0.4$. What appears to happen to μ^- as $\epsilon \to -\infty$?

3. Given that a certain nonlinear hard spring satisfies the initial value problem

$$\begin{cases} u'' + \frac{1}{5}\,u' + \left(u + \frac{1}{5}\,u^3\right) = \cos \omega t \\ u(0) = 0, \quad u'(0) = 0 \end{cases}$$

plot the solution $u(t)$ over the interval $0 \leq t \leq 60$ for $\omega = 0.5, 0.7, 1.0, 1.3, 2.0$. Continue to experiment with various values of ω, where $0.5 \leq \omega \leq 2.0$, to find a value ω^* for which $|u(t)|$ is largest over the interval $40 \leq t \leq 60$.